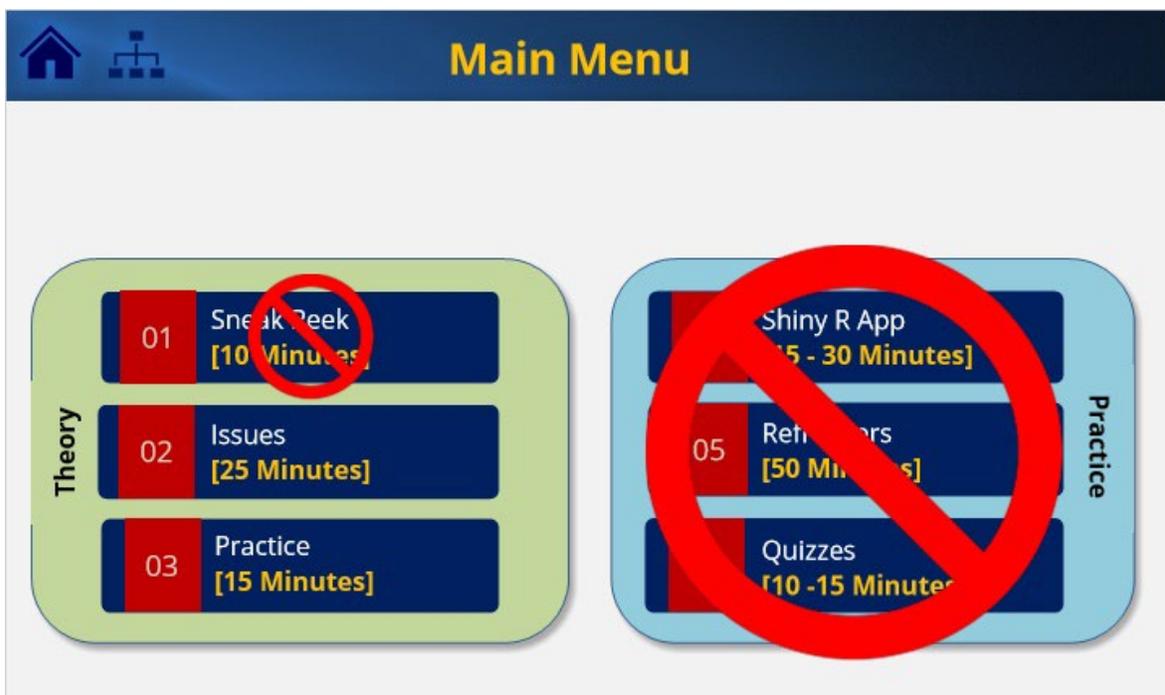


ITEMS Digital Module 02: Scale Reliability in Structural Equation Modeling

This document contains all core content slides from sections 2 & 3 with the exception of slides that show video screens. In the digital module all slides can be accessed individually.

Module Organization

The module starts with an introductory section that leads to the main menu from which learners can select individual content and activity sections:

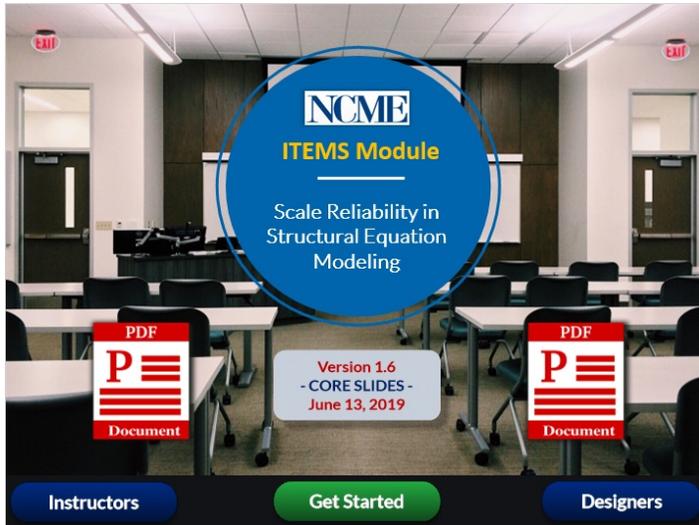


This handout only contains the slides for the introductory section and the *Issues* and *Practice* section due to the interactive nature of many other slides.

ITEMS: Scale Reliability in SEM (Version 1.6)

1. Introduction

1.1 Module Cover (START)



1.2 Instructors



1.3 Designers

Meet the instructional design team:



André A. Rupp



Xi Lu



Special thanks:



Click on the image to get to know them a bit!

1.4 Welcome



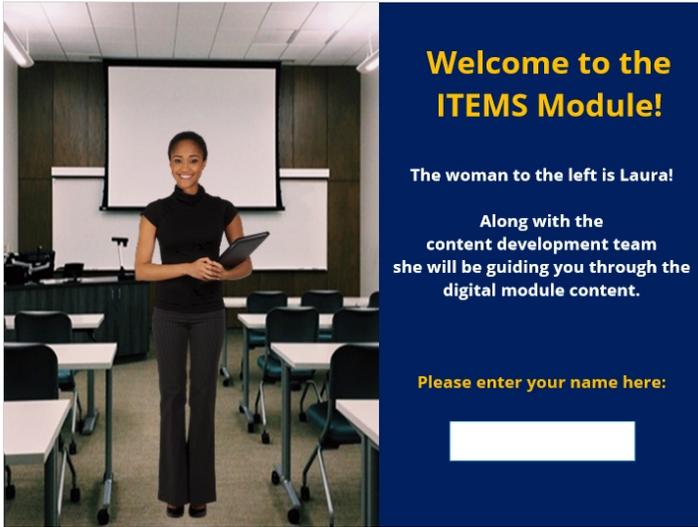
Welcome to the ITEMS Module!

