

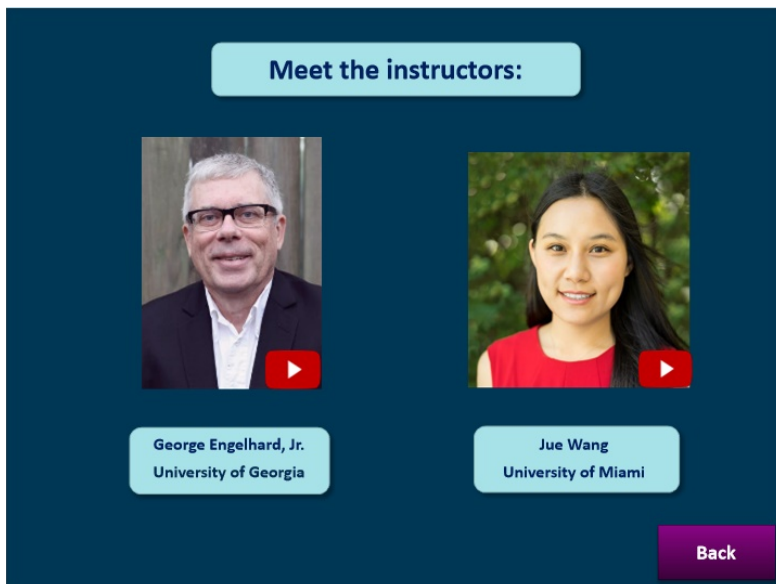
DM10 SLIDES (Rasch Measurement Theory, Version 2.1)

1. Module Overview

1.1 Module Cover (START)





1.2 Instructors





1.3 Designers

Meet the designers:




Jonathan Lehrfeld
ETS



André A. Rupp
Mindful
Measurement

Back

1.4 Welcome



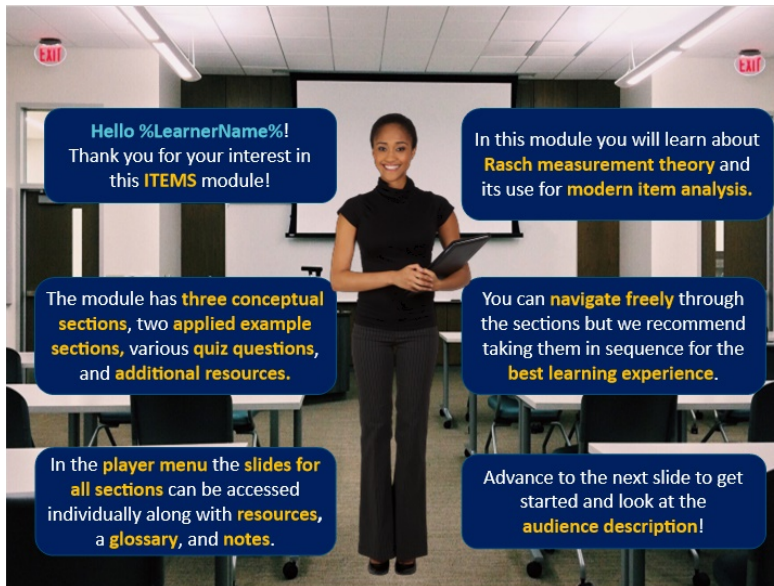
**Welcome to the
ITEMS Module!**

The woman to the left is Laura!

Along with the instructors,
she will be guiding you
through the module content.

Tell us your name here:

1.5 Overview




1.6 Target Audience

Target Audience

Anyone who would like a gentle statistical introduction to this topic:

- graduate students and faculty in Master's, Ph.D., or certificate programs
- psychometricians and other measurement professionals
- data scientists / analysts
- research assistants or research scientists
- technical project directors
- assessment developers



However, we hope that you find the information in this module useful no matter what your official title or role in an organization is!

1.7 Expectations (I)



1.8 Expectations (II)



1.9 Learning Objectives

Learning Objectives



1. Describe key components of Rasch measurement theory
2. Understand invariant measurement and Rasch's philosophy of measurement
3. Apply Rasch measurement theory to scale construction
4. Perform Rasch model analysis using computer programs
5. Evaluate the fit of a Rasch scale and interpret Rasch location measures
6. Make suggestions and implications to scale development in operational practice

1.10 Prerequisites

Prerequisites

Working knowledge of foundational assessment and statistical concepts such as:

- ✓ Basic descriptive statistics (e.g., mean, standard deviation)
- ✓ Basic mathematical functions (e.g., logarithms)
- ✓ Statistical definition of probability
- ✓ Guttman and Thurstone scaling
- ✓ Other measurement theories:
 - classical test theory (CTT)
 - generalizability theory (g-theory)
 - item response theory (IRT)

1.11 Resources

Resources

Module Citation

Wang, J., & Engelhard Jr., G., (2019). Rasch measurement theory (Digital ITEMS Module 10). *Educational Measurement: Issues and Practice*, 38(4), 112-113.





Additional Resources

Resources 2 (Slide Layer)

Resources: Books

Engelhard & Wind (2018). *Invariant Measurement with Raters and Rating Scales: Rasch Models for Rater-Mediated Assessments*.






Wilson (2005). *Constructing Measures: An Item Response Modeling Approach*.

More Books

Back

Resources 1 (Slide Layer)



Resources: Books



Andrich & Marais (2019). *A Course in Rasch Measurement Theory*.



Bond & Fox (2015). *Applying the Rasch Model: Fundamental Measurement in the Human Sciences*.

Engelhard (2013). *Invariant Measurement: Using Rasch Models in the Social, Behavioral, and Health Sciences*.



[More Books](#)[Back](#)

1.12 Main Menu

**Main Menu**

Theory

01 Conceptual Foundations
[20 Minutes]

02 Rasch Measurement Theory
[30 Minutes]

03 Creating a Rasch Scale
[25 Minutes]

04 Winsteps and Shiny ERMA
[20 Minutes]

05 Data Activities
[30 Minutes]

06 Quizzes
[30 Minutes]

Practice

2. Section 1: Conceptual Foundations

2.1 Cover: Section 1

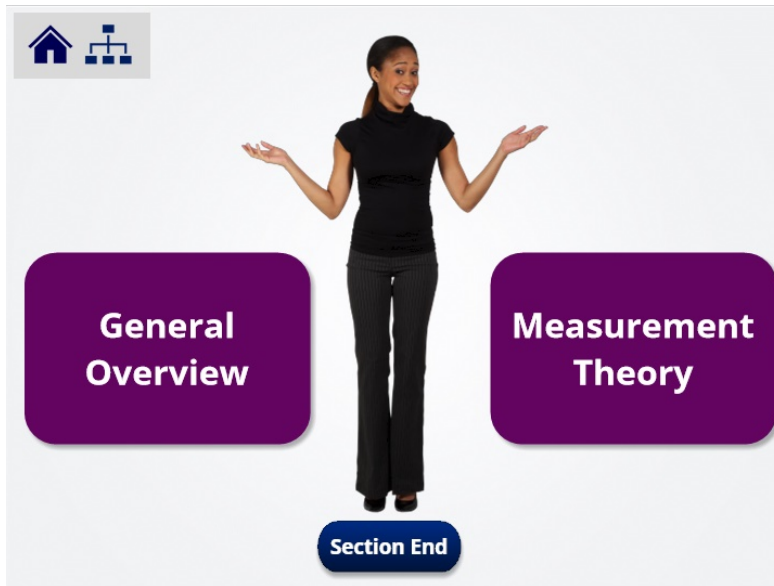


2.2 Objectives: Section 1

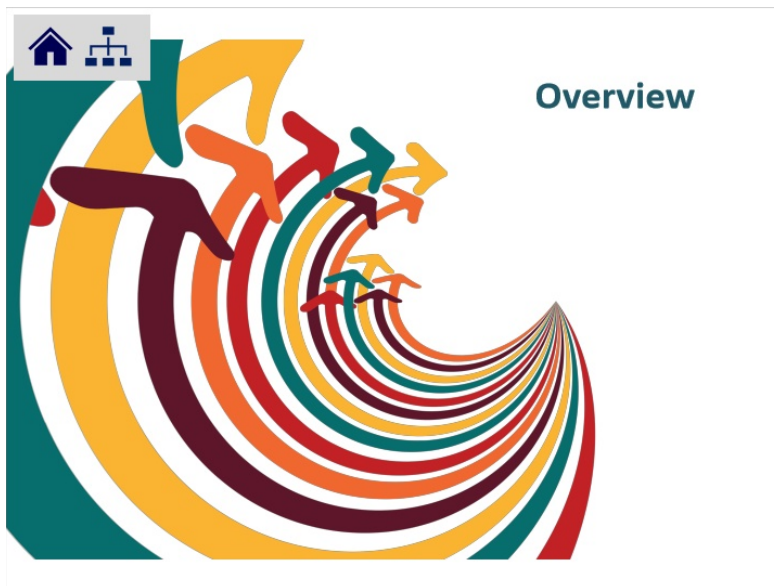
Learning Objectives

1. Describe key perspectives on invariant measurement
2. Describe key differences of invariant measurement in the physical and social sciences
3. Describe Rasch's concept of specific objectivity in measurement
4. Provide a broad definition of measurement theory



2.3 Topic Selection



2.4 Bookmark: Overview



2.5 Perspectives on Invariant Measurement





General Overview

Invariant Measurement



Objectivity

Network of Measures



Click on the buttons to see quotes from seminal researchers 

Invariant Measurement (Slide Layer)



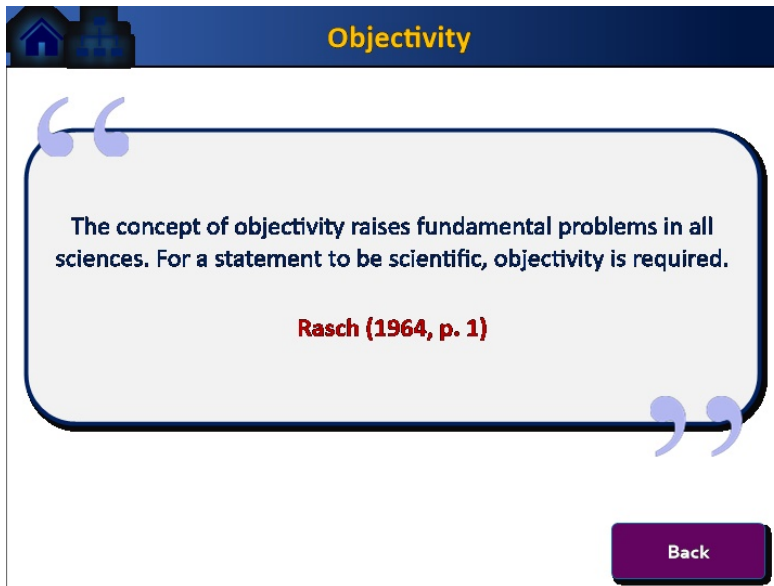
Invariant Measurement

The scientist is usually looking for invariance whether he knows it or not... The scientist seeks measures that will stay put while his back is turned.

Stevens (1951, pp. 20-21)

Back

Objectivity (Slide Layer)



The slide features a dark blue header with a home icon and the title "Objectivity" in yellow. The main content is a light gray rounded rectangle with a dark blue border, containing a quote in dark blue text. The quote is attributed to "Rasch (1964, p. 1)" in red text. A purple "Back" button is located at the bottom right.

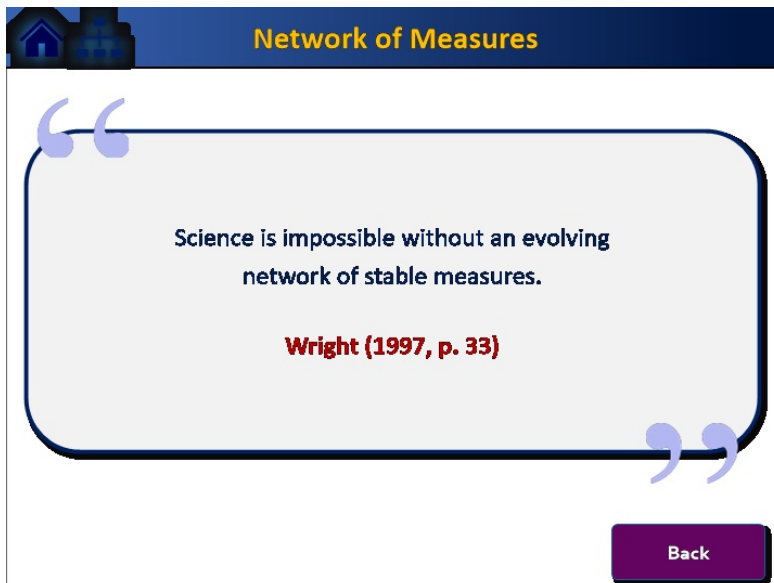
Objectivity

The concept of objectivity raises fundamental problems in all sciences. For a statement to be scientific, objectivity is required.

Rasch (1964, p. 1)

Back

Network of Measures (Slide Layer)



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Network of Measures

Science is impossible without an evolving network of stable measures.

