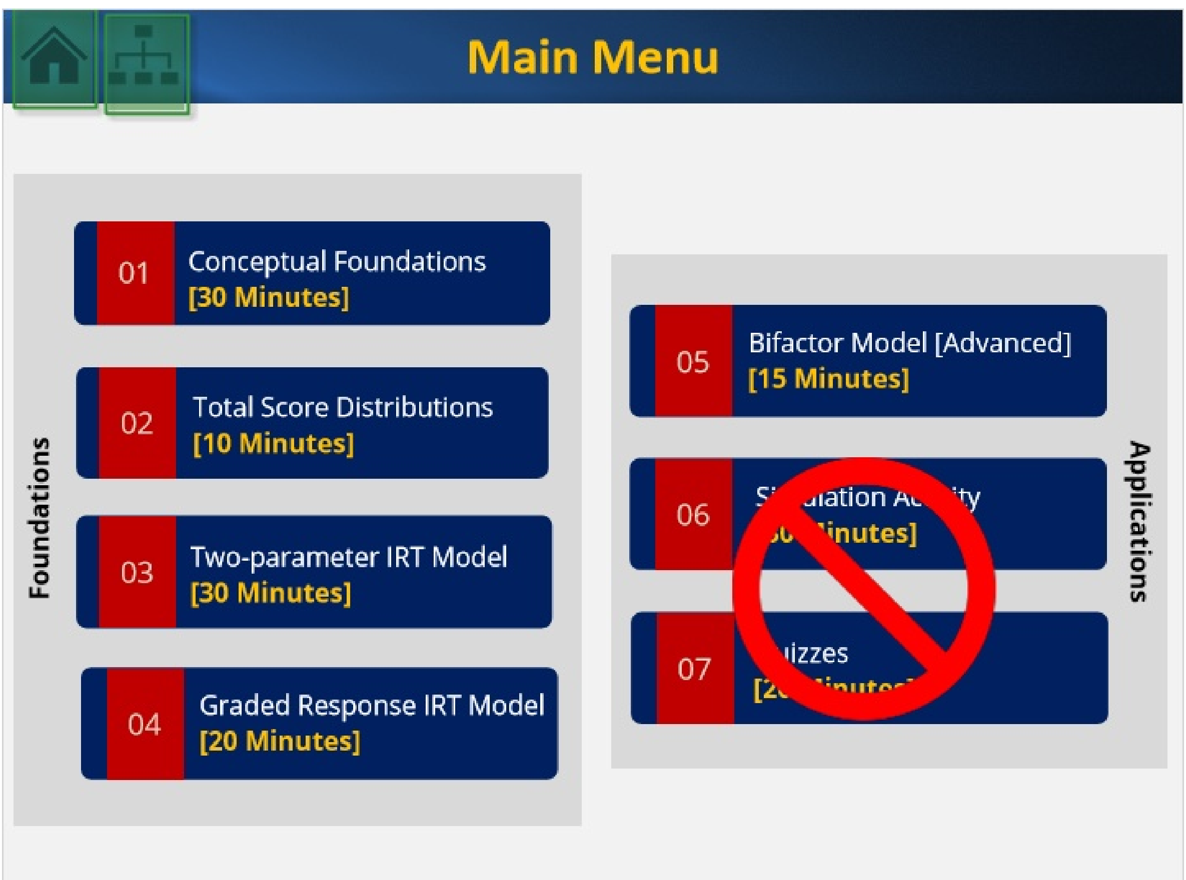
ITEMS DM13: Simulation Studies in Item Response Theory

This document contains only the core content slides from the module. In the digital module all slides can be accessed individually.

Module Organization

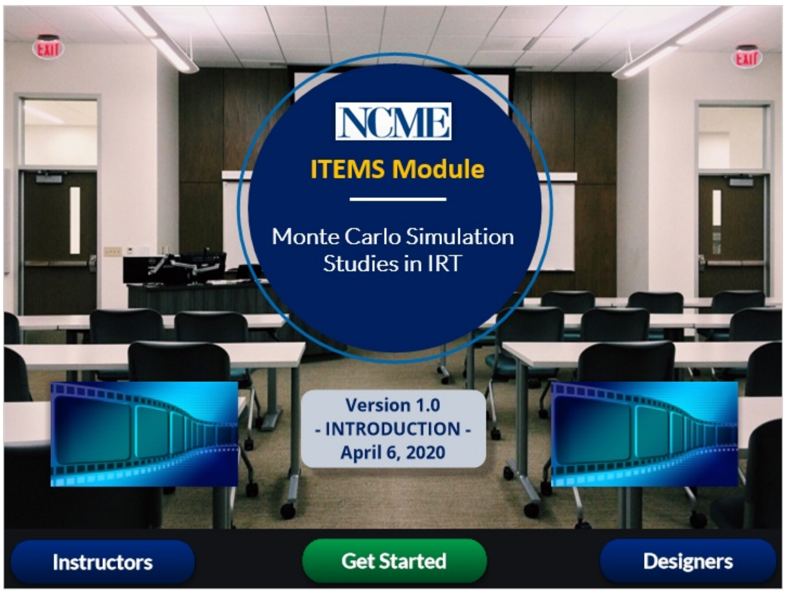
The module starts with an introductory section that leads to the main menu from which learners can select individual sections; this slide deck contains the slides for the five content sections only as shown below; green images in this document are clickable hotspot regions:



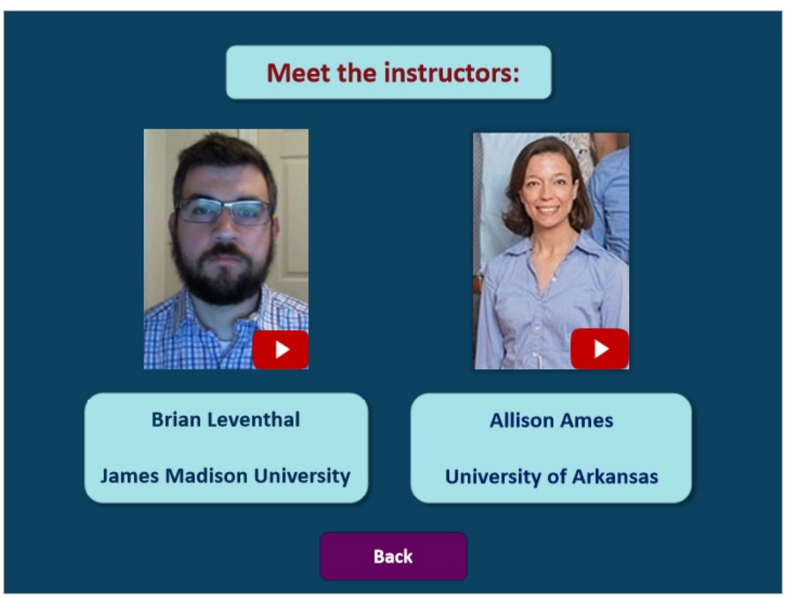
DM13 VIDEO (Section 5, Version 1.0)