The woman to the left is Laura!

Along with the content development team she will be guiding you through the digital module content.

Please enter your name here:

Untitled Layer 1 (Slide Layer)



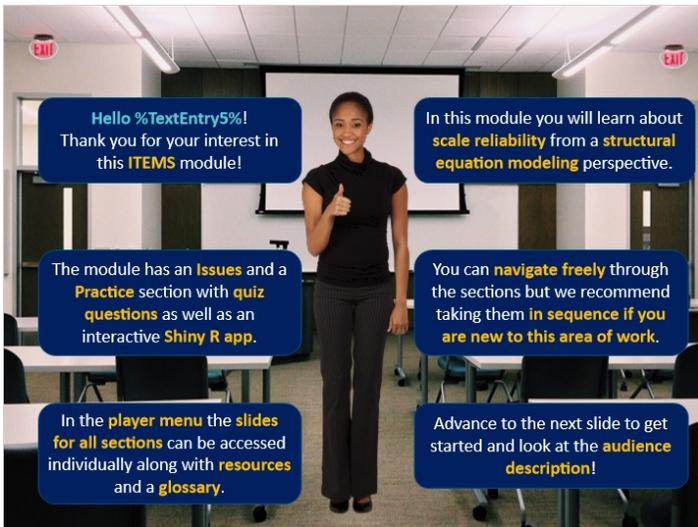
Welcome to the ITEMS Module!

The woman to the left is Laura!

Along with the content development team she will be guiding you through the digital module content.

Please enter your name here:

1.5 Overview



Hello %TextEntry5%! Thank you for your interest in this ITEMS module!

In this module you will learn about **scale reliability** from a **structural equation modeling** perspective.

The module has an **Issues** and a **Practice** section with **quiz questions** as well as an interactive **Shiny R app**.

You can **navigate freely** through the sections but we recommend taking them **in sequence** if you are new to this area of work.

In the **player menu** the **slides for all sections** can be accessed individually along with **resources** and a **glossary**.

Advance to the next slide to get started and look at the **audience description!**

1.6 Target Audience

Target Audience

Anyone who would like a **gentle statistical introduction** to this topic:

- graduate students and faculty in Master's, Ph.D., or certificate programs
- psychometricians and other measurement professionals
- data scientists / analysts
- research assistants or research scientists
- technical project directors
- assessment developers



However, we hope that you find the information in this module **useful no matter what your official title or role** in an organization is!

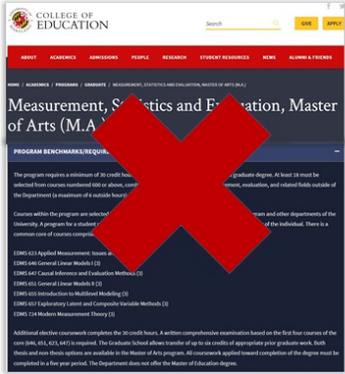
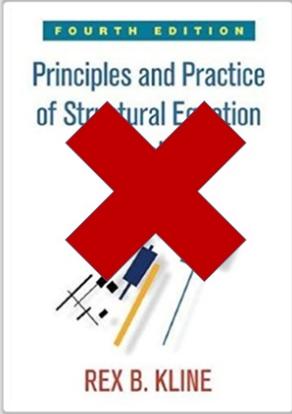
1.7 Expecations (I)



Let's discuss expectations....

1.8 Expectations (II)

ITEMS Modules in Context



1.9 Prerequisites

Prerequisites

- **working knowledge of foundational statistical concepts such as:**
 - variable scales and distributions
 - means, variances / standard deviations, and covariances / correlations
 - confidence intervals and hypothesis tests
- **working knowledge of foundational measurement concepts such as**
 - assessments / tests, items / tasks, and scales / instruments
 - basic notion of reliability
 - distinction between reliability and validity
- **optional: basic experience with R or Mplus** for the **practice exercises** although not needed as introductory videos are provided

Wikipedia: [Intro to Statistics](#) Wikipedia: [Intro to Assessment](#) Software: [Mplus](#) Software: [R](#)

1.10 Resources

Resources



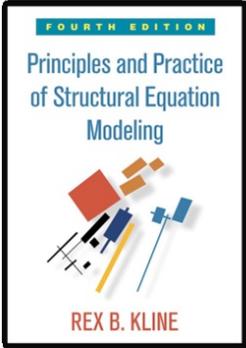
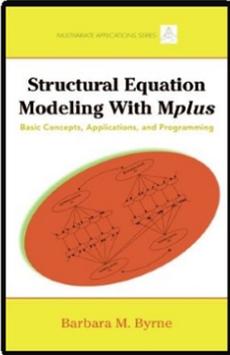
Selected **free materials** are in the 'Resource' section of the **player interface**.

Links to **key reference books** are provided via the button below.