Wright (1997, p. 33)


Back

2.6 Invariant Measurement in Physical Sciences




Invariant Measurement: Physical Sciences


Measurement of length




Measurement of weight





Measurement of time



Measurement of temperature









2.7 Invariant Measurement in Social Sciences 1



Invariant Measurement: Social Sciences

Wong-Baker FACES® Pain Rating Scale

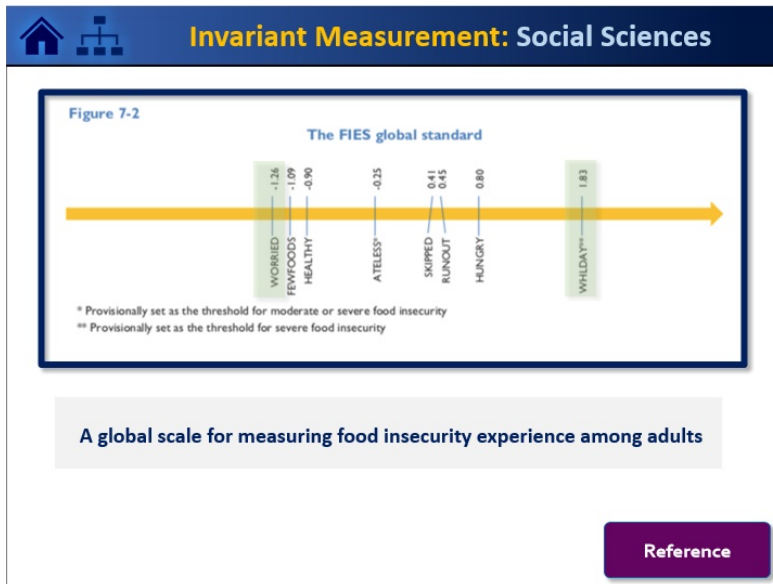
					
0	2	4	6	8	10
No Hurt	Hurts Little Bit	Hurts Little More	Hurts Even More	Hurts Whole Lot	Hurts Worst

- developed by Donna Wong and Connie Baker
- originally created for children, but now used with patients age 3 and above across the world



(Source: <https://wongbakerfaces.org/>)

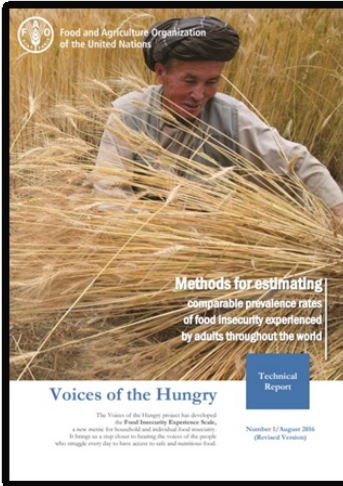
WONG-BAKER
FACES
FOUNDATION

2.8 Invariant Measurement in Social Sciences 2



References (Slide Layer)



  **Reference**



Food and Agriculture Organization of the United Nations (UN-FAO). (2016). *Voices of the hungry: Methods for estimating comparable rates of food insecurity experienced by adults throughout the world*. Rome, FAO.

Back

2.9 Measurement in Physical vs. Social Sciences




Physical vs. Social Sciences



The measurement of psychological constructs can be more difficult than the measurement of physical objects since constructs cannot be directly observed

Two difficulties for scales:

- Different instruments designed to measure the same construct may provide different measures for the same person
- The same instrument may work in different ways for different persons



2.10 Specific Objectivity of Measurement 1



Invariant Measurement

Thurstone → Guttman → Rasch

Similarities	Differences
Proposed the concept of invariance (specific objective) of comparison.	Thurstone focused on groups, while Rasch developed individual-centered statistics.
Emphasized the demand for unidimensionality for achieving invariance.	Guttman's scale is deterministic, while Rasch formalized a probabilistic model with both items and persons parameterized.

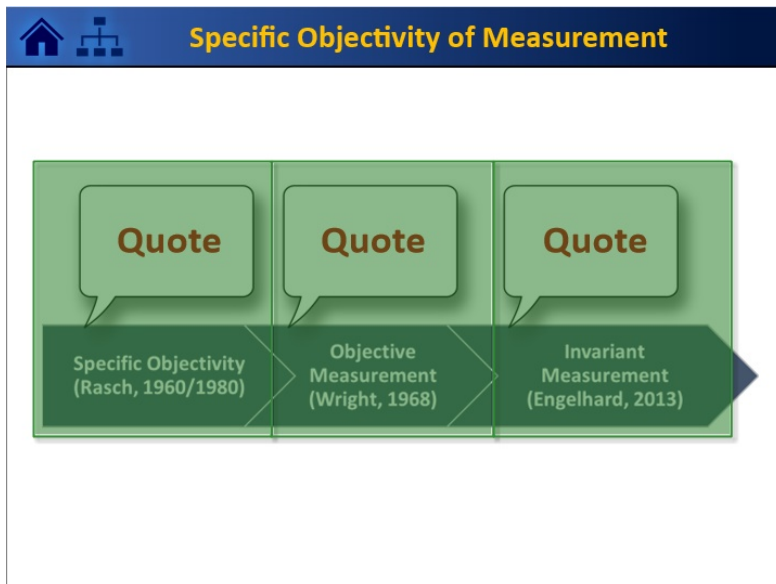
[References](#)

References (Slide Layer)

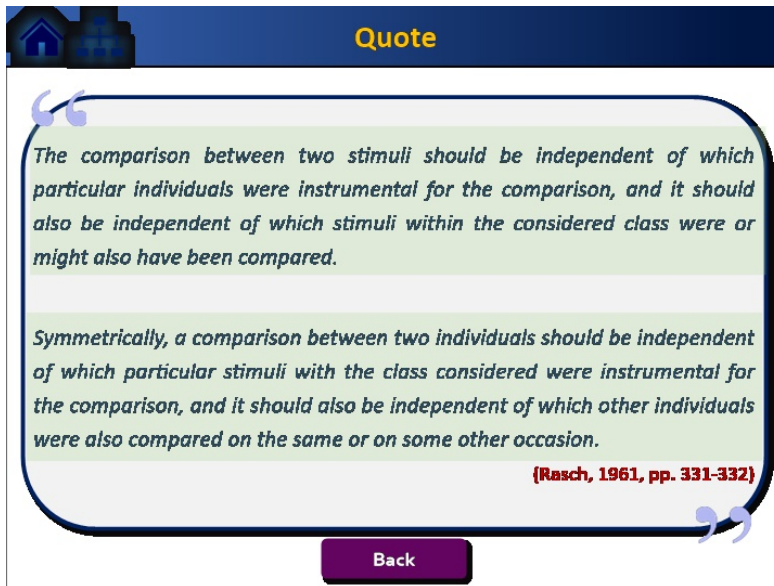
**References**

Back

2.11 Specific Objectivity of Measurement 2



Rasch Quote (Slide Layer)



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Quote

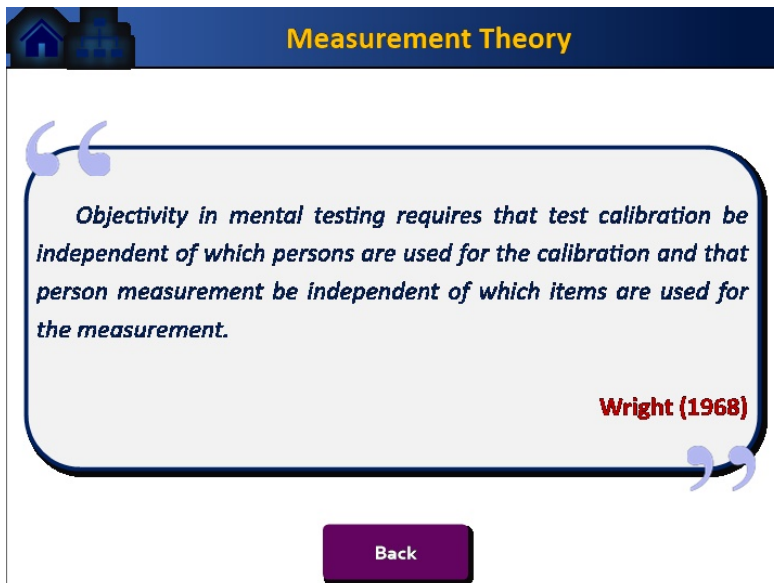
The comparison between two stimuli should be independent of which particular individuals were instrumental for the comparison, and it should also be independent of which stimuli within the considered class were or might also have been compared.

Symmetrically, a comparison between two individuals should be independent of which particular stimuli with the class considered were instrumental for the comparison, and it should also be independent of which other individuals were also compared on the same or on some other occasion.

(Rasch, 1961, pp. 331-332)

Back

Wright Quote (Slide Layer)



The slide features a dark blue header with a home icon and the title "Measurement Theory" in yellow. The main content is enclosed in a light blue rounded rectangle with a dark blue border and purple quotation marks. It contains one paragraph of italicized text and a citation. A purple "Back" button is at the bottom.

Measurement Theory

Objectivity in mental testing requires that test calibration be independent of which persons are used for the calibration and that person measurement be independent of which items are used for the measurement.

Wright (1968)

Back

Engelhard Quote (Slide Layer)

Five Requirements for Invariant Measurement

Person measurement:

1. The measurement of persons must be independent of the particular items that happen to be used for the measuring: item-invariant measurement of persons.
2. A more able person must always have a better chance of success on an item than a less able person: non-crossing person response functions.

Item calibration:

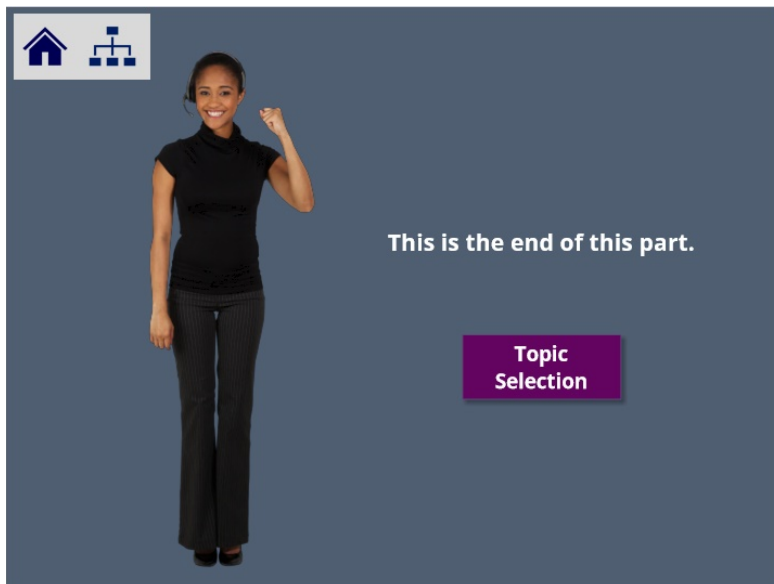
3. The calibration of the items must be independent of the particular persons used for calibration: person-invariant calibration of test items.
4. Any person must have a better chance of success on an easy item than on a more difficult item: non-crossing item response functions.

Variable map:

5. Items and persons must be simultaneously located on a single underlying latent variable: variable map.

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2.12 Bookend: Overview



2.13 Bookmark: Measurement Theory



2.14 Measurement Theory

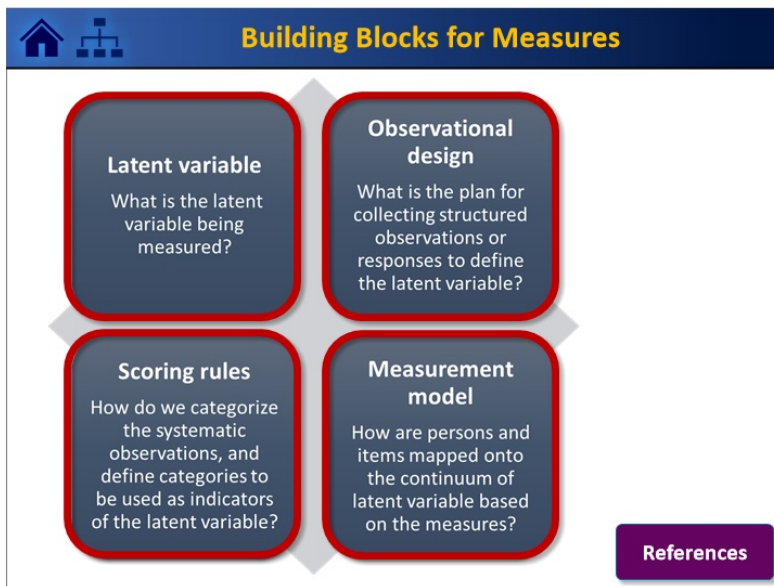


Measurement Theory

*Theories of measurement broadly conceived may be viewed as loosely integrated **conceptual frameworks** within which are embedded rigorously formulated **statistical models** of estimation and inference about the **properties of measurements and scores**.*

Messick (1983, p. 498)

2.15 Building Blocks for Researcher-Constructed Measures





References (Slide Layer)



2.16 Measurement/Statistical Models

Measurement Traditions	
Test-Score Tradition	Scaling Tradition
Key models: <ol style="list-style-type: none">1. Classical test theory (CTT)2. Generalizability theory (GT)3. Factor analysis (FA)	Key models: <ol style="list-style-type: none">1. Psychophysical models (PM)2. Absolute Scaling (AS)3. Item Response Theory (IRT)
Essential features: <p>Test-score focus Linear models Focus on test scores and the estimation of error components</p>	Essential features: <p>Item-person response focus Non-linear models Focus on modeling the responses of persons to items</p>

2.17 Properties of Measurements and Scores

 **Properties of Measurements and Scores**



The meanings and interpretations of scores are essential

- Determine impact of research in the social, behavioral, and health sciences
- Frame the substantive conclusions and inferences that we draw
- Delineate and limit policies and practices derived from research work

Validity is not a property of the test or assessment as such, but rather of the meaning of the test scores. These scores are a function not only of the items or stimulus conditions, but also of the persons responding as well as the context of the assessment...