1. Module Overview

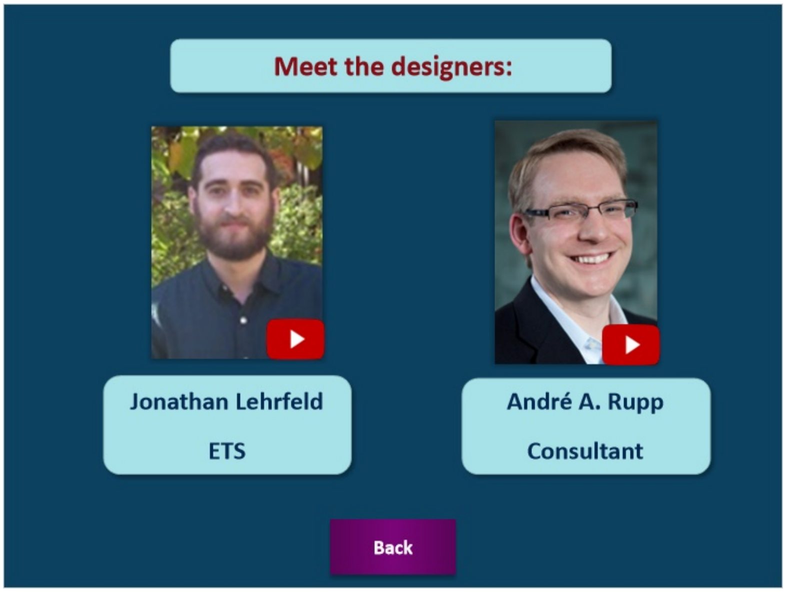
1.1 Module Cover (START)



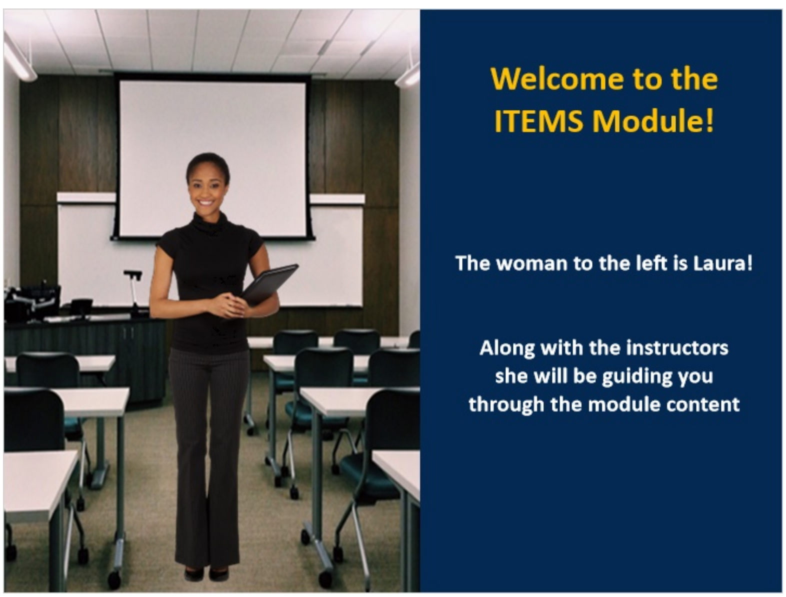
1.2 Content Team



1.3 Design Team



1.4 Welcome



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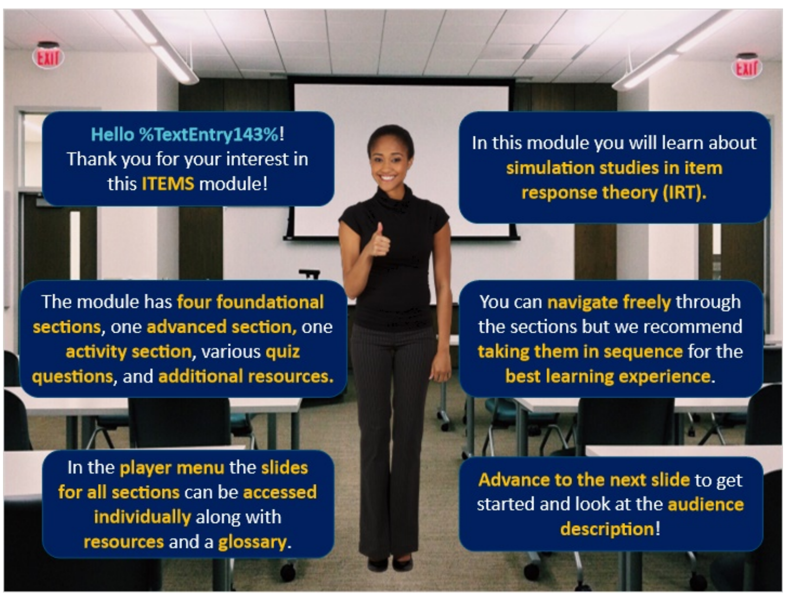
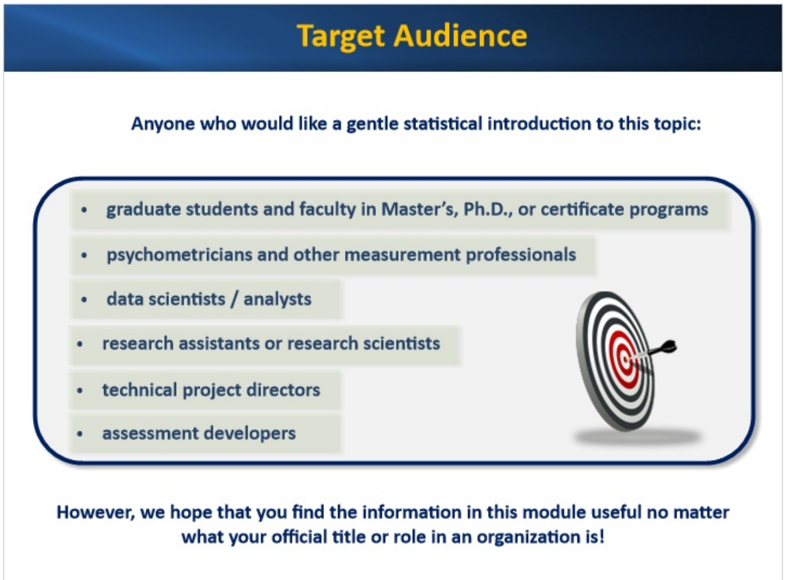
1.5 Overview