Reference Books

Resources Detail (Slide Layer)

Resources: Books

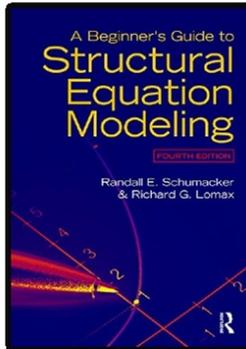


More Books

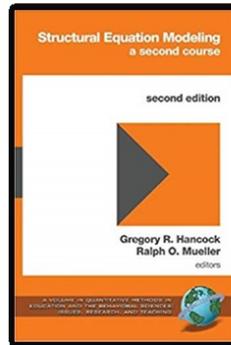
Back to Main Slide

Resources Detail 2 (Slide Layer)

Resources: Books



Back to
Other Books



Back to
Main Slide

1.11 Learning Objectives

Learning Objectives



Express measurement models for single and multiple items as path models

Perform relative model fit comparisons for a sample scale in Mplus

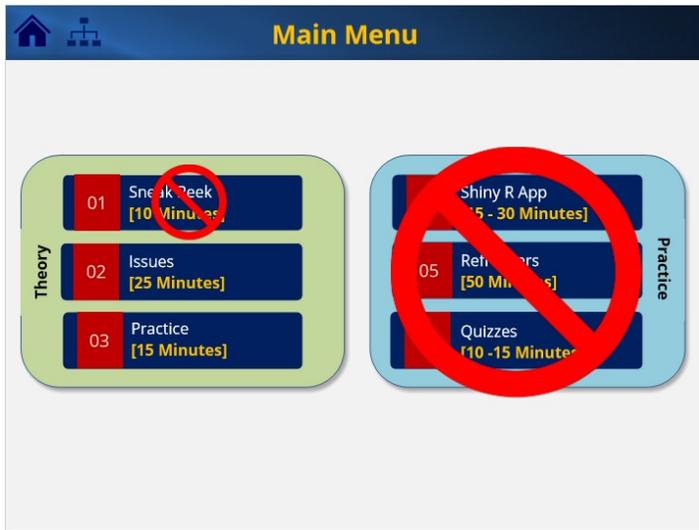
Differentiate between parallel, tau-equivalent, and congeneric models

Estimate Cronbach's α and McDonald's ω in Mplus

Compute Cronbach's α and McDonald's ω from model parameters

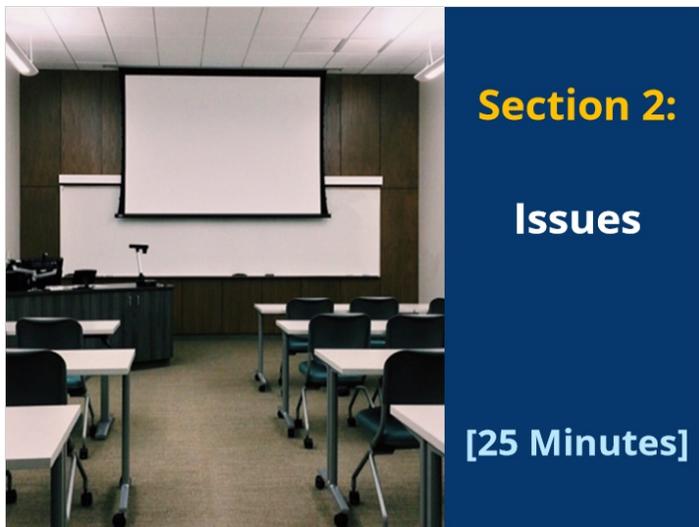
Estimate bootstrap confidence intervals in Mplus

1.12 Main Menu



2. Section 2: Issues

2.1 Cover: Issues



2.2 Objectives: Issues



Learning Objectives



1. Understand the basic conceptual foundations of the SEM framework
2. Understand the relationship between observed item scores and latent variables
3. Express the basic CTT models using path models
4. Compute Cronbach's α and McDonald's ω from path model coefficients

2.3 Intro to Issues (I)



Advantages of SEM

1. Help to clarify the roles of measured scale items and the latent constructs they intend to measure
2. Facilitate a more principled and formal practice of scale reliability assessment



2.4 Intro to Issues (II)



Module Structure

ISSUES

- Single scale item within the SEM framework
- Multi-item instrument within the SEM framework
- Formal assessment and comparison of different measurement structures
- Computation of appropriate measures of scale reliability

PRACTICE

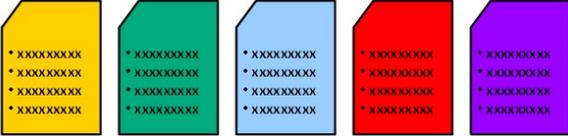
- Example developed using Mplus
- Recommendations for practitioners

2.5 Scales



Scales

- **Encounter scales every day:** attitudes, beliefs, achievement
- Composed of **items** to provide insight into some underlying **construct(s)**



2.6 Goal Orientation Scale (I)

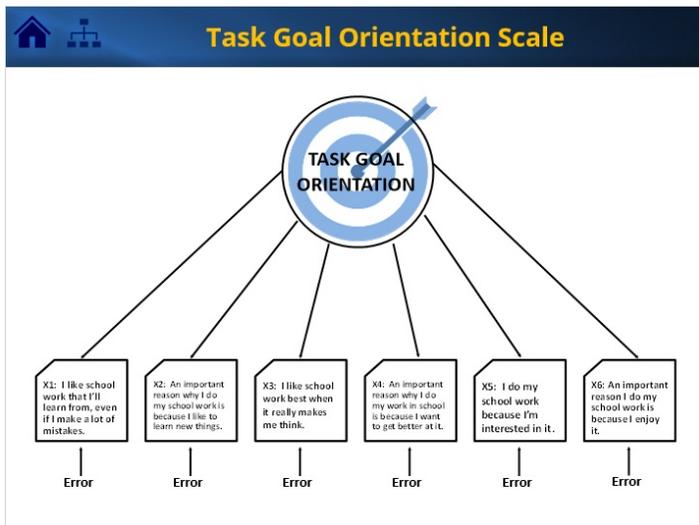
Task Goal Orientation Scale

TASK GOAL ORIENTATION SCALE
(Midgley et al., 1998)

X1: I like school work that I'll learn from, even if I make a lot of mistakes.
 X2: An important reason why I do my school work is because I like to learn new things.
 X3: I like school work best when it really makes me think.
 X4: An important reason why I do my work in school is because I want to get better at it.
 X5: I do my school work because I'm interested in it.
 X6: An important reason I do my school work is because I enjoy it.