Messick (1995, p. 741)

2.18 Scores on Educational Assessments



Scores on Educational Assessments

The key purpose of educational assessments is to make decisions on the basis of scores



- Levels (international, national, state, school, teacher, or student)
- Operational definitions (of the outcomes that we value as a society)

Classroom assessments focus on individual students

- Before instruction: readiness, placement
- During instruction: formative, diagnostic
- After instruction: summative, strategic

Quote

Quote (Slide Layer)



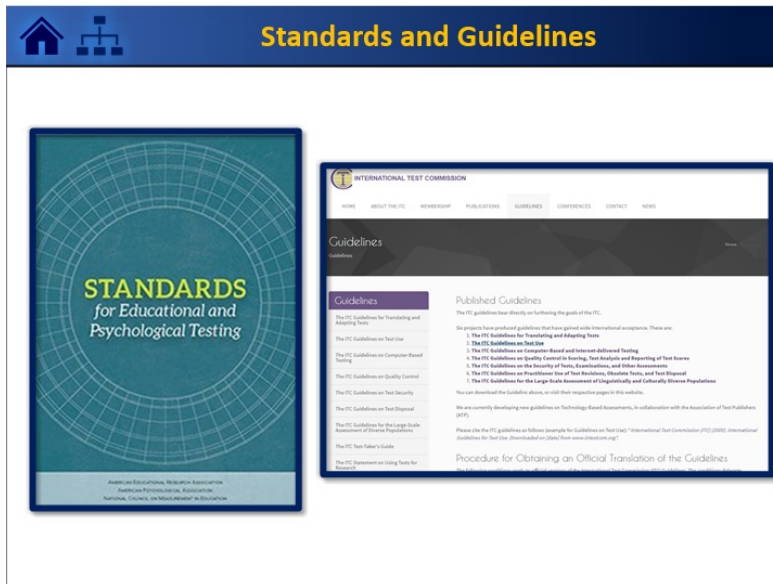
Quote

... the term score is used generically in its broadest sense to mean any coding or summarization of observed consistencies or performance regularities on a test, questionnaire, observation procedure, or other assessment devices such as work samples, portfolios, and realistic problem simulations.

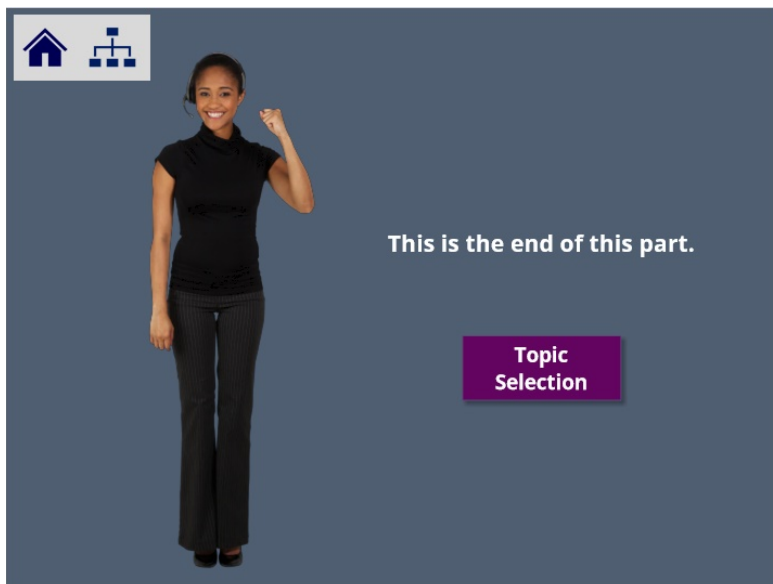
Messick (1995, p. 741)

Back



2.19 Standards & Guidelines



2.20 Bookend: Measurement Theory



2.21 Summary I





Summary (I)

- Invariant measurement is crucial for **developing measures** for the social sciences, but it is hard to achieve since **constructs are latent**
- Rasch's concept of **specific objectivity** emphasizes the **invariance of comparisons** between stimuli and individuals
- His **theory of measurement** includes the formulation of a **measurement model** based on a conceptual framework
- The scores from this model can be used to make **inferences and predictions** about human characteristics and behaviors

Quote

Quote (Slide Layer)





Quote

Looking then for concepts [of measurement] that could possibly be taken as primary it seems worth-while to concentrate upon two essential characteristics of scientific statements: 1. they are concerned with comparisons; 2. the statements are claimed to be objective; both terms of course calling for precise qualifications.

Rasch (1964, p. 2)

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2.22 Summary II



Summary (II)



Item-invariant measurement of persons:


- Measurement of persons shall be free of test forms being used
- Relevant measurement problems include test equating, item parameter linking, and computerized adaptive testing

Person-invariant calibration of items:

- Calibration of items shall be independent of specific samples
- Relevant measurement problem include differential item functioning and measurement invariance

2.23 Bookend: Section 1





This is the end of this section.

[Quiz](#) [Main Menu](#)

3. Section 2: Rasch Measurement Theory

3.1 Cover: Section 2





3.2 Objectives: Section 2


Learning Objectives

1. Explain Rasch's motivation for his measurement theory
2. Describe the fundamental principles of Rasch measurement theory
3. List the Rasch-based fit indices and explain their underlying structure
4. List commonly used Rasch models and explain their structure

3.3 Rasch's Motivation





Rasch's Motivation



Georg Rasch, 1901-1980

Quote

Quote (Slide Layer)



Rasch's Motivation

“Lately the statistical methods of psychometrics have been severely criticized in psychological quarters. Thus Skinner maintains that if order is to be found in human and animal behavior, then it should be extracted from investigations into individuals, and that psychometric methods are inadequate for such purposes since they deal with groups of individuals. And as regards abnormal psychology Zubin states: "Recourse must be had to individual statistics, treating each patient as a separate universe. Unfortunately, present day statistical methods are entirely group-centered, so that there is a real need for developing individual-centered statistics.”

Rasch, G. (1961). On general laws and meaning of measurement in psychology. In J. Neyman (Ed.), *Proceedings of the fourth Berkeley Symposium on mathematical statistics and probability* (pp 321-333). Berkeley: University of California Press.

”

Back

3.4 Rasch's Example I

Rasch's Example: Setup							
	Item ID						
Person ID	1	2	3	4	5	6	# Correct (Person) / Raw Score
1	+	+	o	+	o	+	4
2	o	o	o	+	o	o	1
3	+	+	+	+	+	+	6
4	o	+	o	+	o	o	2
5	+	+	+	+	+	+	6
6	+	+	o	+	o	+	4
7	+	+	o	+	+	+	5
8	o	o	o	o	o	o	0
9	+	+	o	+	+	+	5
10	o	+	o	+	o	o	2
# Correct (Item) / Difficulty	6	8	2	9	4	6	

A fictitious example of **10 persons** solving **6 items** (+: correct; o: incorrect)

3.5 Rasch's Example II

Rasch's Example: Item Ordering							
	Item ID						
Person ID	4	2	1	6	5	3	# Correct (Person) / Raw Score
1	+	+	+	+	o	o	4
2	+	o	o	o	o	o	1
3	+	+	+	+	+	+	6
4	+	+	o	o	o	o	2
5	+	+	+	+	+	+	6
6	+	+	+	+	o	o	4
7	+	+	+	+	+	o	5
8	o	o	o	o	o	o	0
9	+	+	+	+	+	o	5
10	+	+	o	o	o	o	2
# Correct (Item) / Difficulty	9	8	6	6	4	2	

Order items according to difficulty by the **number of correct answers** to each item

3.6 Rasch's Example III


Rasch's Example: Person Ordering							
	Item ID						
Person ID	4	2	1	6	5	3	# Correct (Person) / Raw Score
3	+	+	+	+	+	+	6
5	+	+	+	+	+	+	6
7	+	+	+	+	+	0	5
9	+	+	+	+	+	0	5
1	+	+	+	+	0	0	4
6	+	+	+	+	0	0	4
4	+	+	0	0	0	0	2
10	+	0	0	0	0	0	2
2	+	0	0	0	0	0	1
8	0	0	0	0	0	0	0
# Correct (Item) / Difficulty	9	8	6	6	4	2	

Order persons by their "ability" to solve each single item by using their raw scores

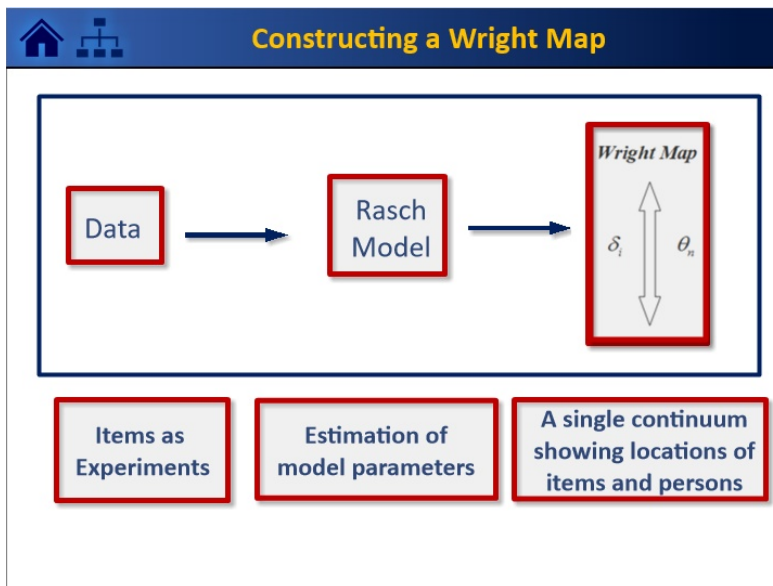
3.7 Guttman Scale

Deterministic Guttman Scales

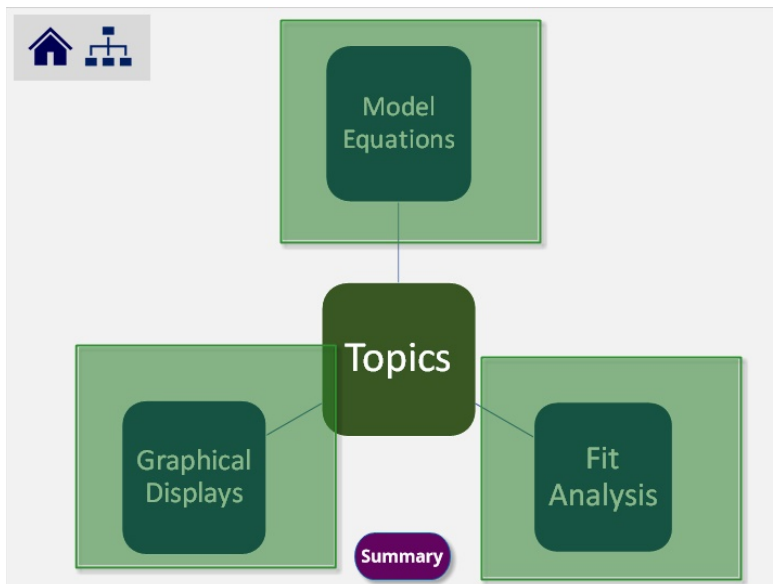
- Designed to create a **single continuum** for the **joint ordering** of persons and items
- Response patterns are **completely determined**
- Sets the foundation for item analysis that is closely related to **probabilistic scaling**
- We want to produce a probabilistic **Guttman response pattern**
- The reproduced **Rasch-based probabilities** can be used to calculate **residuals** and **fit indices**



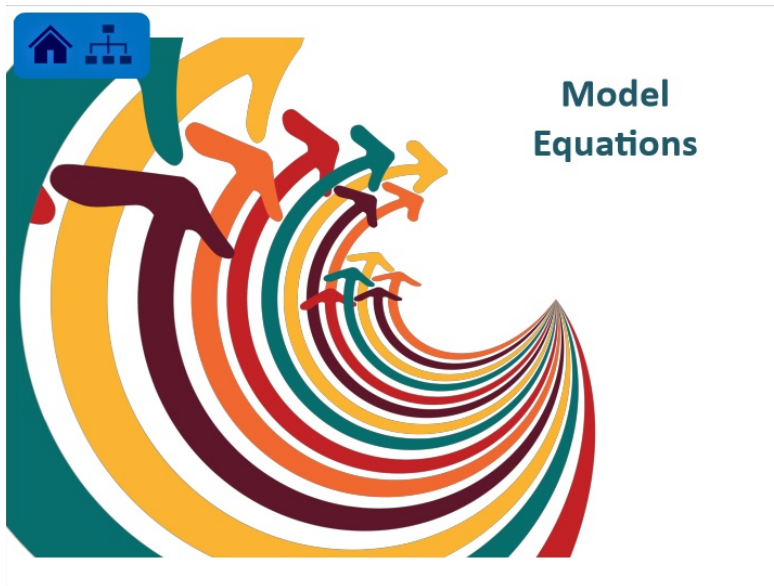
3.8 Constructing a Wright Map





3.9 Topic Selection



3.10 Bookmark: Equations




3.11 Original Form of Rasch Model





Original Form of Rasch Model

- Probability of solving an item is determined by the degree of **ability of the person** (ξ_n) and the degree of **difficulty of the item** (δ_i)
- Probability is a **joint function** of ξ_n/δ_i instead of ξ_n and δ_i separately
- The simplest **probability function** is:

$$P_{ni1} = \frac{\frac{\xi_n}{\delta_i}}{1 + \frac{\xi_n}{\delta_i}} = \frac{\xi_n}{\xi_n + \delta_i}$$