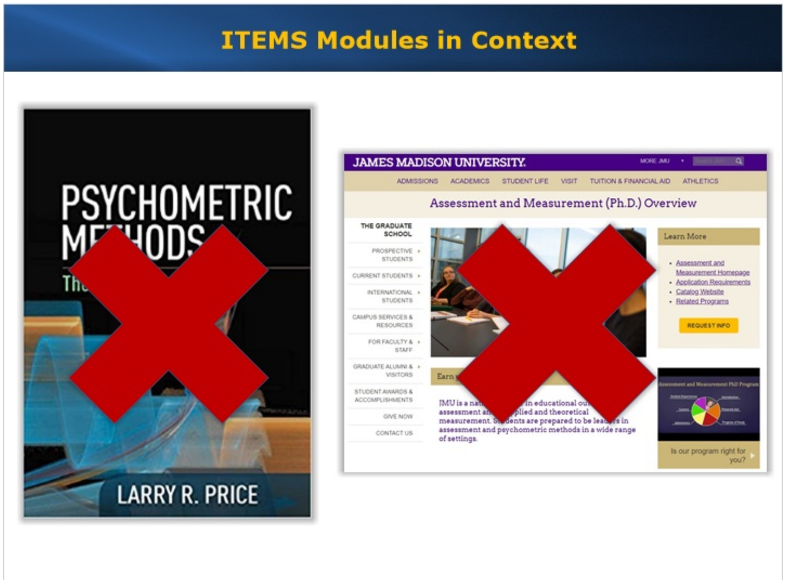


1.6 Target Audience

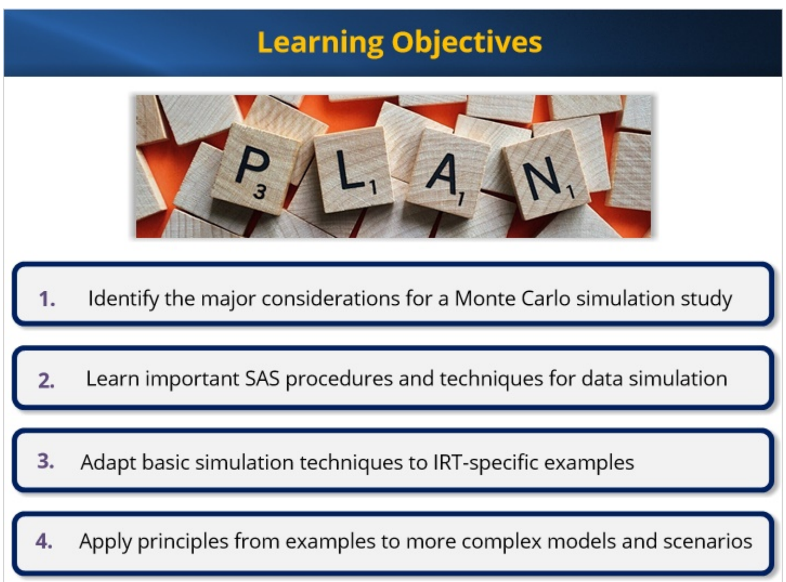


1.7 Expecations (I)

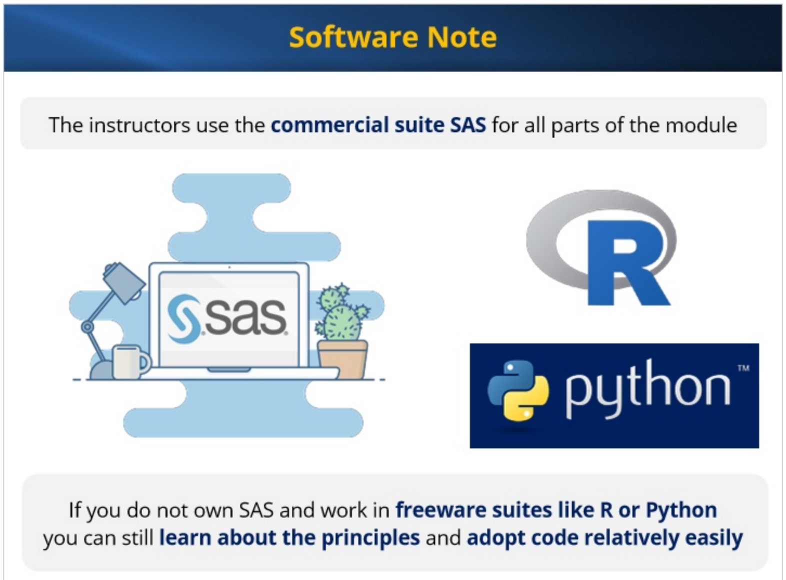
1.8 Expectations (II)



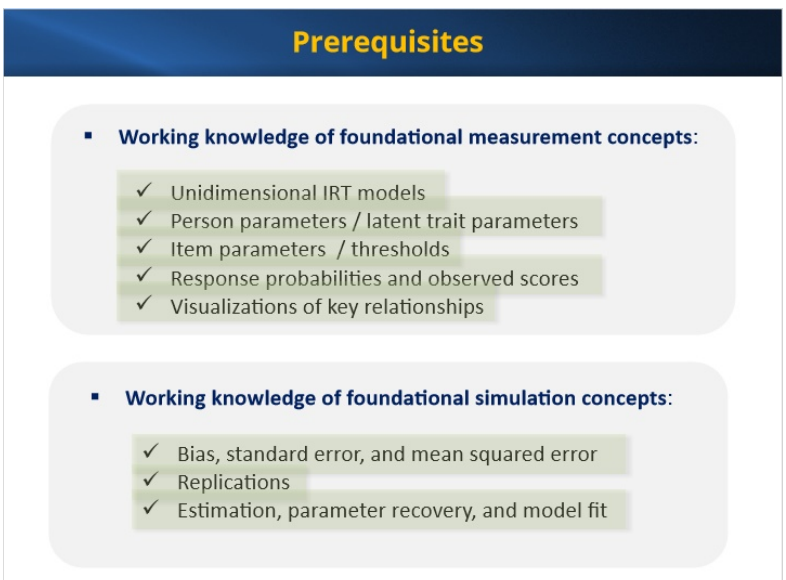
1.9 Learning Objectives



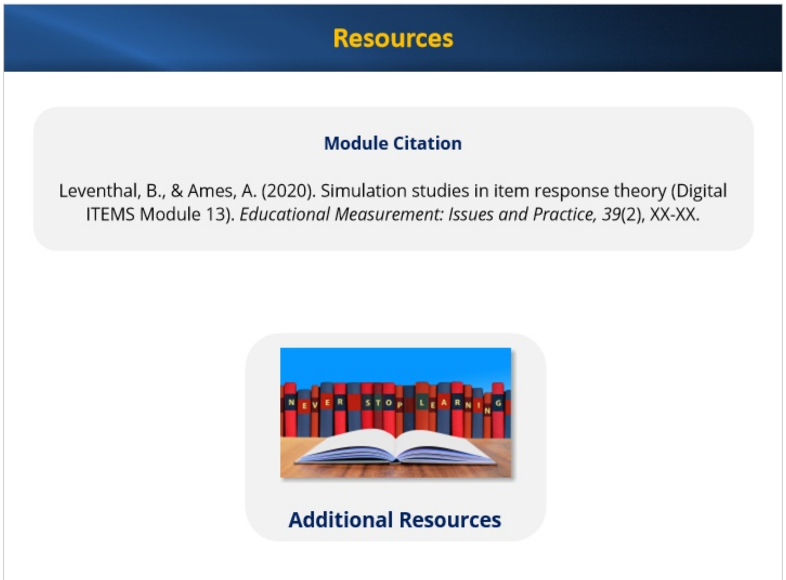
1.10 Software Note



1.11 Prerequisites



1.12 Resources



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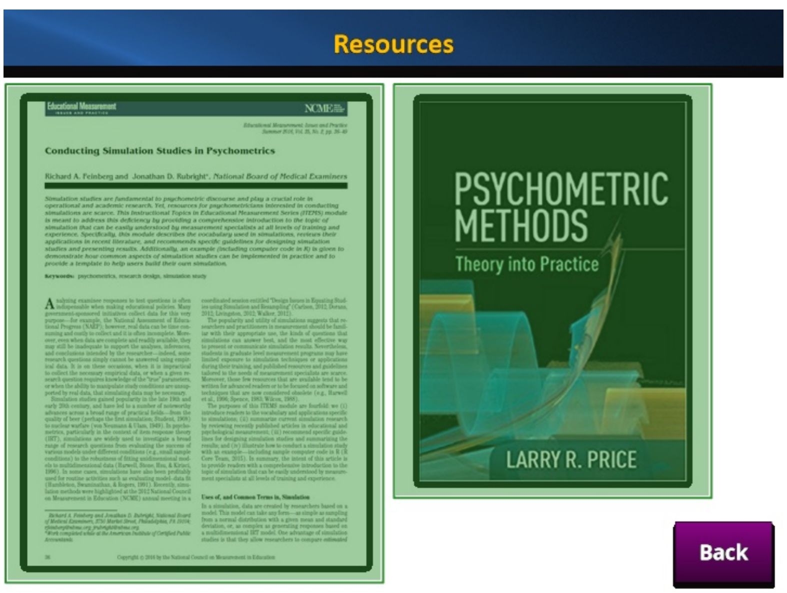
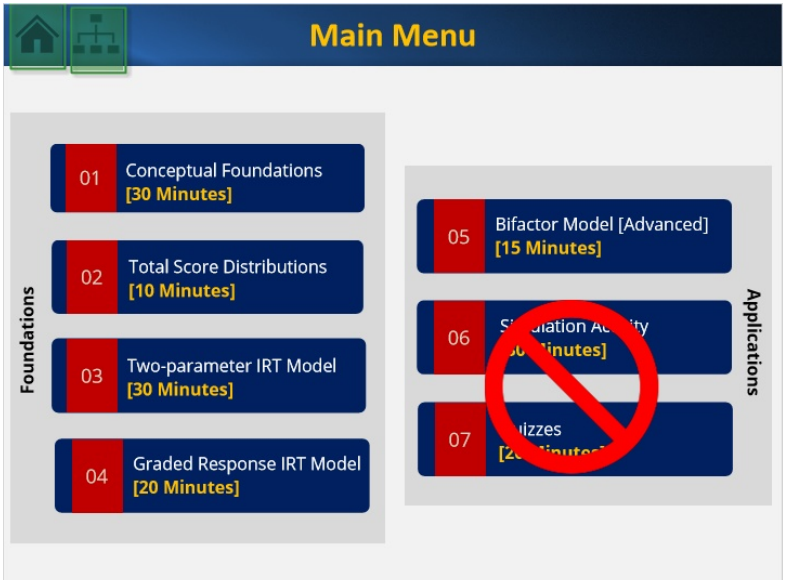