21 (co)variances

2.7 Goal Orientation Scale (II)

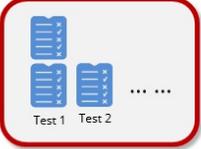


2.8 CTT Model

CTT Model

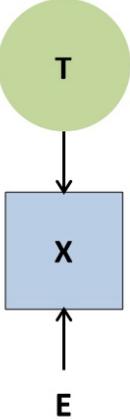
$$X_i = T_i + E_i$$

- X_i is a score on a scale item
- T_i is the long run expected value of X_i
- E_i is the remainder or error term



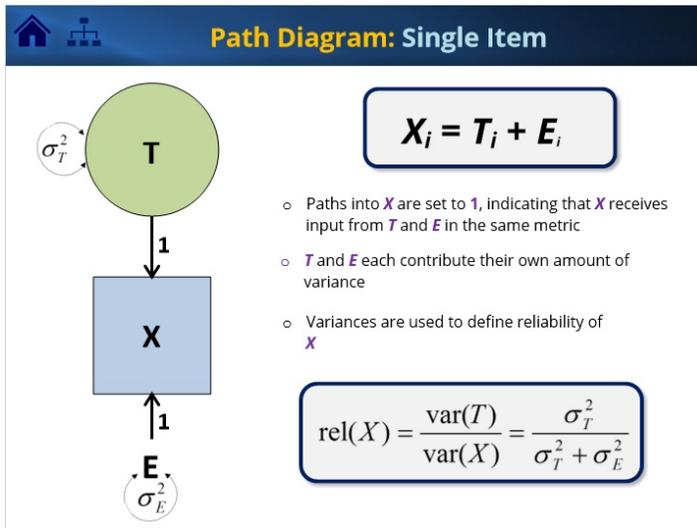
2.9 Path Diagram: Single Item (I)

Path Diagram: Single Item

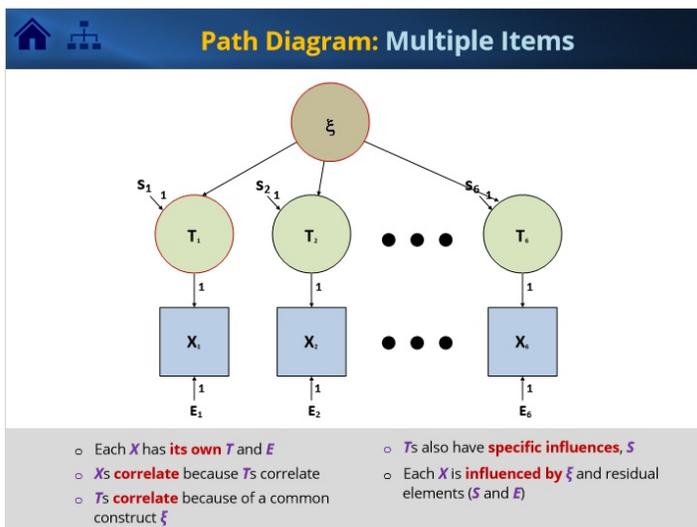

$$X_i = T_i + E_i$$

- X is a measured variable, represented by a box
- T is a theoretical score on an underlying construct of interest, represented by a circle
- E is other unrelated contributing agents
- **Arrows** represent hypothesized causal influence, typically assumed to be linear
- T and E are not directly connected, an assumption that the residual influences on X are unrelated to its underlying construct.

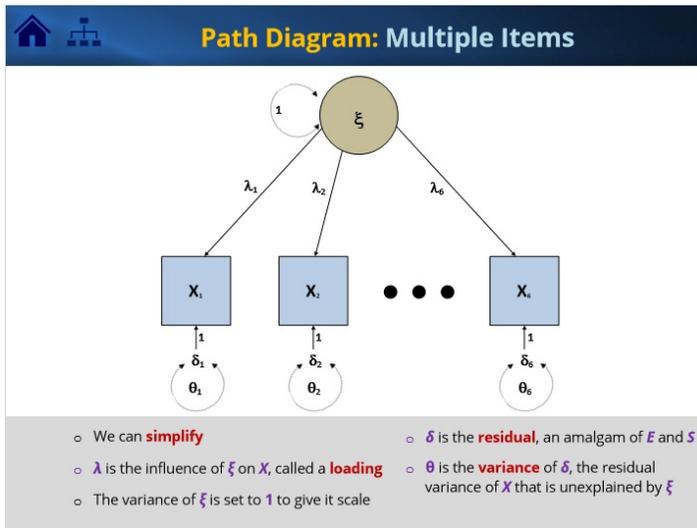
2.10 Path Diagram: Single Item (II)