Hover over parameters to see explanation 

ksi (Slide Layer)




Original Form of Rasch Model



- Probability of solving an item is determined by the degree of **ability of the person** (ξ_n) and the degree of **difficulty of the item** (δ_i)
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$$P_{ni1} = \frac{\frac{\xi_n}{\delta_i}}{1 + \frac{\xi_n}{\delta_i}} = \frac{\xi_n}{\xi_n + \delta_i}$$

Ability of person n

Hover over parameters to see explanation 

delta (Slide Layer)




Original Form of Rasch Model



- Probability of solving an item is determined by the degree of **ability of the person** (ξ_n) and the degree of **difficulty of the item** (δ_i)
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Difficulty of item i

Hover over parameters to see explanation 

Pni1 (Slide Layer)




Original Form of Rasch Model



- Probability of solving an item is determined by the degree of **ability of the person** (ξ_n) and the degree of **difficulty of the item** (δ_i)
- Probability is a **joint function** of ξ_n/δ_i instead of ξ_n and δ_i separately
- The simplest **probability function** is:

$$P_{ni1} = \frac{\frac{\xi_n}{\delta_i}}{1 + \frac{\xi_n}{\delta_i}} = \frac{\xi_n}{\xi_n + \delta_i}$$

Probability of person n answering item i correctly

Parameters explanation 


3.12 Original Form of Rasch Model





Sample Probabilities

- A **very able** person solving a **very easy** item $\rightarrow 1$
- A **least able** person solving a **very difficult** item $\rightarrow 0$
- An **average ability** person solving an **average difficulty** item ($\xi_n = \delta_i$) = **.50**

$$P_{ni1} = \frac{\frac{\xi_n}{\delta_i}}{1 + \frac{\xi_n}{\delta_i}} = \frac{\xi_n}{\xi_n + \delta_i}$$

Hover over parameters to see explanation 

ksi (Slide Layer)




Sample Probabilities



- A **very able** person solving a **very easy** item $\rightarrow 1$
- A **least able** person solving a **very difficult** item $\rightarrow 0$
- An **average ability** person solving an **average difficulty** item ($\xi_n = \delta_i$) = .50

$$P_{ni1} = \frac{\frac{\xi_n}{\delta_i}}{1 + \frac{\xi_n}{\delta_i}} = \frac{\xi_n}{\xi_n + \delta_i}$$

Ability of person n

Hover over parameters to see explanation 

delta (Slide Layer)




Sample Probabilities

- A **very able** person solving a **very easy** item $\rightarrow 1$
- A **least able** person solving a **very difficult** item $\rightarrow 0$
- An **average ability** person solving an **average difficulty** item ($\xi_n = \delta_i$) = .50

$$P_{ni1} = \frac{\frac{\xi_n}{\delta_i}}{1 + \frac{\xi_n}{\delta_i}} = \frac{\xi_n}{\xi_n + \delta_i}$$

Difficulty of item i

Hover over parameters to see explanation 

Pni1 (Slide Layer)

Sample Probabilities

- A **very able** person solving a **very easy** item $\rightarrow 1$
- A **least able** person solving a **very difficult** item $\rightarrow 0$
- An **average ability** person solving an **average difficulty** item ($\xi_n = \delta_i$) = .50

Probability of person n answering item i correctly
Parameters explanation

3.13 Dichotomous Rasch Model I

Rasch Model (I)

Rasch's Original Formulas

- Correct response: $P_{ni1} = \frac{\xi_n}{\xi_n + \delta_i}$
- Incorrect response: $P_{ni0} = 1 - \frac{\xi_n}{\xi_n + \delta_i} = \frac{\delta_i}{\xi_n + \delta_i}$
- Odds ratio: $\frac{P_{ni1}}{P_{ni0}} = \frac{\xi_n}{\delta_i}$
- Log-odds / Logits: $\ln \left[\frac{P_{ni1}}{P_{ni0}} \right] = \ln \left[\frac{\xi_n}{\delta_i} \right] = \ln(\xi_n) - \ln(\delta_i)$

Modern Formula

- Log-odds / Logits: $\ln \left[\frac{P_{ni1}}{P_{ni0}} \right] = \theta_n - \delta_i$

Hover over parameters to see explanation

ksi-1 (Slide Layer)

Rasch Model (I)

Rasch's Original Formulas

- Correct response: $P_{ni1} = \frac{\xi_n}{\xi_n + \delta_i}$ Ability of person n
- Incorrect response: $P_{ni0} = 1 - \frac{\xi_n}{\xi_n + \delta_i} = \frac{\delta_i}{\xi_n + \delta_i}$
- Odds ratio: $\frac{P_{ni1}}{P_{ni0}} = \frac{\xi_n}{\delta_i}$
- Log-odds / Logits: $\ln \left[\frac{P_{ni1}}{P_{ni0}} \right] = \ln \left[\frac{\xi_n}{\delta_i} \right] = \ln(\xi_n) - \ln(\delta_i)$

Modern Formula

- Log-odds / Logits: $\ln \left[\frac{P_{ni1}}{P_{ni0}} \right] = \theta_n - \delta_i$ Hover over parameters to see explanation

delta-1 (Slide Layer)

Rasch Model (I)

Rasch's Original Formulas

- Correct response: $P_{ni1} = \frac{\xi_n}{\xi_n + \delta_i}$ Difficulty of item i
- Incorrect response: $P_{ni0} = 1 - \frac{\xi_n}{\xi_n + \delta_i} = \frac{\delta_i}{\xi_n + \delta_i}$
- Odds ratio: $\frac{P_{ni1}}{P_{ni0}} = \frac{\xi_n}{\delta_i}$
- Log-odds / Logits: $\ln \left[\frac{P_{ni1}}{P_{ni0}} \right] = \ln \left[\frac{\xi_n}{\delta_i} \right] = \ln(\xi_n) - \ln(\delta_i)$

Modern Formula

- Log-odds / Logits: $\ln \left[\frac{P_{ni1}}{P_{ni0}} \right] = \theta_n - \delta_i$ Hover over parameters to see explanation

Pni1-1 (Slide Layer)

Rasch Model (I)

Rasch's Original Formulas

- Correct response: $P_{ni1} = \frac{\xi_n}{\xi_n + \delta_i}$
- Incorrect response: $P_{ni0} = 1 - \frac{\xi_n}{\xi_n + \delta_i} = \frac{\delta_i}{\xi_n + \delta_i}$
- Odds ratio: $\frac{P_{ni1}}{P_{ni0}} = \frac{\xi_n}{\delta_i}$
- Log-odds / Logits: $\ln \left[\frac{P_{ni1}}{P_{ni0}} \right] = \ln \left[\frac{\xi_n}{\delta_i} \right] = \ln(\xi_n) - \ln(\delta_i)$

Modern Formula

- Log-odds / Logits: $\ln \left[\frac{P_{ni1}}{P_{ni0}} \right] = \theta_n - \delta_i$

Probability of person n answering item i correctly

Hover over parameters to see explanation

delta-2 (Slide Layer)

Rasch Model (I)

Rasch's Original Formulas

- Correct response: $P_{ni1} = \frac{\xi_n}{\xi_n + \delta_i}$
- Incorrect response: $P_{ni0} = 1 - \frac{\xi_n}{\xi_n + \delta_i} = \frac{\delta_i}{\xi_n + \delta_i}$
- Odds ratio: $\frac{P_{ni1}}{P_{ni0}} = \frac{\xi_n}{\delta_i}$
- Log-odds / Logits: $\ln \left[\frac{P_{ni1}}{P_{ni0}} \right] = \ln \left[\frac{\xi_n}{\delta_i} \right] = \ln(\xi_n) - \ln(\delta_i)$

Modern Formula

- Log-odds / Logits: $\ln \left[\frac{P_{ni1}}{P_{ni0}} \right] = \theta_n - \delta_i$

Difficulty of item i

Hover over parameters to see explanation

ksi-2 (Slide Layer)

Rasch Model (I)

Rasch's Original Formulas

- Correct response: $P_{ni1} = \frac{\xi_n}{\xi_n + \delta_i}$
- Incorrect response: $P_{ni0} = 1 - \frac{\xi_n}{\xi_n + \delta_i} = \frac{\delta_i}{\xi_n + \delta_i}$ Ability of person n
- Odds ratio: $\frac{P_{ni1}}{P_{ni0}} = \frac{\xi_n}{\delta_i}$
- Log-odds / Logits: $\ln \left[\frac{P_{ni1}}{P_{ni0}} \right] = \ln \left[\frac{\xi_n}{\delta_i} \right] = \ln(\xi_n) - \ln(\delta_i)$

Modern Formula

- Log-odds / Logits: $\ln \left[\frac{P_{ni1}}{P_{ni0}} \right] = \theta_n - \delta_i$

Hover over parameters to see explanation

Pni0-2 (Slide Layer)

Rasch Model (I)

Rasch's Original Formulas

- Correct response: $P_{ni1} = \frac{\xi_n}{\xi_n + \delta_i}$
- Incorrect response: $P_{ni0} = 1 - \frac{\xi_n}{\xi_n + \delta_i} = \frac{\delta_i}{\xi_n + \delta_i}$ Probability of person n answering item i incorrectly
- Odds ratio: $\frac{P_{ni1}}{P_{ni0}} = \frac{\xi_n}{\delta_i}$
- Log-odds / Logits: $\ln \left[\frac{P_{ni1}}{P_{ni0}} \right] = \ln \left[\frac{\xi_n}{\delta_i} \right] = \ln(\xi_n) - \ln(\delta_i)$

Modern Formula

- Log-odds / Logits: $\ln \left[\frac{P_{ni1}}{P_{ni0}} \right] = \theta_n - \delta_i$

Hover over parameters to see explanation

odds-ratio-3 (Slide Layer)

Rasch Model (I)

Rasch's Original Formulas

- Correct response: $P_{ni1} = \frac{\xi_n}{\xi_n + \delta_i}$
- Incorrect response: $P_{ni0} = 1 - \frac{\xi_n}{\xi_n + \delta_i} = \frac{\delta_i}{\xi_n + \delta_i}$
- Odds ratio: $\frac{P_{ni1}}{P_{ni0}} = \frac{\xi_n}{\delta_i}$

Odds ratio of person n answering item i correctly
- Log-odds / Logits: $\ln \left[\frac{P_{ni1}}{P_{ni0}} \right] = \ln \left[\frac{\xi_n}{\delta_i} \right] = \ln(\xi_n) - \ln(\delta_i)$

Modern Formula

- Log-odds / Logits: $\ln \left[\frac{P_{ni1}}{P_{ni0}} \right] = \theta_n - \delta_i$

Hover over parameters to see explanation

log-odds-4 (Slide Layer)

Rasch Model (I)

Rasch's Original Formulas

- Correct response: $P_{ni1} = \frac{\xi_n}{\xi_n + \delta_i}$
- Incorrect response: $P_{ni0} = 1 - \frac{\xi_n}{\xi_n + \delta_i} = \frac{\delta_i}{\xi_n + \delta_i}$
- Odds ratio: $\frac{P_{ni1}}{P_{ni0}} = \frac{\xi_n}{\delta_i}$
- Log-odds / Logits: $\ln \left[\frac{P_{ni1}}{P_{ni0}} \right] = \ln \left[\frac{\xi_n}{\delta_i} \right] = \ln(\xi_n) - \ln(\delta_i)$

Modern Formula

- Log-odds / Logits: $\ln \left[\frac{P_{ni1}}{P_{ni0}} \right] = \theta_n - \delta_i$

Log-odds of correct response to item i

Hover over parameters to see explanation

log-odds-5 (Slide Layer)

Rasch Model (I)

Rasch's Original Formulas

- Correct response: $P_{ni1} = \frac{\xi_n}{\xi_n + \delta_i}$
- Incorrect response: $P_{ni0} = 1 - \frac{\xi_n}{\xi_n + \delta_i} = \frac{\delta_i}{\xi_n + \delta_i}$
- Odds ratio: $\frac{P_{ni1}}{P_{ni0}} = \frac{\xi_n}{\delta_i}$
- Log-odds / Logits: $\ln \left[\frac{P_{ni1}}{P_{ni0}} \right] = \ln \left[\frac{\xi_n}{\delta_i} \right] = \ln(\xi_n) - \ln(\delta_i)$

Modern Formula

- Log-odds / Logits: $\ln \left[\frac{P_{ni1}}{P_{ni0}} \right] = \theta_n - \delta_i$

Log-odds of correct response to item i

to see explanation

theta-5 (Slide Layer)

Rasch Model (I)

Rasch's Original Formulas

- Correct response: $P_{ni1} = \frac{\xi_n}{\xi_n + \delta_i}$
- Incorrect response: $P_{ni0} = 1 - \frac{\xi_n}{\xi_n + \delta_i} = \frac{\delta_i}{\xi_n + \delta_i}$
- Odds ratio: $\frac{P_{ni1}}{P_{ni0}} = \frac{\xi_n}{\delta_i}$
- Log-odds / Logits: $\ln \left[\frac{P_{ni1}}{P_{ni0}} \right] = \ln \left[\frac{\xi_n}{\delta_i} \right] = \ln(\xi_n) - \ln(\delta_i)$

Modern Formula

- Log-odds / Logits: $\ln \left[\frac{P_{ni1}}{P_{ni0}} \right] = \theta_n - \delta_i$

Ability of person n

to see explanation

delta-5 (Slide Layer)

Rasch Model (I)