1.13 Main Menu



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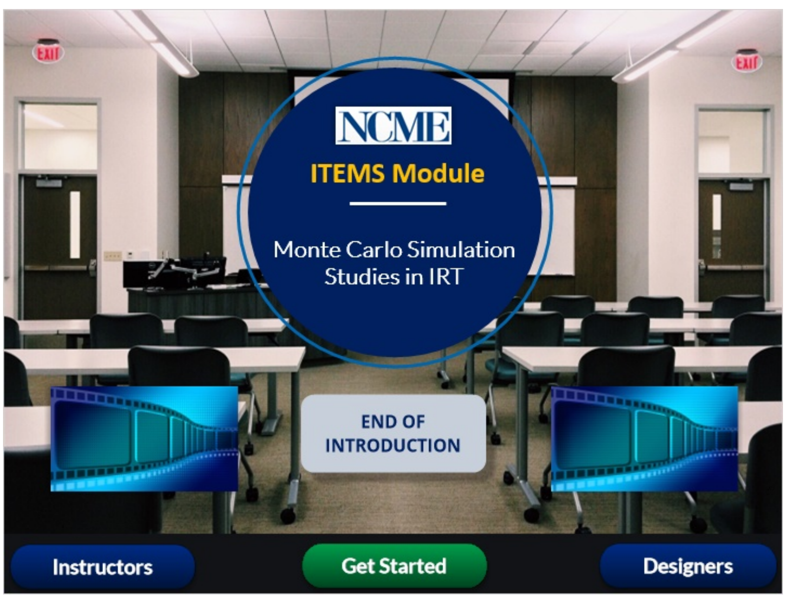
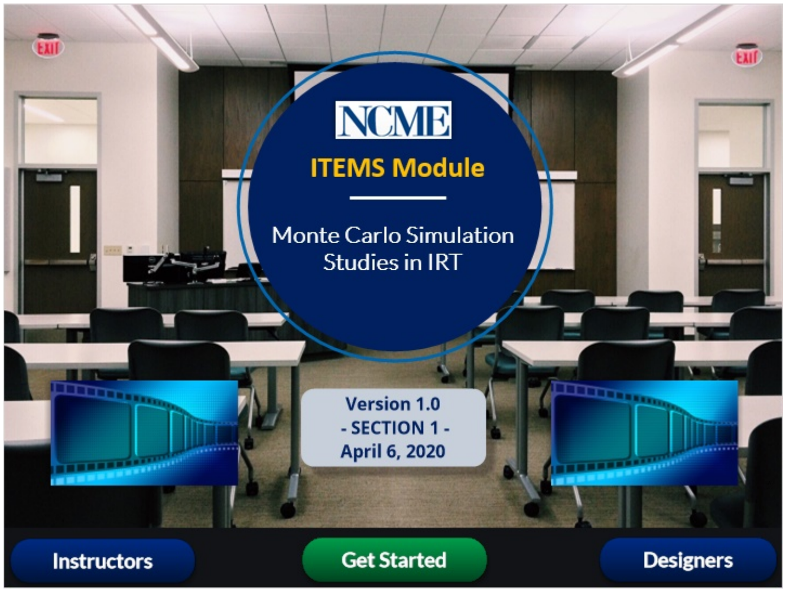


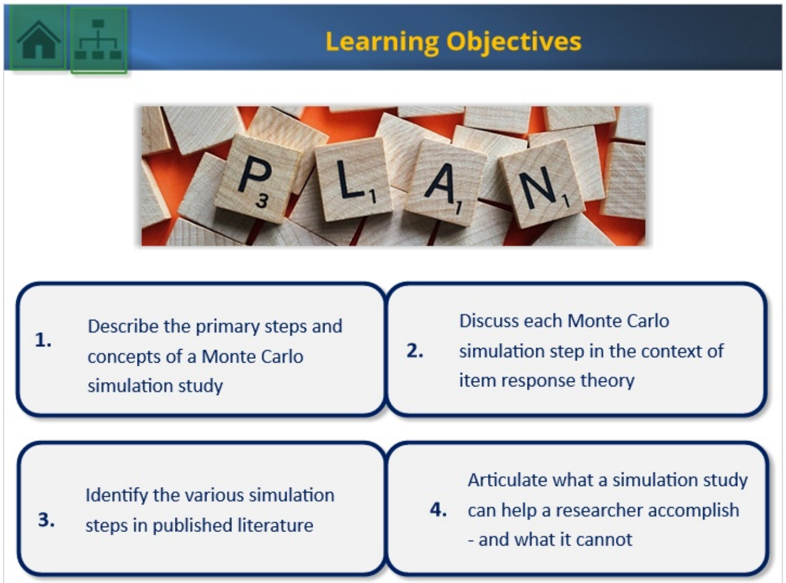
2. Section 1: Conceptual Foundations

2.1 Module Cover (START)

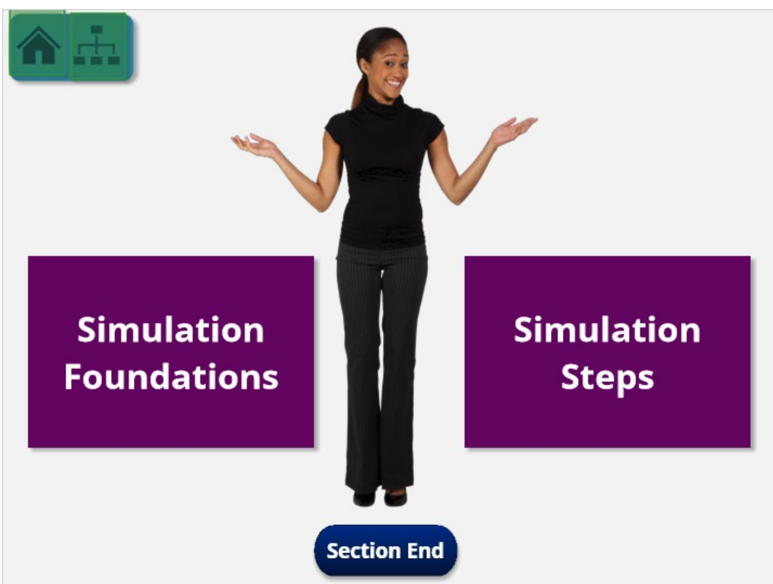


2.2 Cover: Section 1

2.3 Objectives



2.4 Topic Selection

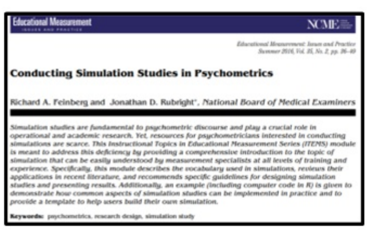


2.5 Bookmark: Foundations

2.6 Important Resources

Important Resources

- General resources:
  - Feinberg, R.R., & Rubright, J.D. (2016). Conducting simulation studies in psychometrics. *Educational Measurement: Issues and Practice, 35*(2), 36–49.
  - Harwell, M., Stone, C.A., Hsu, T., & Kirisci, L. (1996). Monte Carlo studies in item response theory. *Applied Psychological Measurement, 20*(2), 101–125.
- Application of a simulation:
  - Drasgow, F. (1989). An evaluation of marginal maximum likelihood estimation for the two-parameter logistic model. *Applied Psychological Measurement, 13*, 77-90.