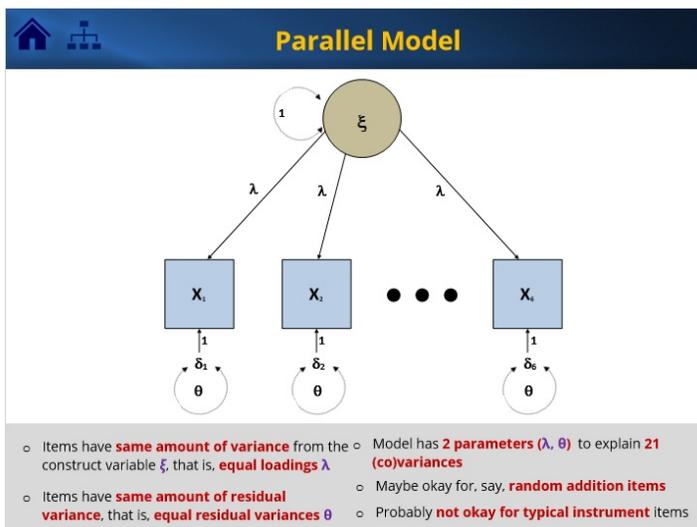
2.11 Path Diagram: Multiple Items (I)



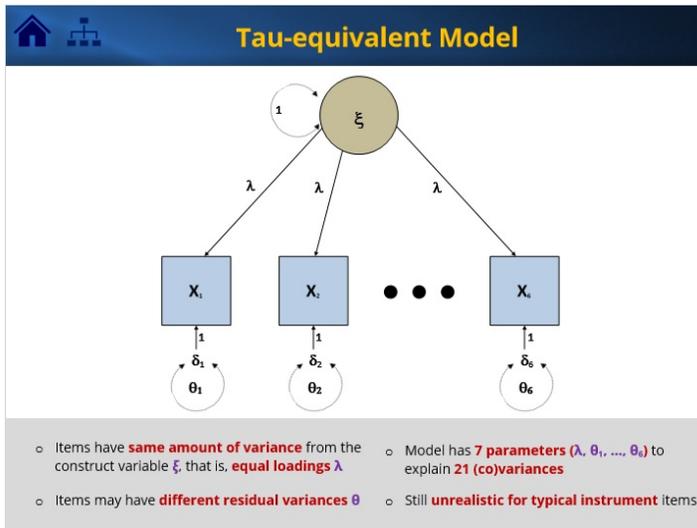
2.12 Path Diagram: Multiple Items (II)



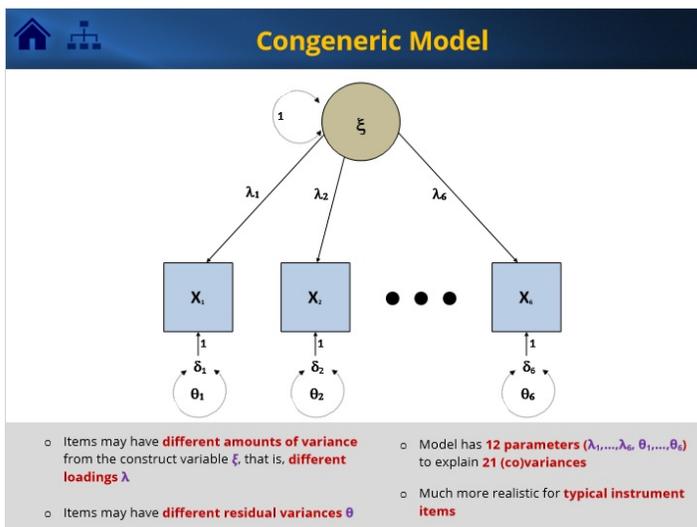
2.13 Parallel Model



2.14 Tau-equivalent Model



2.15 Congeneric Model

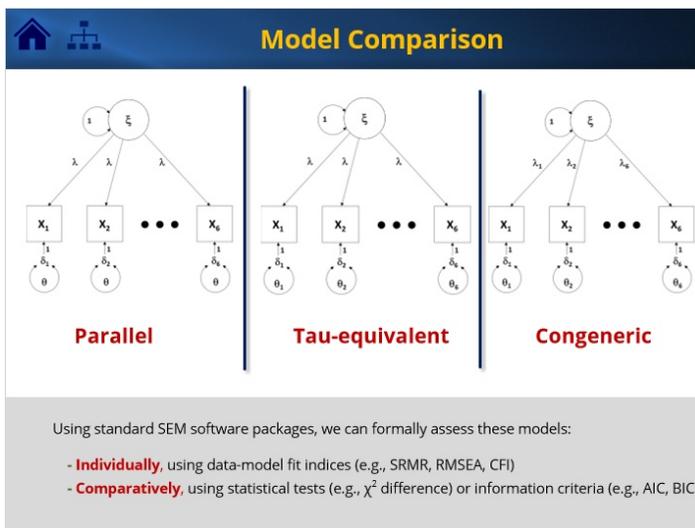


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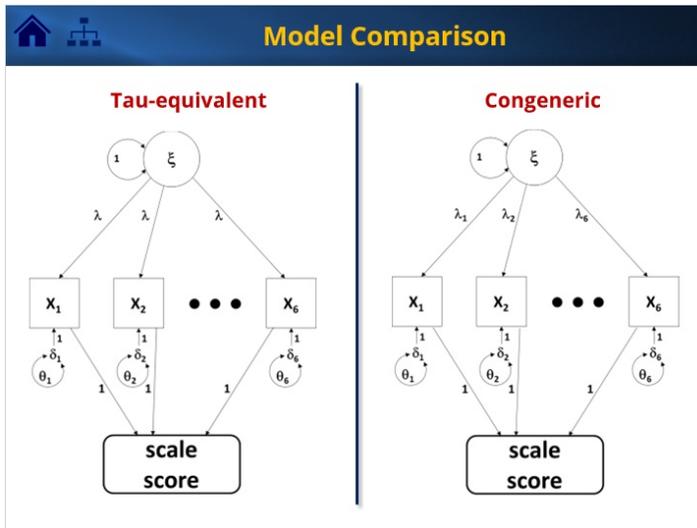
A Summary