Rasch's Original Formulas

- Correct response: $P_{ni1} = \frac{\xi_n}{\xi_n + \delta_i}$
- Incorrect response: $P_{ni0} = 1 - \frac{\xi_n}{\xi_n + \delta_i} = \frac{\delta_i}{\xi_n + \delta_i}$
- Odds ratio: $\frac{P_{ni1}}{P_{ni0}} = \frac{\xi_n}{\delta_i}$
- Log-odds / Logits: $\ln \left[\frac{P_{ni1}}{P_{ni0}} \right] = \ln \left[\frac{\xi_n}{\delta_i} \right] = \ln(\xi_n) - \ln(\delta_i)$

Modern Formula

- Log-odds / Logits: $\ln \left[\frac{P_{ni1}}{P_{ni0}} \right] = \theta_n - \delta_i$

Difficulty of item i

to see explanation

3.14 Dichotomous Rasch Model II

Rasch Model (II)

Probability Function:

$$P_{nix} = \frac{\exp[x_{ni}(\theta_n - \delta_i)]}{\sum_{x_n} \exp[x_{ni}(\theta_n - \delta_i)]}$$

Response Probability:



Correctly Answer

Incorrect Answer

Equal
Difficulty and Ability

Hover over parameters to see explanation

Solving a problem (Slide Layer)



Rasch Model (II)

Probability Function:

$$p_{nix} = \frac{\exp[x_{ni}(\theta_n - \delta_i)]}{\sum_{x_{ni}} \exp[x_{ni}(\theta_n - \delta_i)]}$$

Response Probability:


Correctly Answer

Incorrect Answer



Equal Difficulty and Ability

$$p_{ni1} = \frac{\exp(\theta_n - \delta_i)}{1 + \exp(\theta_n - \delta_i)}$$

Hover over parameters to see explanation



Not solving a problem (Slide Layer)



Rasch Model (II)

Probability Function:

$$p_{nix} = \frac{\exp[x_{ni}(\theta_n - \delta_i)]}{\sum_{x_{ni}} \exp[x_{ni}(\theta_n - \delta_i)]}$$

Response Probability:


Correctly Answer

Incorrect Answer



Equal Difficulty and Ability

$$p_{ni0} = \frac{1}{1 + \exp(\theta_n - \delta_i)}$$

Hover over parameters to see explanation



Equal amounts (Slide Layer)



Rasch Model (II)

Probability Function:

$$p_{nix} = \frac{\exp[x_{ni}(\theta_n - \delta_i)]}{\sum_{x_{ni}} \exp[x_{ni}(\theta_n - \delta_i)]}$$

Response Probability:


Correctly Answer

Incorrect Answer



Equal Difficulty and Ability

$$P_{ni1} = P_{ni0} = \frac{1}{2}$$

Hover over parameters to see explanation



pnix (Slide Layer)



Rasch Model (II)

Probability Function:

$$p_{nix} = \frac{\exp[x_{ni}(\theta_n - \delta_i)]}{\sum_{x_{ni}} \exp[x_{ni}(\theta_n - \delta_i)]}$$

Probability of person n answering item i with response x (1 = correct, 0 = incorrect)


Response Probability:

Correctly Answer



Incorrect Answer

Equal Difficulty and Ability

Hover over parameters to see explanation



xni (Slide Layer)



Rasch Model (II)

Probability Function:

$$p_{nix} = \frac{\exp(x_{ni}(\theta_n - \delta_i))}{\sum_{x_{ni}} \exp(x_{ni}(\theta_n - \delta_i))}$$

Scored response to item i
(1 = correct, 0 = incorrect)


Response Probability:

Correctly Answer



Incorrect Answer

Equal Difficulty and Ability

Hover over parameters to see explanation



theta (Slide Layer)



Rasch Model (II)

Probability Function:

$$p_{nix} = \frac{\exp(x_{ni}(\theta_n - \delta_i))}{\sum_{x_{ni}} \exp(x_{ni}(\theta_n - \delta_i))}$$

Ability of person n


Response Probability:

Correctly Answer



Incorrect Answer

Equal Difficulty and Ability

Hover over parameters to see explanation



delta (Slide Layer)



Rasch Model (II)

Probability Function:

$$p_{nix} = \frac{\exp[x_{ni}(\theta_n - \delta_i)]}{\sum_{x_{ni}} \exp[x_{ni}(\theta_n - \delta_i)]}$$

Difficulty of item i


Response Probability:

Correctly Answer



Incorrect Answer

Equal Difficulty and Ability

Hover over parameters to see explanation



exp (Slide Layer)



Rasch Model (II)

Probability Function:

$$p_{nix} = \frac{\exp[x_{ni}(\theta_n - \delta_i)]}{\sum_{x_{ni}} \exp[x_{ni}(\theta_n - \delta_i)]}$$

Exponential function


Response Probability:

Correctly Answer

Incorrect Answer

Equal Difficulty and Ability

Hover over parameters to see explanation



sum (Slide Layer)

Rasch Model (II)

Probability Function:

$$p_{nix} = \frac{\exp(x_{ni}(\theta_n))}{\sum_{x_n} \exp(x_{ni}(\theta_n))}$$

Summation of probabilities
(over all / both response categories)

Response Probability:

Correctly Answer

Incorrect Answer

Equal
Difficulty and Ability

Hover over parameters
to see explanation

3.15 Commonly Used Rasch Measurement Models

Model Name	Model Equation (Log-odds Formula)
Single-facet Models	
Dichotomous Model	$\ln \left(\frac{p_{ni1}}{p_{ni0}} \right) = \theta_n - \delta_i$
Partial Credit Model	$\ln \left(\frac{p_{nik}}{p_{nik-1}} \right) = \theta_n - \delta_{ik}$
Rating Scale Model	$\ln \left(\frac{p_{nik}}{p_{nik-1}} \right) = \theta_n - (\delta_i + \tau_k) = \theta_n - \delta_i - \tau_k$
Many-facet Models	
Many-facet Partial Credit Model	$\ln \left(\frac{p_{nmik}}{p_{nmik-1}} \right) = \theta_n - \lambda_m - \delta_{ik}$
Many-facet Rating Scale Model	$\ln \left(\frac{p_{nmik}}{p_{nmik-1}} \right) = \theta_n - \lambda_m - (\delta_i + \tau_k) = \theta_n - \lambda_m - \delta_i - \tau_k$

ability (Slide Layer)

Common Rasch Models	
Model Name	Model Equation (Log-odds Formula)
Single-facet Models	
Dichotomous Model	$\ln \left(\frac{P_{ni1}}{P_{ni0}} \right) = \theta_n - \delta_i$
Partial Credit Model	$\ln \left(\frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - \delta_{ik}$
Rating Scale Model	$\ln \left(\frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - (\delta_i + \tau_k) = \theta_n - \delta_i - \tau_k$
Many-facet Models	
Many-facet Partial Credit Model	$\ln \left(\frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - \delta_{ik}$
Many-facet Rating Scale Model	$\ln \left(\frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - (\delta_i + \tau_k) = \theta_n - \lambda_m - \delta_i - \tau_k$

θ_n = ability of person n

difficulty (Slide Layer)

Common Rasch Models	
Model Name	Model Equation (Log-odds Formula)
Single-facet Models	
Dichotomous Model	$\ln \left(\frac{P_{ni1}}{P_{ni0}} \right) = \theta_n - \delta_i$
Partial Credit Model	$\ln \left(\frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - \delta_{ik}$
Rating Scale Model	$\ln \left(\frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - (\delta_i + \tau_k) = \theta_n - \delta_i - \tau_k$
Many-facet Models	
Many-facet Partial Credit Model	$\ln \left(\frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - \delta_{ik}$
Many-facet Rating Scale Model	$\ln \left(\frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - (\delta_i + \tau_k) = \theta_n - \lambda_m - \delta_i - \tau_k$

δ_i = difficulty of item i

delta ik (Slide Layer)

Common Rasch Models	
Model Name	Model Equation (Log-odds Formula)
Single-facet Models	
Dichotomous Model	$\ln \left(\frac{P_{ni1}}{P_{ni0}} \right) = \theta_n - \delta_i$
Partial Credit Model	$\ln \left(\frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - \delta_{ik}$
Rating Scale Model	$\ln \left(\frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - (\delta_i + \tau_k) = \theta_n - \delta_i - \tau_k$
Many-facet Models	
Many-facet Partial Credit Model	$\ln \left(\frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - \delta_{ik}$
Many-facet Rating Scale Model	$\ln \left(\frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - (\delta_i + \tau_k) = \theta_n - \lambda_m - \delta_i - \tau_k$

δ_{ik} = difficulty of step k of item i (assuming unique scale structure of each item)

tau k (Slide Layer)

Common Rasch Models	
Model Name	Model Equation (Log-odds Formula)
Single-facet Models	
Dichotomous Model	$\ln \left(\frac{P_{ni1}}{P_{ni0}} \right) = \theta_n - \delta_i$
Partial Credit Model	$\ln \left(\frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - \delta_{ik}$
Rating Scale Model	$\ln \left(\frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - (\delta_i + \tau_k) = \theta_n - \delta_i - \tau_k$
Many-facet Models	
Many-facet Partial Credit Model	$\ln \left(\frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - \delta_{ik}$
Many-facet Rating Scale Model	$\ln \left(\frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - (\delta_i + \tau_k) = \theta_n - \lambda_m - \delta_i - \tau_k$

τ_k = difficulty of step k (assuming common scale structure among all items)

lamda m (Slide Layer)

Common Rasch Models	
Model Name	Model Equation (Log-odds Formula)
Single-facet Models	
Dichotomous Model	$\ln \left(\frac{P_{ni1}}{P_{ni0}} \right) = \theta_n - \delta_i$
Partial Credit Model	$\ln \left(\frac{P_{nik}}{P_{ni(k-1)}} \right) = \theta_n - \delta_{ik}$
Rating Scale Model	$\ln \left(\frac{P_{nik}}{P_{ni(k-1)}} \right) = \theta_n - (\delta_i + \tau_k) = \theta_n - \delta_i - \tau_k$
Many-facet Models	
Many-facet Partial Credit Model	$\ln \left(\frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - \delta_{ik}$
Many-facet Rating Scale Model	$\ln \left(\frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - (\delta_i + \tau_k) = \theta_n - \lambda_m - \delta_i - \tau_k$

λ_m = scoring severity of rater m

P ni1 (Slide Layer)

Common Rasch Models	
Model Name	Model Equation (Log-odds Formula)
Single-facet Models	
Dichotomous Model	$\ln \left(\frac{P_{ni1}}{P_{ni0}} \right) = \theta_n - \delta_i$
Partial Credit Model	$\ln \left(\frac{P_{nik}}{P_{ni(k-1)}} \right) = \theta_n - \delta_{ik}$
Rating Scale Model	$\ln \left(\frac{P_{nik}}{P_{ni(k-1)}} \right) = \theta_n - (\delta_i + \tau_k) = \theta_n - \delta_i - \tau_k$
Many-facet Models	
Many-facet Partial Credit Model	$\ln \left(\frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - \delta_{ik}$
Many-facet Rating Scale Model	$\ln \left(\frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - (\delta_i + \tau_k) = \theta_n - \lambda_m - \delta_i - \tau_k$

P_{ni1} = probability of a correct response to item i by person n

log (Slide Layer)

Common Rasch Models	
Model Name	Model Equation (Log-odds Formula)
Single-facet Models	
Dichotomous Model	$\ln \left(\frac{P_{ni1}}{P_{ni0}} \right) = \theta_n - \delta_i$
Partial Credit Model	$\ln \left(\frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - \delta_{ik}$
Rating Scale Model	$\ln \left(\frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - (\delta_i + \tau_k) = \theta_n - \delta_i - \tau_k$
Many-facet Models	
Many-facet Partial Credit Model	$\ln \left(\frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - \delta_{ik}$
Many-facet Rating Scale Model	$\ln \left(\frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - (\delta_i + \tau_k) = \theta_n - \lambda_m - \delta_i - \tau_k$
$\ln = \text{logarithm}$	

P_{ni0} (Slide Layer)

Common Rasch Models	
Model Name	Model Equation (Log-odds Formula)
Single-facet Models	
Dichotomous Model	$\ln \left(\frac{P_{ni1}}{P_{ni0}} \right) = \theta_n - \delta_i$
Partial Credit Model	$\ln \left(\frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - \delta_{ik}$
Rating Scale Model	$\ln \left(\frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - (\delta_i + \tau_k) = \theta_n - \delta_i - \tau_k$
Many-facet Models	
Many-facet Partial Credit Model	$\ln \left(\frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - \delta_{ik}$
Many-facet Rating Scale Model	$\ln \left(\frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - (\delta_i + \tau_k) = \theta_n - \lambda_m - \delta_i - \tau_k$
$P_{ni0} = \text{probability of an incorrect response to item } i \text{ by person } n$	

P_{nik} (Slide Layer)

Common Rasch Models	
Model Name	Model Equation (Log-odds Formula)
Single-facet Models	
Dichotomous Model	$\ln \left(\frac{P_{ni1}}{P_{ni0}} \right) = \theta_n - \delta_i$
Partial Credit Model	$\ln \left(\frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - \delta_{ik}$
Rating Scale Model	$\ln \left(\frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - (\delta_i + \tau_k) = \theta_n - \delta_i - \tau_k$
Many-facet Models	
Many-facet Partial Credit Model	$\ln \left(\frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - \delta_{ik}$
Many-facet Rating Scale Model	$\ln \left(\frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - (\delta_i + \tau_k) = \theta_n - \lambda_m - \delta_i - \tau_k$