Educational Measurement ISSUES AND PRACTICE

NCME

Educational Measurement: Issues and Practice
Summer 2016, Vol. 35, No. 2, pp. 36–49

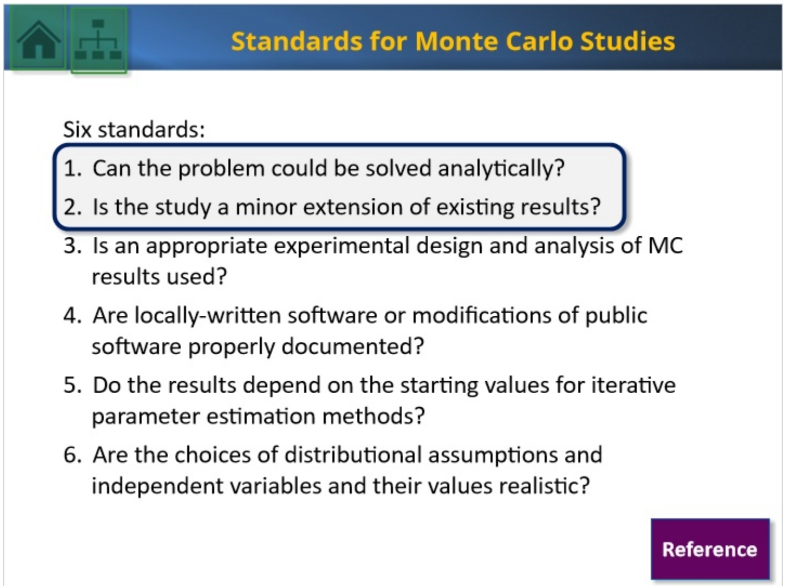
Conducting Simulation Studies in Psychometrics

Richard A. Feinberg and Jonathan D. Rubright*, *National Board of Medical Examiners*

Simulation studies are fundamental to psychometric discourse and play a crucial role in operational and academic research. Yet, resources for psychometricians interested in conducting simulations are scarce. This Instructional Topics in Educational Measurement Series (ITEMS) module is meant to address this deficiency by providing a comprehensive introduction to the topic of simulation that can be easily understood by measurement specialists at all levels of training and experience. Specifically, this module describes the vocabulary used in simulations, reviews their applications in recent literature, and recommends specific guidelines for designing simulation studies and presenting results. Additionally, an example (including computer code in R) is given to demonstrate how common aspects of simulation studies can be implemented in practice and to provide a template to help users build their own simulation.

Keywords: psychometrics, research design, simulation study

2.7 Standards



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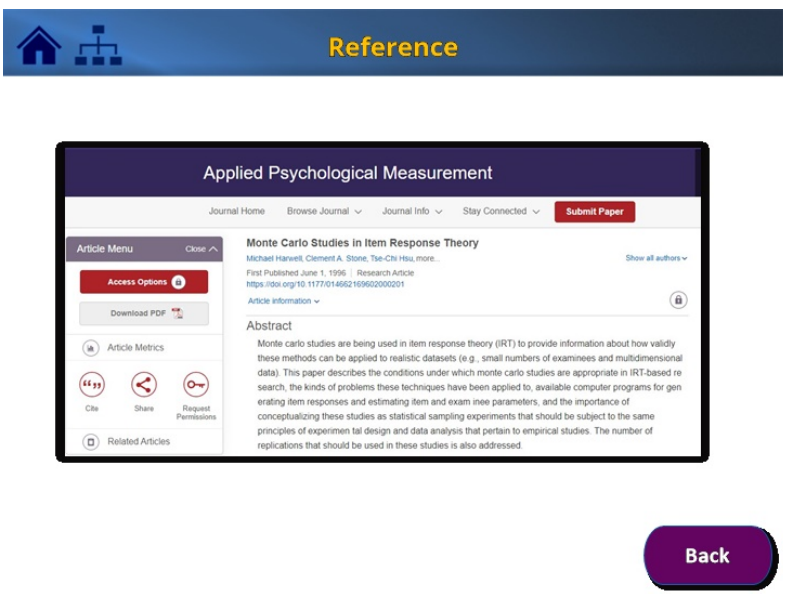
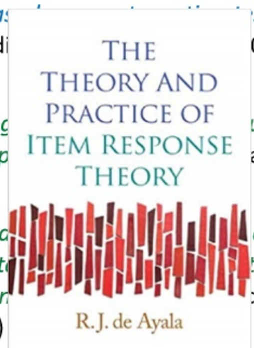


2.8 Introduction (I)

Introduction to Simulation Studies

❖*"As few as 200 persons and five items were required for 'essentially' unbias* … *s [for the 2PL]*." (Drasgow, 1989; di … 09)



❖*"The 2PL equating* … *ust to the violation of local item indep* … ao, 2014)

❖*"A 2:1 or larger ra* … *arameters produced stable ite* … *ter estimates [for the partial credit m* … ck, 1990; discussed in de Ayala, 2009)

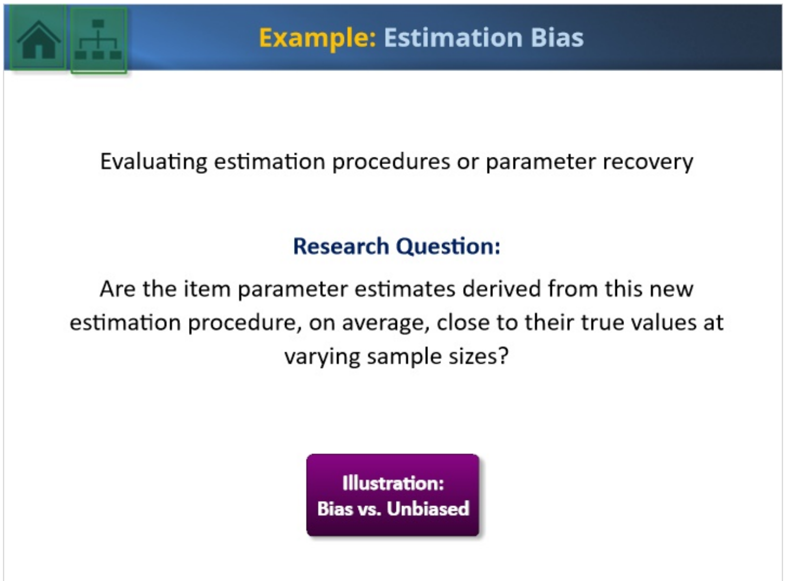
2.9 Introduction (II)