	Loadings	Residual variances	Degree of freedom	Parameters
Parallel	equal	equal	19	2
Tau-equivalent	equal	different	14	7
Congeneric	different	different	9	12

2.16 Model Comparison (I)



2.17 Model Comparison (II)



2.18 Coefficient Alpha

Coefficient α

$\text{var}(\text{tau-equivalent scale}) = \text{var}(\xi) + \text{var}(\text{errors})$
 $= J^2 \lambda^2 + \sum_{i=1}^J \theta_i$

$\text{rel}(\text{tau-equivalent scale}) = \frac{\text{variance explained by } \xi}{\text{total scale variance}}$
 $= \frac{J^2 \lambda^2}{J^2 \lambda^2 + \sum_{i=1}^J \theta_i}$

- model-based estimate of Cronbach's α for total scale score
- restrictive as it assumes tau-equivalence

2.19 McDonald's Omega (I)

🏠
McDonald's ω

var(congeneric scale) = var(ξ) + var(errors)

$$= \sum_{i,j=1}^J \lambda_i \lambda_j + \sum_{i=1}^J \theta_i$$

$$= (\sum_{i=1}^J \lambda_i)^2 + \sum_{i=1}^J \theta_i$$

rel(congeneric scale) = $\frac{\text{variance explained by } \xi}{\text{total scale variance}}$

$$= \frac{(\sum_{i=1}^J \lambda_i)^2}{(\sum_{i=1}^J \lambda_i)^2 + \sum_{i=1}^J \theta_i}$$

- Model-based estimate of McDonald's ω for total scale score
- Does *not* assume tau-equivalence

2.20 McDonald's Omega (II)

🏠
McDonald's ω

$$\omega = \frac{(\sum_{i=1}^J \lambda_i)^2}{(\sum_{i=1}^J \lambda_i)^2 + \sum_{i=1}^J \theta_i}$$

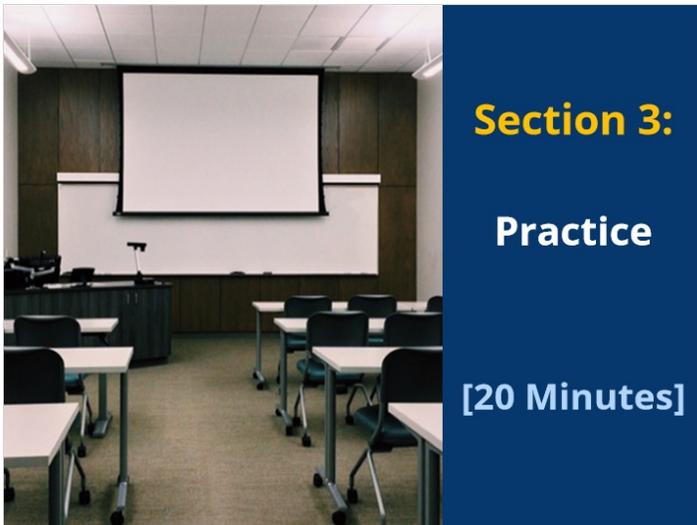
- McDonald's ω is a superior alternative to Cronbach's α
- ω currently has no closed-form estimate; it must be modeled
- SEM framework also allows for
 - bootstrap confidence intervals
 - modeling local dependence
 - nonnormality
 - complex/multilevel samples
 - missing data
- PRACTICE video will show Mplus

2.21 Bookend: Issues



3. Section 3: Practice

3.1 Cover: Practice



3.2 Objectives: Practice

  **Learning Objectives**



1. Specify the core CTT measurement models in Mplus
2. Perform relative model-data fit assessment using suitable indices
3. Compute Cronbach's α and McDonald's ω from parameter estimates
4. Compute and interpret a bootstrap confidence interval for McDonald's ω

3.3 Intro to Practice (I)

  **Advantages of SEM**

- 1 Help to clarify the roles of measured scale items and the latent constructs they intend to measure
- 2 Facilitate a more principled and formal practice of scale reliability assessment



3.4 Intro to Practice (II)

  **Module Structure**

ISSUES

- single scale item within the SEM framework
- multi-item instrument within the SEM framework
- formal assessment and comparison of different measurement structures
- computation of appropriate measures of scale reliability

PRACTICE

- example developed using Mplus
- recommendations for practitioners

3.5 Task Goal Scale

  **Task Goal Orientation Scale**

X1: I like school work that I'll learn from, even if I make a lot of mistakes.

X2: An important reason why I do my school work is because I like to learn new things.

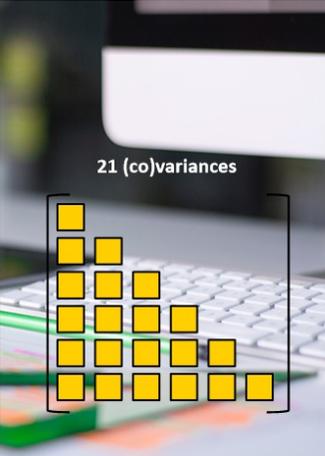
X3: I like school work best when it really makes me think.

X4: An important reason why I do my work in school is because I want to get better at it.