P_{nik} = probability of a response in category k of item i by person n

P_{nik-1} (Slide Layer)

Common Rasch Models	
Model Name	Model Equation (Log-odds Formula)
Single-facet Models	
Dichotomous Model	$\ln \left(\frac{P_{ni1}}{P_{ni0}} \right) = \theta_n - \delta_i$
Partial Credit Model	$\ln \left(\frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - \delta_{ik}$
Rating Scale Model	$\ln \left(\frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - (\delta_i + \tau_k) = \theta_n - \delta_i - \tau_k$
Many-facet Models	
Many-facet Partial Credit Model	$\ln \left(\frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - \delta_{ik}$
Many-facet Rating Scale Model	$\ln \left(\frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - (\delta_i + \tau_k) = \theta_n - \lambda_m - \delta_i - \tau_k$

P_{nik} = probability of a response in category $(k-1)$ of item i by person n

P_{nmik} (Slide Layer)

Common Rasch Models	
Model Name	Model Equation (Log-odds Formula)
Single-facet Models	
Dichotomous Model	$\ln \left(\frac{P_{ni1}}{P_{ni0}} \right) = \theta_n - \delta_i$
Partial Credit Model	$\ln \left(\frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - \delta_{ik}$
Rating Scale Model	$\ln \left(\frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - (\delta_i + \tau_k) = \theta_n - \delta_i - \tau_k$
Many-facet Models	
Many-facet Partial Credit Model	$\ln \left(\frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - \delta_{ik}$
Many-facet Rating Scale Model	$\ln \left(\frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - (\delta_i + \tau_k) = \theta_n - \lambda_m - \delta_i - \tau_k$



P_{nmik} = probability of a response in category k of item i by rater m for person n

P_{nmik-1} (Slide Layer)

Common Rasch Models	
Model Name	Model Equation (Log-odds Formula)
Single-facet Models	
Dichotomous Model	$\ln \left(\frac{P_{ni1}}{P_{ni0}} \right) = \theta_n - \delta_i$
Partial Credit Model	$\ln \left(\frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - \delta_{ik}$
Rating Scale Model	$\ln \left(\frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - (\delta_i + \tau_k) = \theta_n - \delta_i - \tau_k$
Many-facet Models	
Many-facet Partial Credit Model	$\ln \left(\frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - \delta_{ik}$
Many-facet Rating Scale Model	$\ln \left(\frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - (\delta_i + \tau_k) = \theta_n - \lambda_m - \delta_i - \tau_k$

P_{nmik} = probability of a response in category $(k-1)$ of item i by rater m for person n

3.16 Rasch Model Fits Inside the General 3PL Model

 **Rasch Model Fits Inside the General 3PL**

The Rasch Model is a special case of the more general 3-parameter logistic (3PL) model with $c = 0.00$ and $a = 1.00$.

$$P_{ni1} = c_i + (1 + c_i) \frac{\exp[a_i(\theta_n - b_i)]}{1 + \exp[a_i(\theta_n - b_i)]}$$

P_{ni1} = probability of correct response
 b_i = item difficulty parameter
 a_i = item discrimination parameter
 c_i = item lower asymptote (pseudo-guessing) parameter
 θ_n = latent proficiency of person

- An Excel file for generating item response functions is in the Resources section of this player.

3.17 Bookend: Equations





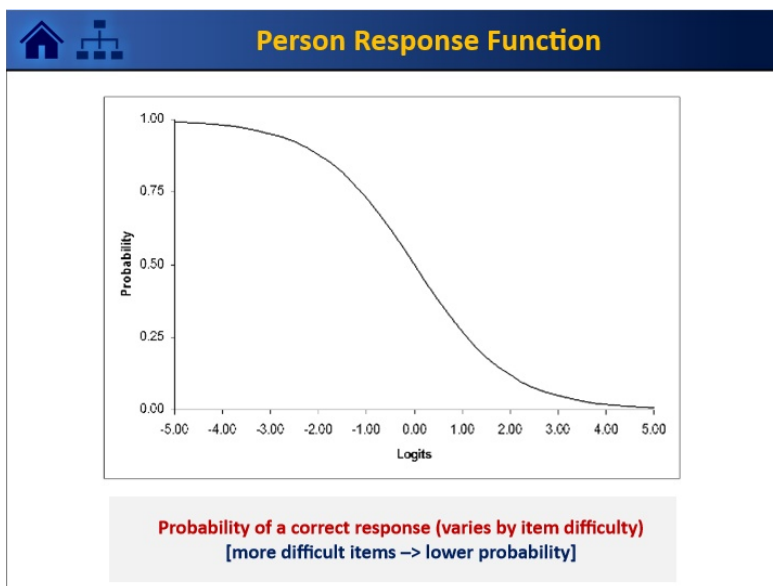
This is the end of this part.

Topic Selection

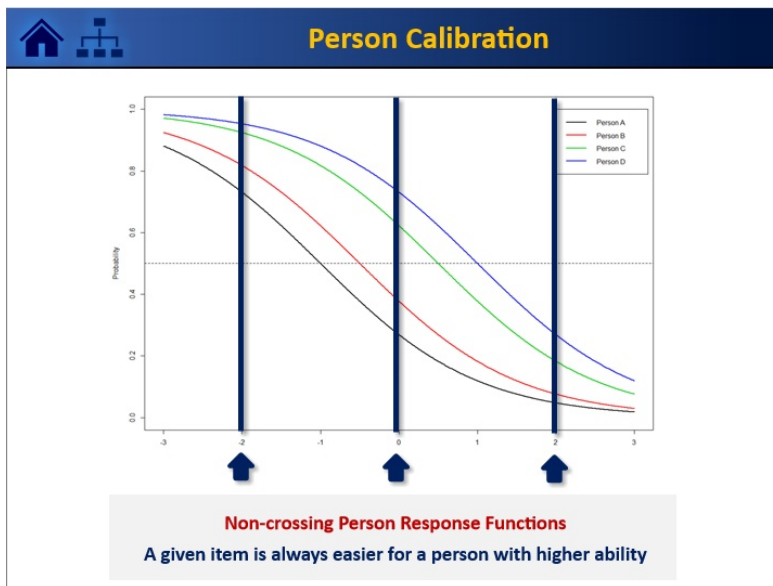
3.18 Bookmark: Graphs



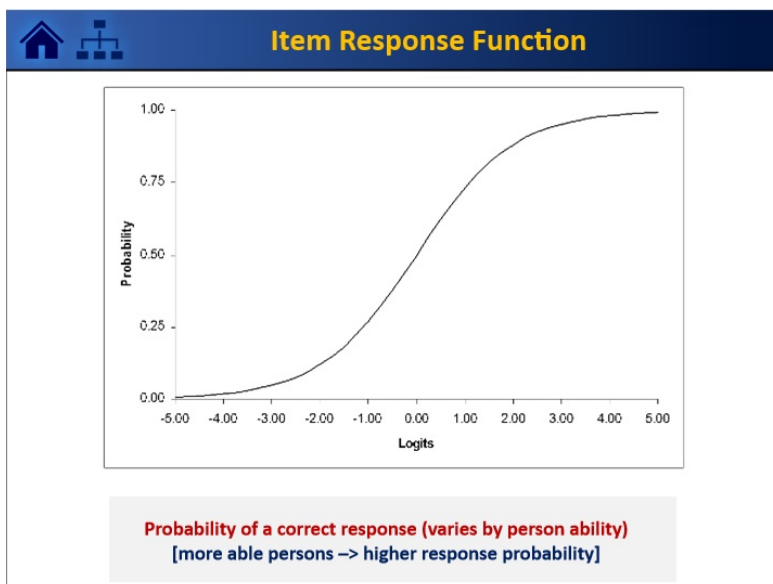
3.19 Person Response Function



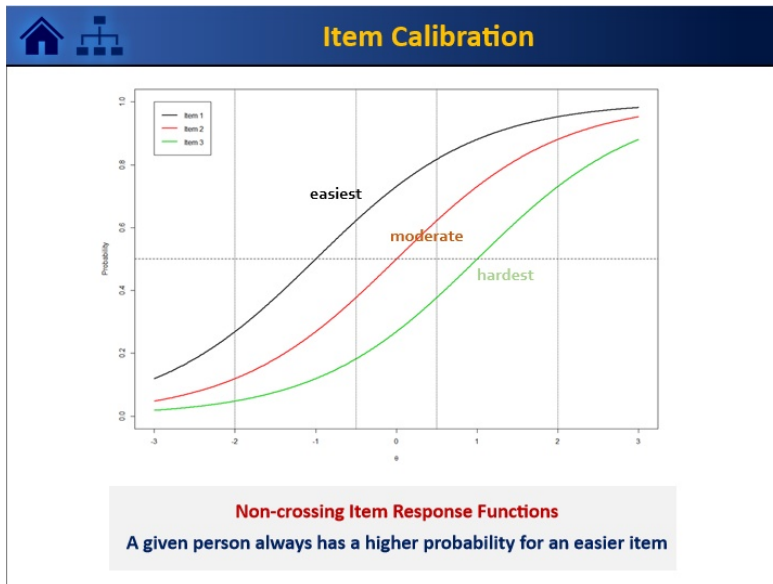
3.20 Person Measurement



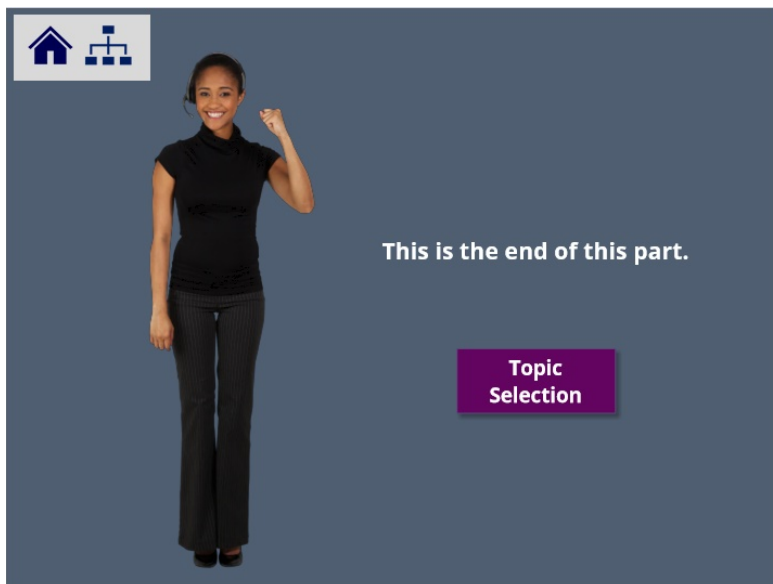
3.21 Item Response Function



3.22 Item Calibration



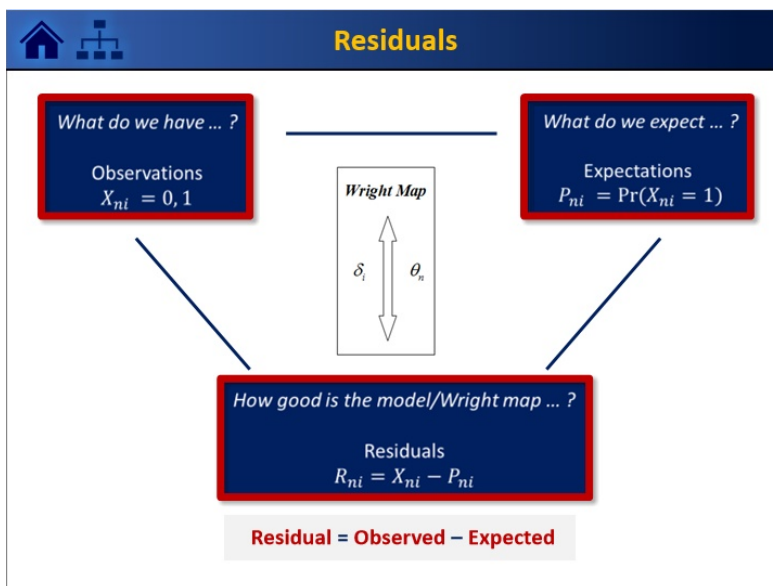
3.23 Bookend: Graphs



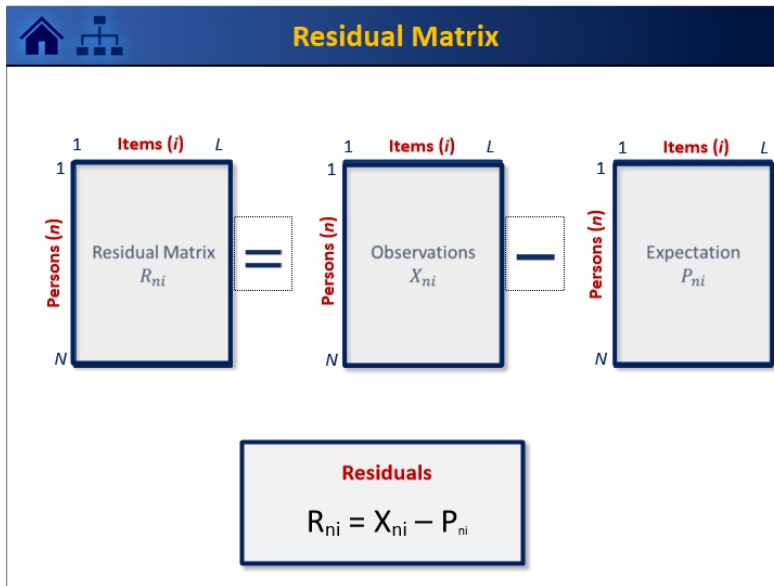
3.24 Bookmark: Fit



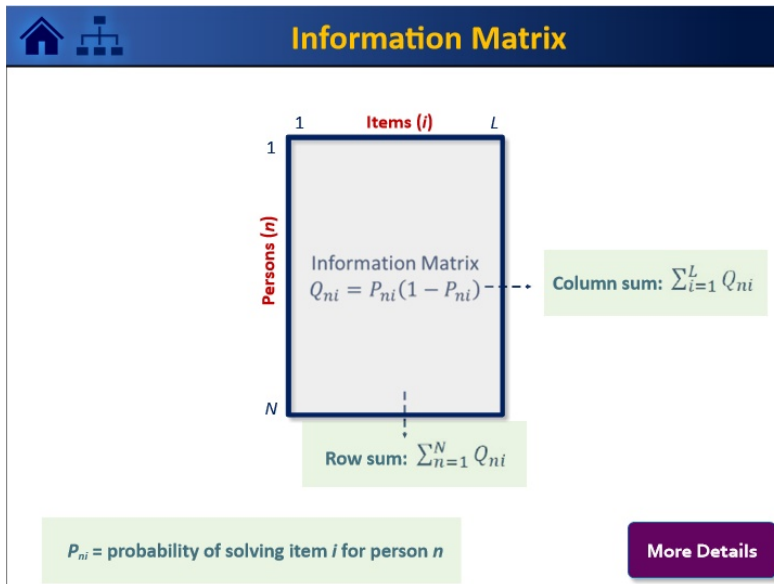
3.25 Residuals



3.26 Residual Matrix



3.27 Information Matrix



References (Slide Layer)