Introduction to Simulation Studies

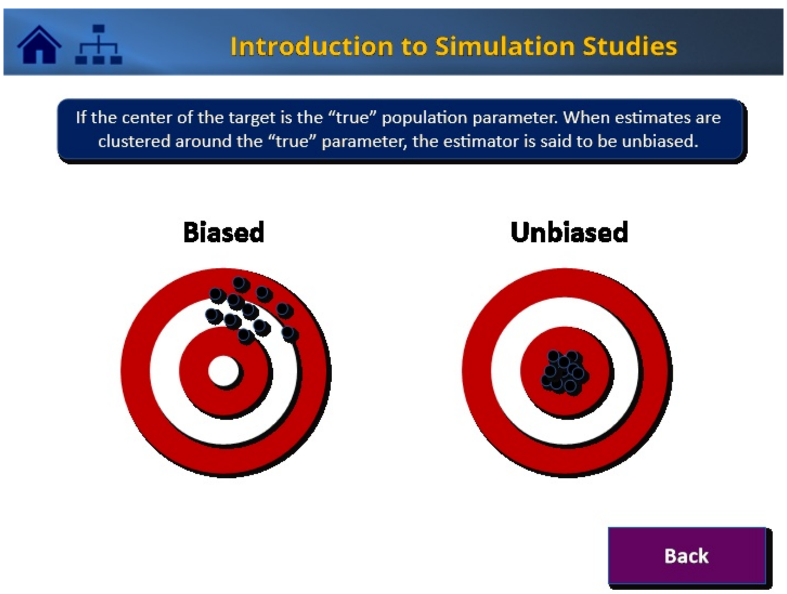
- **How were these sample size heuristics and claims determined?**
  - ✓ Monte Carlo simulation (MCS) techniques were used.
  - ✓ In MCS, data are created by researchers based on a model and used to answer a methodological research question.

- **Applications of MCS techniques in IRT:**
  - ✓ Evaluating estimation procedures or parameter recovery,
  - ✓ Evaluating the statistical properties of an IRT-based statistic, or
  - ✓ Comparing methodologies used in conjunction with IRT.

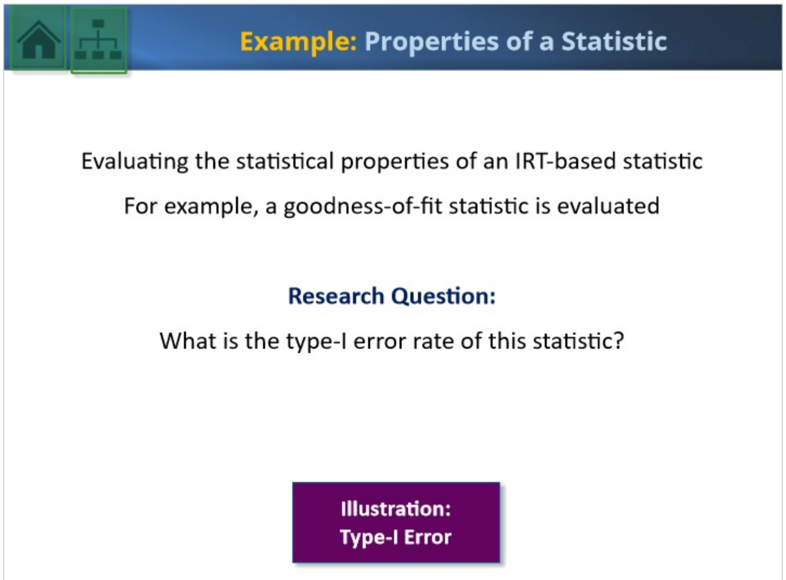
2.10 Example: Bias



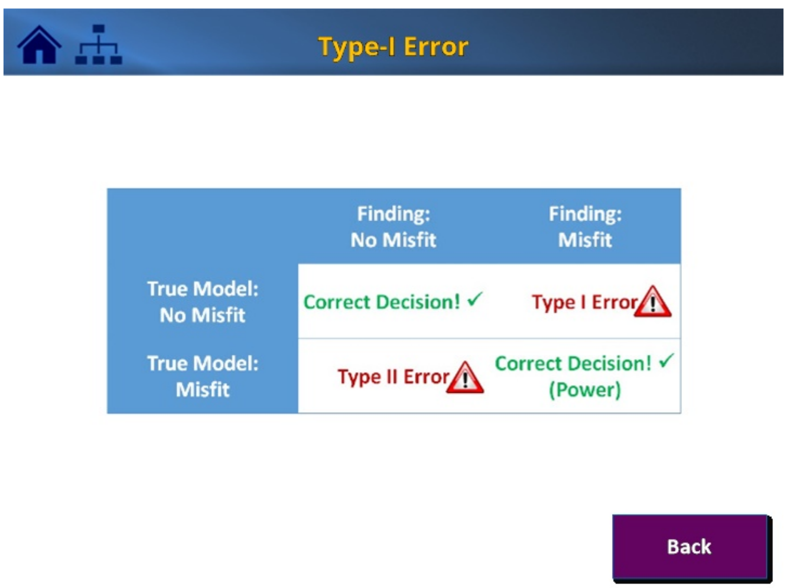
Introduction to Simulation Studies (Slide Layer)



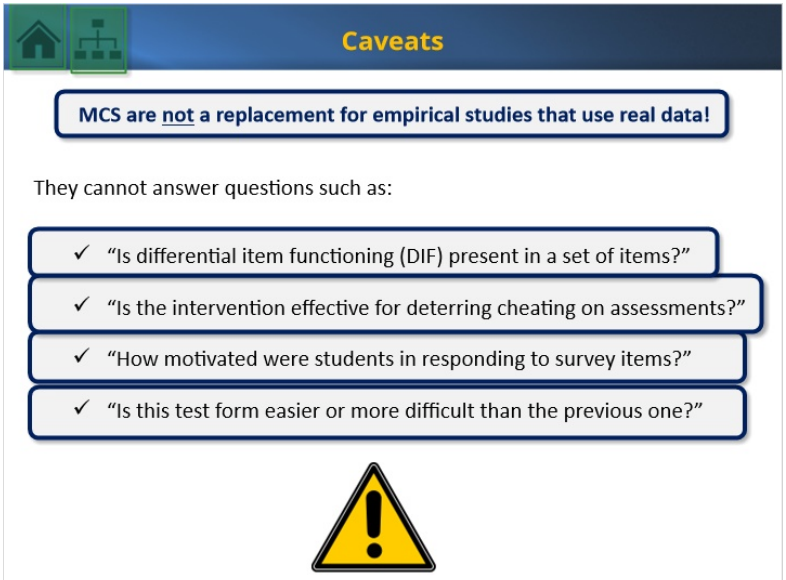
2.11 Example: Type-I Error



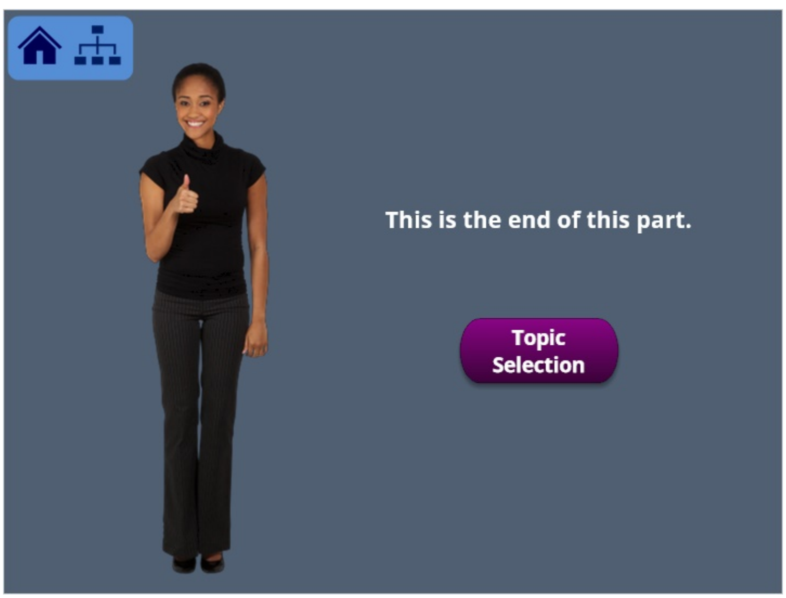
Type-I Error (Slide Layer)