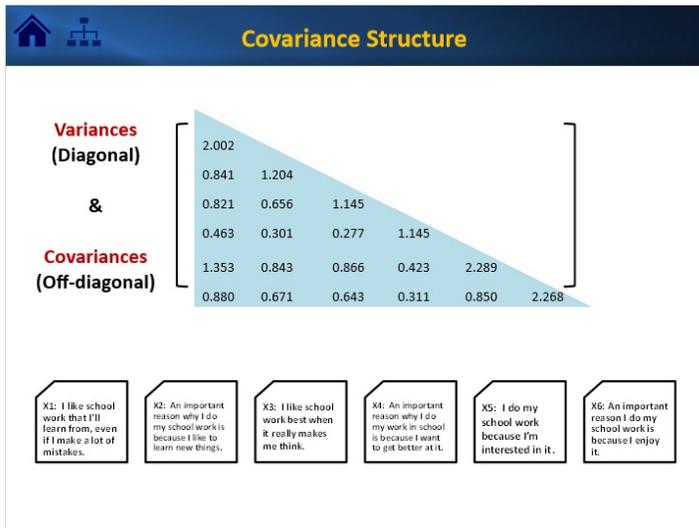
X5: I do my school work because I'm interested in it.

X6: An important reason I do my school work is because I enjoy it.

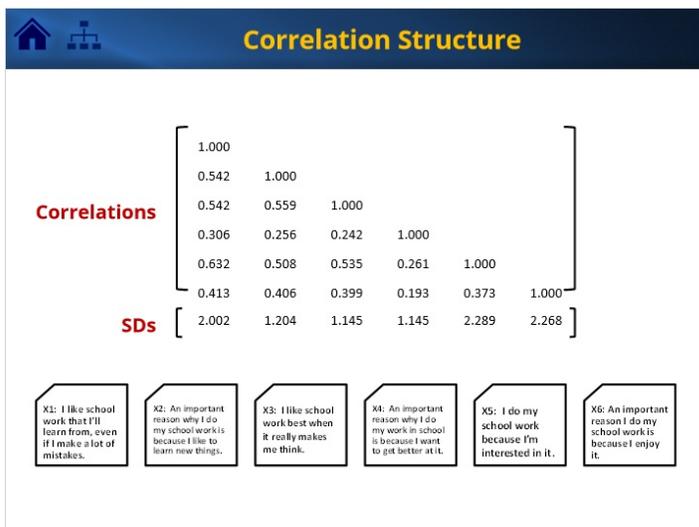
21 (co)variances



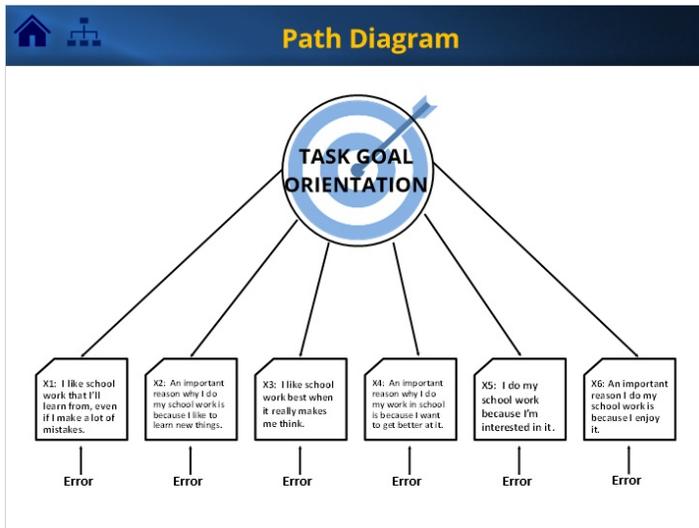
3.6 Covariance Structure



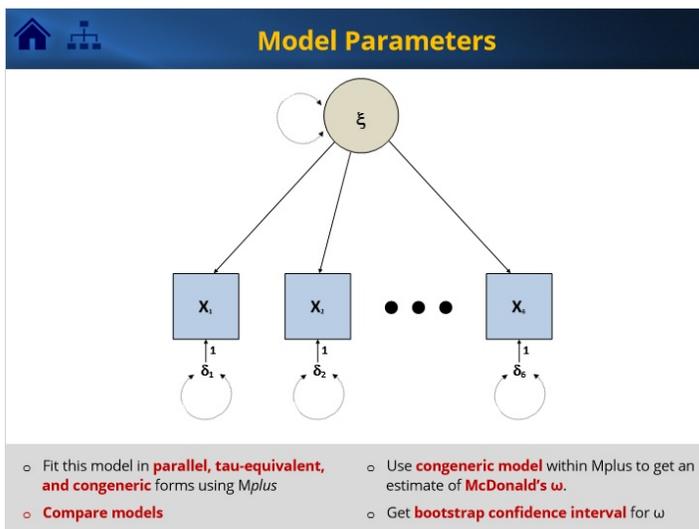
3.7 Correlation Structure



3.8 Path Diagram



3.9 Model Parameters



3.10 Parallel Model

🏠
Parallel Model

○ Items have **equal loadings, λ**

○ Items have **equal residual variance, θ**

○ Model has **2 parameters** to explain 21 (co)variances – **19 degrees of freedom**

Mplus Syntax

TITLE:
Parallel model

DATA:
FILE IS myData.csv;

VARIABLE:
NAMES ARE X1-X6;

MODEL:
TGO@1;
TGO BY X1-X6* (lambda);
X1-X6 (theta);

OUTPUT:
SAMPSTAT STDYX;

3.11 Tau-equivalent Model

🏠
Tau-equivalent Model

○ Items have **equal loadings, λ**

○ Items have **varied residual variance, θ**

○ Model has **7 parameters** to explain 21 (co)variances – **14 degrees of freedom**

Mplus Syntax

TITLE:
Tau-equivalent model

DATA:
FILE IS myData.csv;