Information Matrix





<https://www.rasch.org/rmt/rmt231j.htm>

Engelhard (2013, p.18)

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3.28 Mean Square Outfit



Mean Square Outfit

Items (*i*)

Persons (*n*)

1

L

1

N

Outfit Square residuals

$$Z_{ni}^2 = \frac{R_{ni}^2}{Q_{ni}}$$

Row sum: $\sum_{n=1}^N Z_{ni}^2$

Item Outfit (row average):

$$U_i = \frac{\sum_{n=1}^N Z_{ni}^2}{N}$$

Column sum: $\sum_{i=1}^L Z_{ni}^2$

Person Outfit (column average):

$$U_n = \frac{\sum_{i=1}^L Z_{ni}^2}{L}$$



Residual: $R_{ni} = X_{ni} - P_{ni}$

Information: $Q_{ni} = P_{ni}(1 - P_{ni})$


- Expected value = 1
- Values close to 1 = good item / person fit
- Values away from 1 = misfit / investigation

Reference

References (Slide Layer)



Mean Square Outfit





<https://www.rasch.org/rmt/rmt231j.htm>

Engelhard (2013, p.18)

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3.29 Mean Square Infit



Mean Square Infit

1 Items (*i*) *L*

1

Persons (*n*)

N

Infit Square residuals
 $R_{ni}^2 = (X_{ni} - P_{ni})^2$

Column sum: $\sum_{i=1}^L R_{ni}^2$

Person Infit: $V_n = \frac{\sum_{i=1}^L R_{ni}^2}{\sum_{i=1}^L Q_{ni}}$



Row sum: $\sum_{n=1}^N R_{ni}^2$

Item Infit: $V_i = \frac{\sum_{n=1}^N R_{ni}^2}{\sum_{n=1}^N Q_{ni}}$


Residual: $R_{ni} = X_{ni} - P_{ni}$
Information: $Q_{ni} = P_{ni}(1 - P_{ni})$

Reference

References (Slide Layer)



Mean Square Infit





<https://www.rasch.org/rmt/rmt231j.htm>

Engelhard (2013, p.18)

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3.30 Interpretation of Mean Square Statistics



Interpretation of Mean Square Statistics

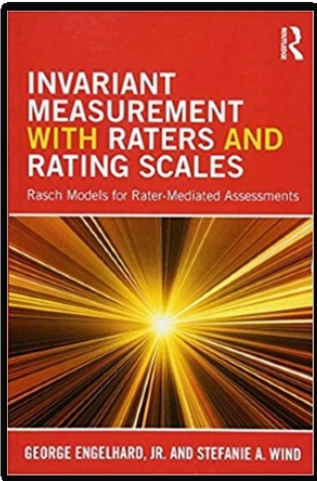
Mean Square Error (MSE)	Interpretation for Infit and Outfit	Fit Category
$.50 \leq MSE < 1.50$	Productive for measurement	A
$MSE < .50$	Less productive for measurement, but not distorting of measures	B
$1.50 \leq MSE < 2.00$	Unproductive for measurement, but not distorting of measures	C
$2.00 \leq MSE$	Unproductive for measurement, and distorting of measures	D

Note: These guidelines reflect rules of thumb.
In practice, users should inspect the distributions of Infit and Outfit indicators.

Reference

References (Slide Layer)

Reference





INvariant
MEASUREMENT
WITH RATERS AND
RATING SCALES


Rasch Models for Rater-Mediated Assessments

GEORGE ENGELHARD, JR. AND STEFANIE A. WIND

Back

3.31 Bookend: Fit







This is the end of this part.

Topic Selection

3.32 Summary (I)




Summary (I)



- **Rasch measurement models** are **confirmatory** in nature
- **Unidimensional** Rasch models assume a **single measurement continuum** for ability
- **Unidimensional** Rasch models capture only **one item characteristic** (e.g., difficulty) and **one person characteristic** (e.g., ability)
- A large **proportion of variance explained** indicates that a **common construct** can be well measured by the items
- **Unexplained variance** in Rasch models can be used to detect **additional dimensions** via **fit statistics**

3.33 Summary (II)



Summary (II)

Rasch models provide an ordering of items and persons along a common continuum / on an underlying scale

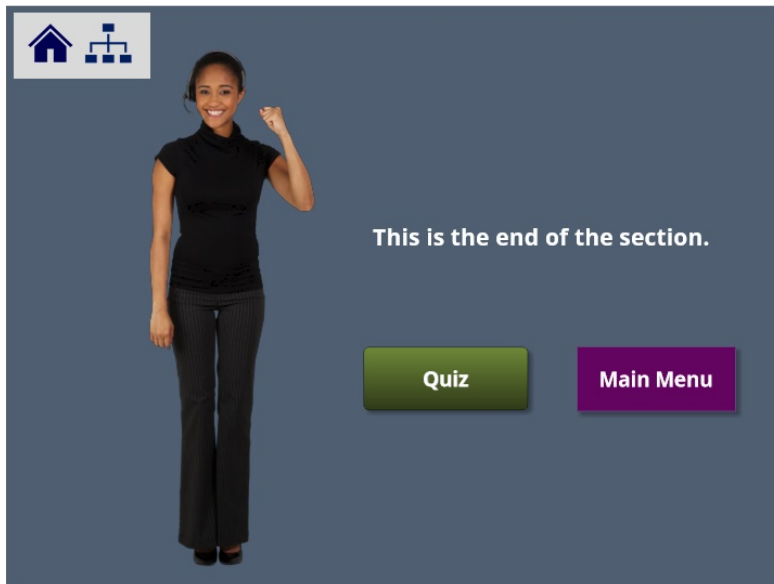
When good fit of items and persons is achieved:

- Ordering of items is invariant across person samples (e.g., **subpopulations**)
- Ordering of persons is invariant across item samples (e.g., **test forms**)

Rasch models have non-crossing person and item response functions:

- A more able person always has a better chance of success on an item than a less able person (**person response functions**)
- Any person always has a better chance of success on an easier item than on a more difficult item (**item response functions**)

3.34 Bookend: Section 2



4. Section 3: Creating a Rasch Scale

4.1 Cover: Section 3



4.2 Objectives: Section 3





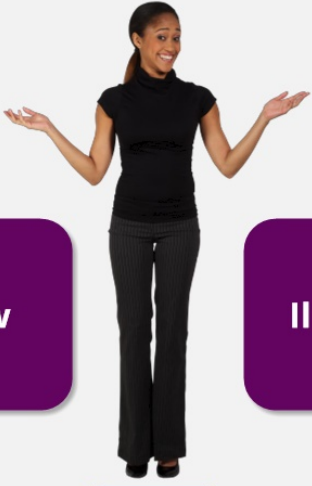
Learning Objectives



1. Explain the key steps for conceptualizing a construct
2. Describe the key steps for a Rasch model analysis
3. Interpret a Rasch scale for empirical examples

4.3 Topic Selection





Overview



Illustration

Section End

4.4 Bookmark: Overview



4.5 Conceptualizing a Construct




Conceptualizing a Construct (I)

Latent variable



- What is the construct?
- Is it unidimensional?
- Can person responses to items represent different levels of the construct?

Observational design

- What are the categories for the construct and item formats?
- What is the plan for collecting person responses?



4.6 Conceptualizing a Construct




Conceptualizing a Construct

Scoring rules



- What is the interpretation of each category?
- How do we score the categories?

Rasch measurement model

- Can we map persons and items onto an invariant and unidimensional scale?
- How well do persons and items fit the Rasch measurement model?




4.7 Creating a Hypothesized Wright Map



Creating a Hypothesized Wright Map

How do we use it?

- Step 1:** Define latent variable
- Step 2:** Map items along the logit scale based on hypothesized item locations
- Step 3:** Determine the response format




[Open Checklist](#)

What is the latent variable (construct)?		
Logit Scale	Persons	Items
-5.00		[Hard items]
-4.00	High values on the latent variable	
-3.00		
-2.00		
-1.00		
0.00	Midrange values on the latent variable	[Moderately difficult items]
-1.00		
-2.00		
-3.00		
-4.00	Low values on the latent variable	
-5.00		[Easy items]

What is the response format or rating scale used?


- Dichotomous ($x=0, 1$)
- Polytomous ($x=0, 1, 2, 3, \dots$)

Checklist (Slide Layer)




Checklist

- ✓ Make sure every item measures the latent variable
- ✓ Make sure items follow a cumulative scale
- ✓ Make sure items have different degrees of difficulty to target persons with varying locations along the latent variable



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4.8 Procedures for Rasch Model Analysis



Procedures for Rasch Model Analysis

Model


- **Dimensionality of scale** (unidimensional, multidimensional)
- **Response format** (dichotomous, polytomous)
- **Scale structures of items** (common, different)
- **Number of facets** (e.g., item, person, and rater)

Estimation



- **Maximum likelihood** (e.g., joint, conditional, marginal)
- **Bayesian** (e.g., MCMC, prior specification, posterior inference)
- **Other** (e.g., normal approximation, pairwise algorithm)

Software

- **Specialized programs** (e.g., Winsteps, Facets, RUMM2030)
- **General programs** (e.g., SAS, STATA, Mplus, R packages)

www.rasch.org


4.9 Interpreting a Unidimensional Rasch Scale

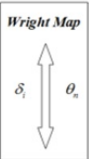


Interpreting a Unidimensional Rasch Scale

Dimensionality analysis

- Scree plot from exploratory factor analysis
- Proportion of variance explained






Wright Map

Empirical Wright map



- Relative ordering of items and persons
- Scale structures for polytomous items

Item and person summary statistics

- Item locations
- Person locations




4.10 Interpreting a Unidimensional Rasch Scale



Interpreting a Unidimensional Rasch Scale


Fit indices & reliability of separation

- Person fit and person reliability
- Item fit and item reliability



Precision

- Standard error of measurement for persons
- Standard error of calibration for items



4.11 Bookend: Overview



4.12 Bookmark: Illustration



4.13 An Illustrative Example

An Illustrative Example

Item	Description of Item
1	Child has toys which teach color, size, and shape.
2	Child has three or more puzzles.
3	Child has record player and at least five children's records.
4	Child has toys permitting free expression
5	Child has toys or games requiring refined movements.
6	Child has toys or games which help teach numbers.
7	Child has at least 10 children's books.
8	At least 10 books are visible in the apartment.
9	Family buys and reads a daily newspaper.
10	Family subscribes to at least one magazine.
11	Child is encouraged to learn shapes.