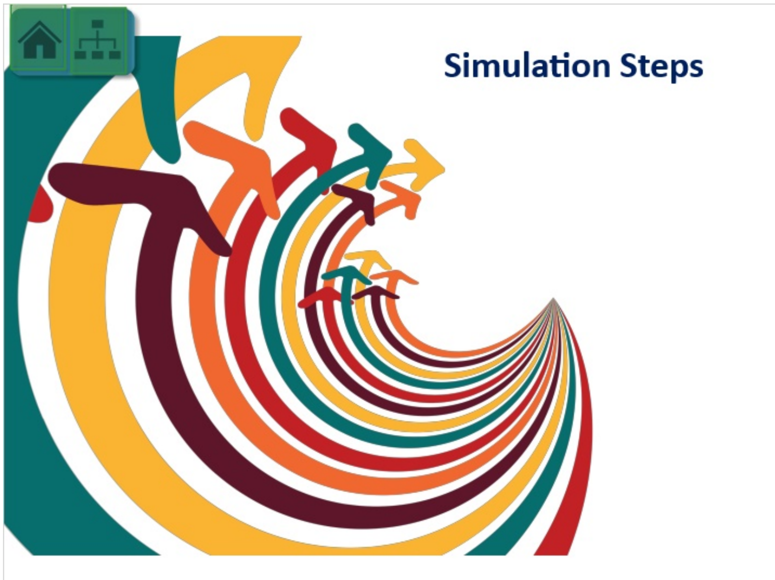
2.12 Caveats



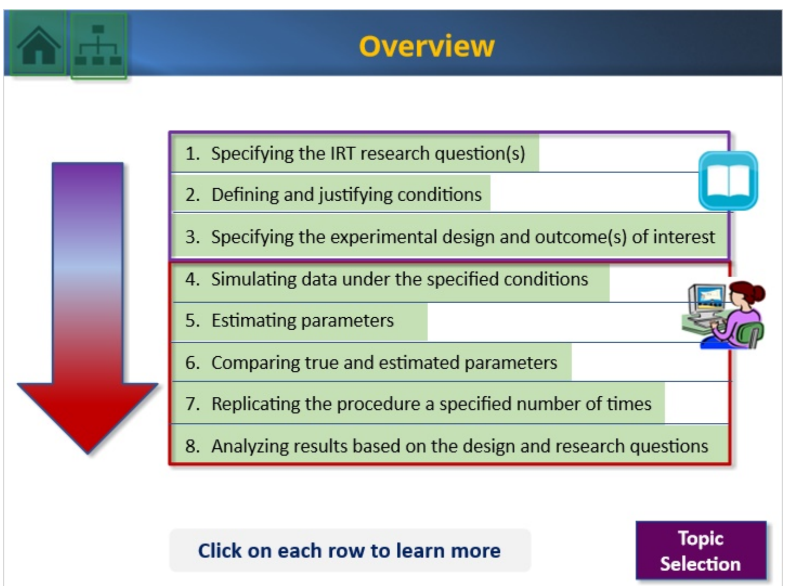
2.13 Bookend: Foundations



2.14 Bookmark: Simulation Steps



2.15 Step Selection



2.16 Research Questions (I)

Step 1: Research Question(s)

The research question will guide all other aspects of the simulation

- *"Articulating a clear research question … forces the researcher to ensure that the exact design choices made align with the question being asked."* Feinberg & Rubright, 2016

- *"… relies heavily on knowledge of a literature."* Harwell, Stone, Hsu, & Kirisci, 1996

2.17 Research Questions (II)

Step 1: Example

Drasgow (1989) provides background from a literature review:

- Large sample sizes and long tests
- Common situations in which these are unrealistic or untenable

Example research question from Drasgow (1989):

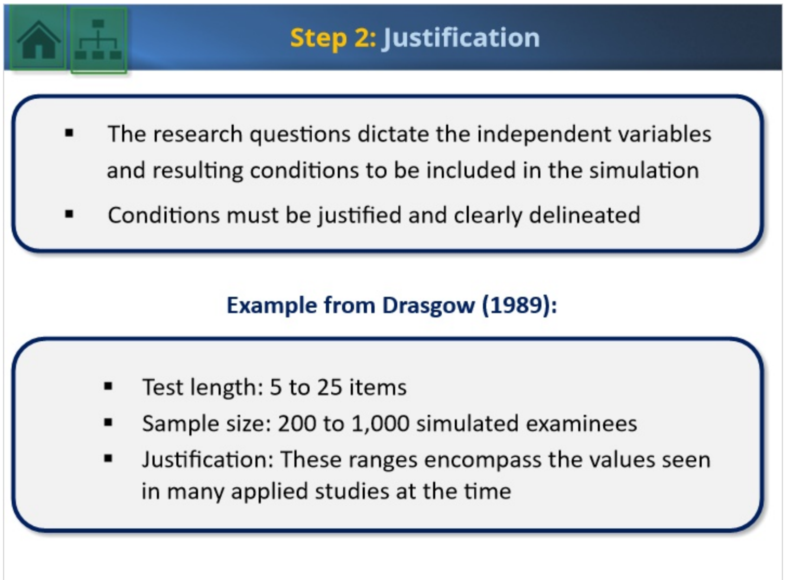
What is the range of conditions that allow accurate calibration of two-parameter logistic items by marginal maximum likelihood estimation?

Reference

Reference (Slide Layer)