VARIABLE:
NAMES ARE X1-X6;

MODEL:
TGO@1;
TGO BY X1-X6* (lambda);
X1-X6;

OUTPUT:
SAMPSTAT STDYX;

3.12 Congeneric Model

🏠
📊
Congeneric Model

- Items have **varied loadings, λ**
- Items may have **varied residual variances, θ**
- Model has **12 parameters** to explain 21 (co)variances – **9 degrees of freedom**

Mplus Syntax

TITLE:
Congeneric model

DATA:
FILE IS myData.csv;

VARIABLE:
NAMES ARE X1-X6;

MODEL:
TGO@1;
TGO BY X1-X6*;
X1-X6;

OUTPUT:
SAMPSTAT STDYX;

3.13 Model Comparisons

🏠
📊
Model Comparison

Parallel

Tau-equivalent

Congeneric

Model	χ^2	AIC	BIC	SRMR	RMSEA	CFI
Parallel (19 df)	507.693	511.693	519.101	0.627	0.293	0.104
Tau-equivalent (14 df)	165.669	179.669	205.595	0.279	0.190	0.722
Congeneric (9 df)	10.178	34.178	78.623	0.017	0.021	0.998

3.14 Parameter Estimates

Parameter Estimates				
MODEL RESULTS				
TGO BY	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
X1	1.581	0.104	15.140	0.000
X2	0.856	0.065	13.121	0.000
X3	0.825	0.062	13.358	0.000
X4	0.413	0.069	5.955	0.000
X5	1.725	0.121	14.252	0.000
X6	1.208	0.131	9.200	0.000
Variances				
TGO	1.000	0.000	999.000	999.000
Residual Variances				
X1	1.510	0.180	8.411	0.000
X2	0.717	0.073	9.838	0.000
X3	0.631	0.065	9.703	0.000
X4	1.142	0.096	11.920	0.000
X5	2.262	0.245	9.252	0.000
X6	3.688	0.325	11.363	0.000

3.15 McDonald's Omega

$$\omega = \frac{(\sum_{i=1}^J \lambda_i)^2}{(\sum_{i=1}^J \lambda_i)^2 + \sum_{i=1}^J \theta_i}$$

$$\hat{\omega} = \frac{6.608^2}{6.608^2 + 9.950}$$

$$= 0.814$$

$(\hat{\alpha} = 0.786)$

3.16 Confidence Interval (I)

  **Confidence Interval Syntax**

```
TITLE:
  Congeneric model, with bootstrap

DATA:
  FILE IS myData.csv;

VARIABLE:
  NAMES ARE X1-X6;

ANALYSIS:
  BOOTSTRAP IS 5000;

MODEL:
  TGO@1;
  TGO BY X1-X6* (L1-L6);
  X1-X6 (TH1-TH6);
```

3.17 Confidence Interval (II)

  **Confidence Interval Syntax**

```
MODEL CONSTRAINT:

  NEW(sumL sumTH omega);

  L1 > 0;
  L2 > 0;
  L3 > 0;
  L4 > 0;
  L5 > 0;
  L6 > 0;

  sumL = L1 + L2 + L3 + L4 + L5 + L6;

  sumTH = TH1 + TH2 + TH3 + TH4 + TH5 + TH6;

  omega = ((sumL)^2) / (((sumL)^2)+sumTH);

OUTPUT:
  CINT(bcbootstrap);
```

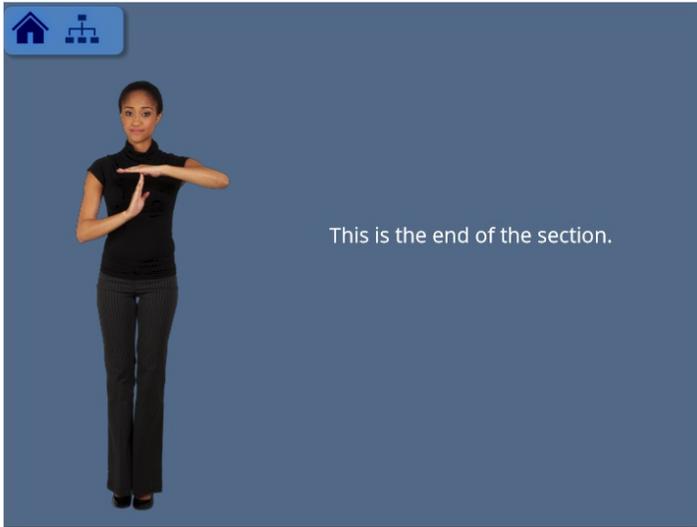
3.18 Confidence Interval (III)

Confidence Interval Estimates				
CONFIDENCE INTERVALS OF MODEL RESULTS				
OMEGA	Lower 5%	Estimate	Upper 5%	90%
	0.784	0.814	0.839	
CONFIDENCE INTERVALS OF MODEL RESULTS				
OMEGA	Lower 2.5%	Estimate	Upper 2.5%	95%
	0.777	0.814	0.844	
CONFIDENCE INTERVALS OF MODEL RESULTS				
OMEGA	Lower .5%	Estimate	Upper .5%	99%
	0.764	0.814	0.853	

3.19 Summary

Summary	
▪ McDonald's ω is a superior alternative to Cronbach's α	
▪ ω currently has no closed-form estimate ; it must be modeled	
▪ Can use SEM software or R packages	
▪ SEM framework also allows for:	
○ bootstrap confidence intervals	
○ modeling local dependence	
○ nonnormality	
○ complex/multilevel samples	
○ missing data	

3.20 Bookend: Practice



3.21 Module Cover (END)