Reference

Instrument (Slide Layer)


Instrument

Back

4.14 Building Blocks for Constructing Measures

 Building Blocks for Constructing Measures			
Building Blocks	Questions	Answers	Learning Stimulation (LS) Scale
Latent variable	What is the latent variable being measured?	Learning Stimulation	The purpose of the LS scale is to measure the degree of learning stimulation in the home environment of preschool children.
Observational design	What is the plan for collecting structured responses or observations from persons?	Items: List of activities available in the home environment	Dichotomous ratings obtained from teachers visiting the homes of preschool children.
Scoring rules	How are responses or observations categorized to represent person levels on the latent variable?	Objects and activities scored not present (x = 0) or present (x = 1)	Activities and objects checked as present indicate a higher level of learning stimulation in the home environment (0 = not present, 1 = present)
Measurement model	How are person and item responses or observations mapped onto the latent variable?	Rasch Model	Dichotomous Rasch Model

4.15 Data Description and Model Selection

 Data and Model	
Data Description	
<ul style="list-style-type: none"> Home environments of 40 preschool children were evaluated based on 11 items on the LSS instrument Teachers responded based on their observations during the home visits 	
Model Selection	
<ul style="list-style-type: none"> Unidimensional construct: Response format: Measurement facets: <ul style="list-style-type: none"> Item difficulty: Stimulation degree: Model choice: 	<ul style="list-style-type: none"> learning stimulation in home environments binary (0 = absence, 1 = presence) difficulty to observe in a home environment degree of enriching and stimulating learning opportunities in home environment Rasch model for dichotomous responses

4.16 Summary Table

Model-fit Summary		
Measures	Person	Item
Mean	.89	.00
Standard Deviation	1.55	1.76
N	40	11
Infit MSE		
Mean	1.00	.98
Standard Deviation	.41	.19
Outfit MSE		
Mean	.86	.86
Standard Deviation	.69	.42
Reliability of Separation	.69	.92
% of Variance Explained:	44.6%	

Item Details
Person Details

Item Breakdown (Slide Layer)


Item Breakdown

Item No.	Description	Proportion	Measure	S.E.	OUTFIT		INFIT	
					MSE	Fit Category	MSE	Fit Category
1	Child has toys which teach color, size, and shape.	0.88	-1.72	0.57	0.84	A	1.01	A
2	Child has three or more puzzles.	0.48	1.34	0.39	1.07	A	0.87	A
3	Child has record player and at least five children's records.	0.38	1.98	0.41	1.06	A	1.18	A
4	Child has toys permitting free expression	0.93	-2.49	0.68	0.20	B	0.67	A
5	Child has toys or games requiring refined movements.	0.60	0.57	0.40	0.75	A	0.94	A
6	Child has toys or games which help teach numbers.	0.83	-1.14	0.51	1.13	A	1.21	A
7	Child has at least 10 children's books.	0.68	0.08	0.41	0.62	A	0.78	A
8	At least 10 books are visible in the apartment.	0.63	0.42	0.40	1.04	A	0.94	A
9	Family buys and reads a daily newspaper.	0.43	1.65	0.40	1.75	C	1.37	A
10	Family subscribes to at least one magazine.	0.33	2.33	0.43	0.80	A	0.95	A
11	shapes.	0.95	-3.03	0.79	0.22	B	0.87	A

Classification
Summary

Back to
Main Slide

Person Breakdown (Slide Layer)

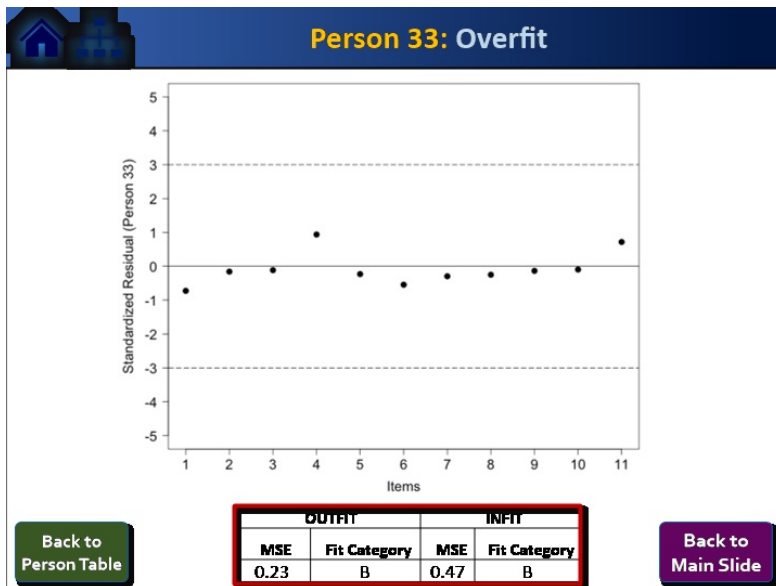
**Person Breakdown**

Person No.	Proportion	Measure	S.E.	OUTFIT		INFIT	
				MSE	Fit Category	MSE	Fit Category

Click on each row to see a residual plot for that person.

Back to Main Slide

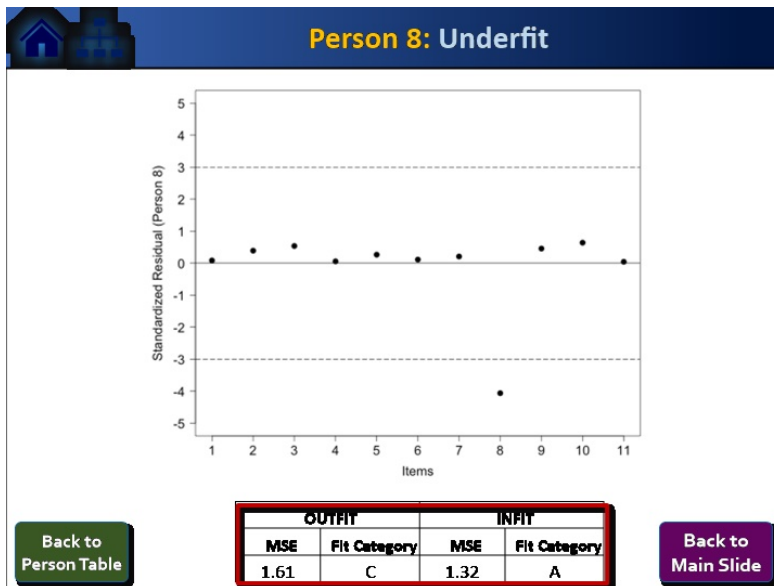
Person 33 (Slide Layer)



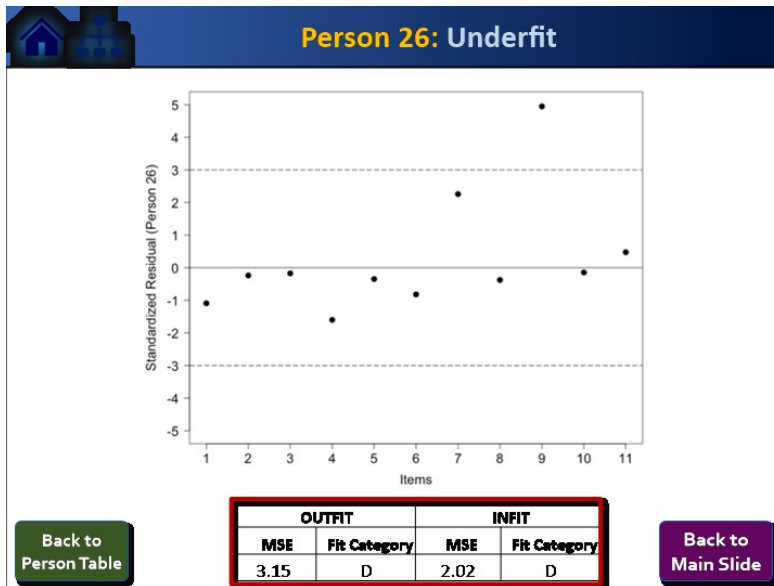
Person 35 (Slide Layer)



Person 8 (Slide Layer)



Person 26 (Slide Layer)



Classification (Slide Layer)

Item Classification

Index	Fit Category			
	A	B	C	D
Person Outfit	.48	.35	.05	.12
Person Infit	.78	.05	.12	.05
Item Outfit	.73	.18	.09	.00
Item Infit	1.00	.00	.00	.00

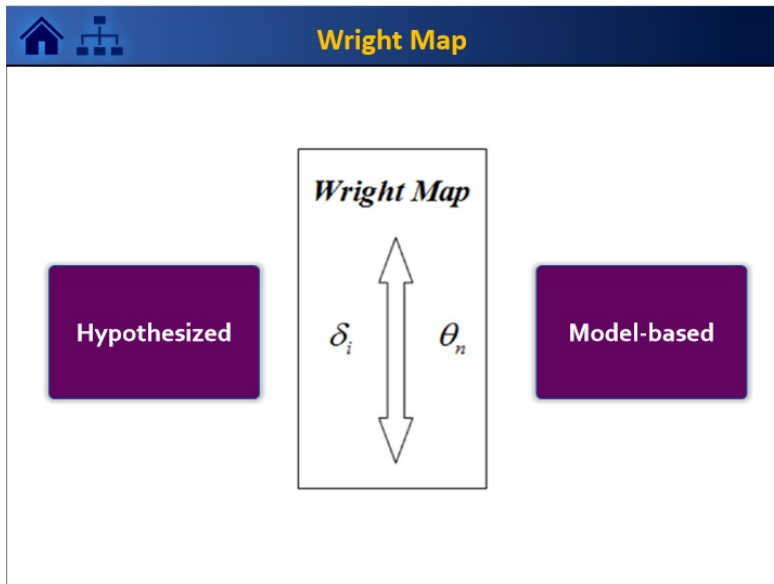
Proportion of cases in each category

Mean Square Error (MSE)	Interpretation	Fit Category
$.50 \leq MSE < 1.50$	Productive for measurement	A
$MSE < .50$	Less productive for measurement, but not distorting of measures	B
$1.50 \leq MSE < 2.00$	Unproductive for measurement, but not distorting of measures	C
$2.00 \leq MSE$	Unproductive for measurement, and distorting of measures	D

General interpretation guidelines

Buttons: [Back to Item Table](#) [Back to Main Slide](#)

4.17 Untitled Slide



Hypothesized (Slide Layer)

Hypothesized

What is the latent construct?

The latent variable is the learning stimulation available in the home environments of preschool children.

Latent Variable	Home Environments of Preschool Children (Latent Variable)	Home Observations (Observed Variables)
0.00		<i>Family subscribes to a magazine</i>
0.00		<i>Family buys and reads a daily newspaper</i>
0.00		
0.00		<i>Ten books visible</i>
0.00	Home environment provides enriching and stimulating learning opportunities <i>[High learning stimulation]</i>	<i>Family subscribes to a magazine</i>
0.00		<i>Family buys and reads a daily newspaper</i>
0.00		
0.00		<i>Ten books visible</i>
0.00	Home environment provides some enriching and stimulating learning opportunities <i>[Midrange learning stimulation]</i>	<i>Ten children's books visible</i>
0.00		<i>Family subscribes to a magazine</i>
0.00		
0.00		<i>Toys to teach numbers</i>
0.00	Home environment provides few enriching and stimulating learning opportunities <i>[Low learning stimulation]</i>	<i>Toys to teach colors and shapes</i>
0.00		<i>Toys to teach shapes</i>
0.00		

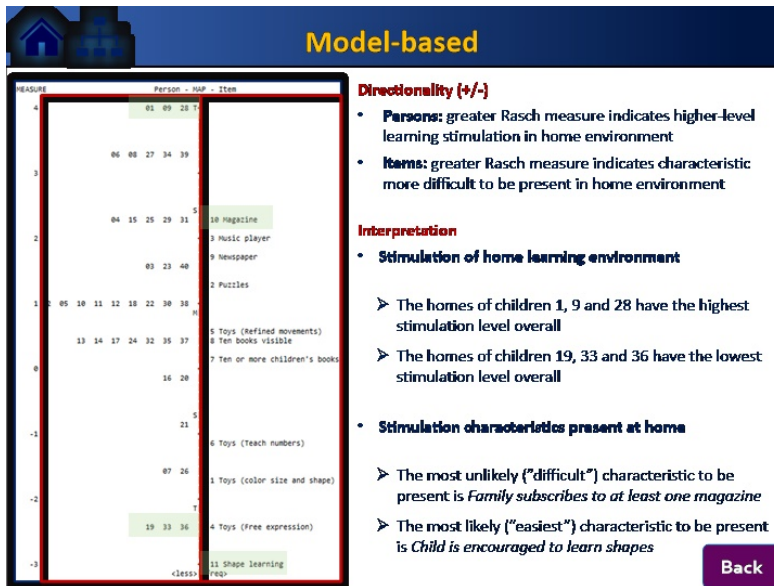
What is the response format or rating scale used?

- Dichotomous scores are used with *not present* coded as zero (x=0) and *present* coded as one (x=1)

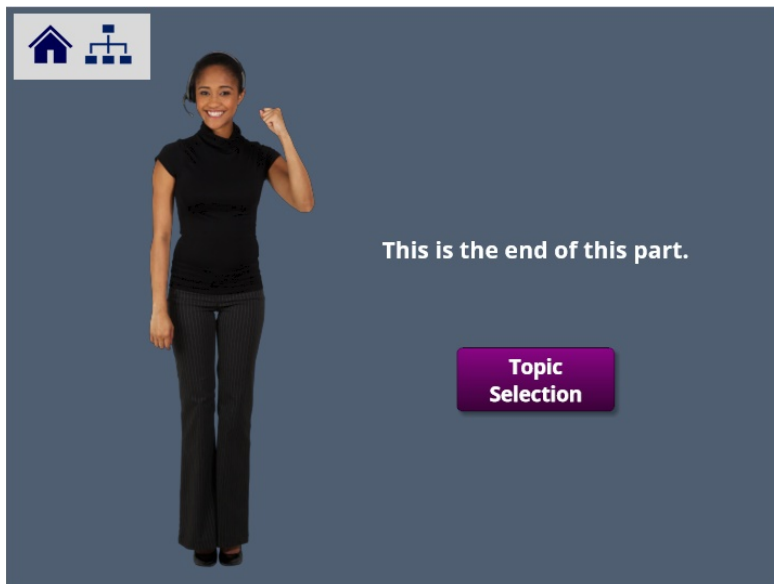
Source: Engelhard (2013, p. 7)

Back



Model-based (Slide Layer)



4.18 Bookend: Illustration





4.19 Summary




Summary

- **Conceptualizing** the underlying construct
- **Creating** hypothesized Wright Map
- **Selecting** an appropriate Rasch measurement model
- **Interpreting** the resulting Rasch scale
- **Next steps:**
 - ✓ Item invariance between subgroups
 - ✓ Person invariance between parallel test forms

4.20 Bookend: Section 3





This is the end of the section.

[Quiz](#)[Main Menu](#)

4.21 Module Cover (END)