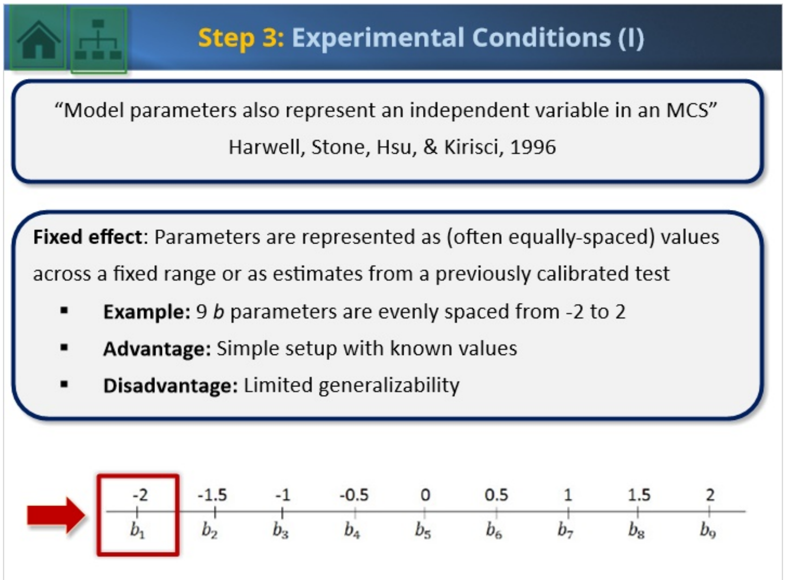
2.18 Bookend: Step 1

2.19 Justifying Conditions

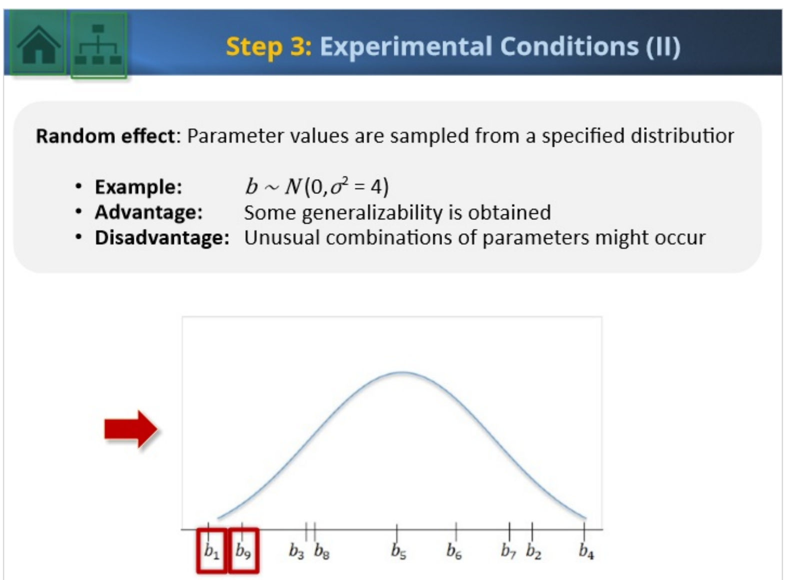


2.20 Bookend: Step 2

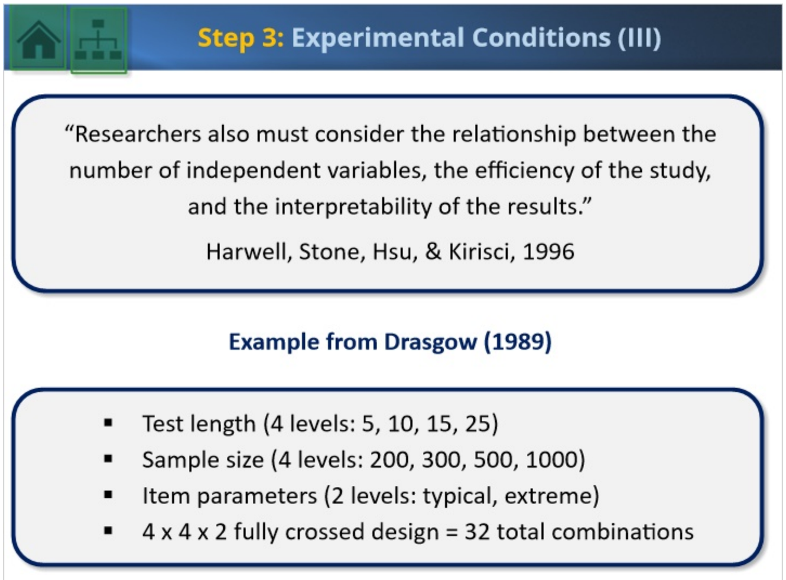
2.21 Experimental Design (I)



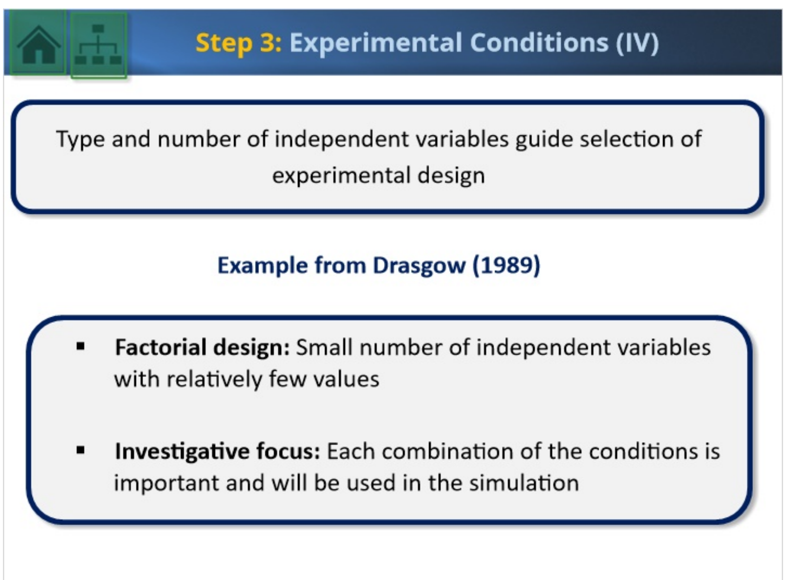
2.22 Experimental Design (II)



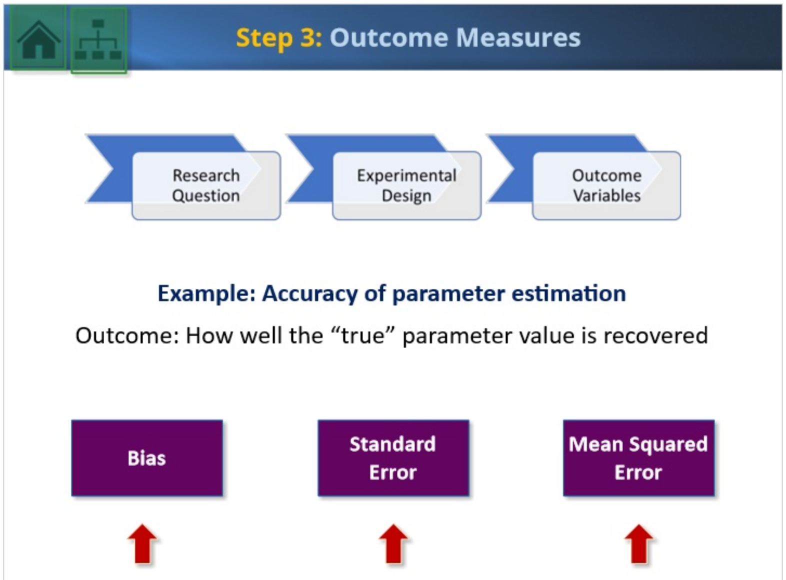
2.23 Experimental Design (III)



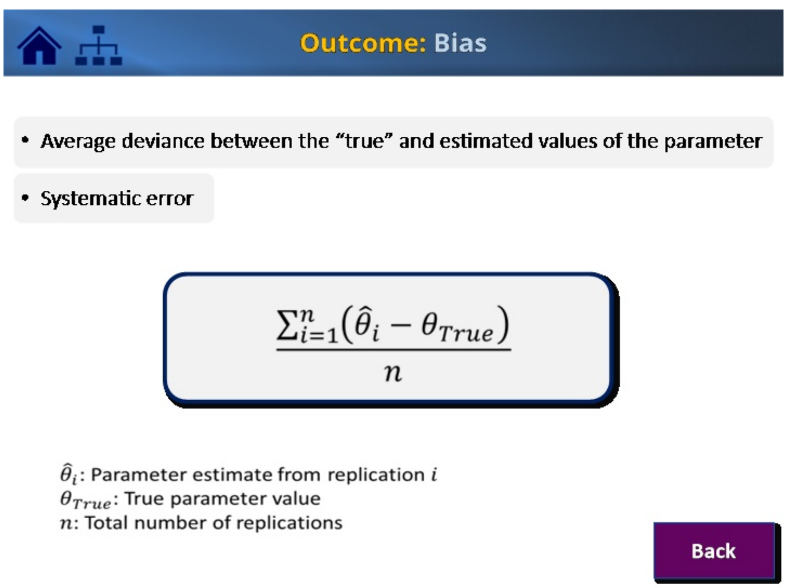
2.24 Experimental Design (IV)



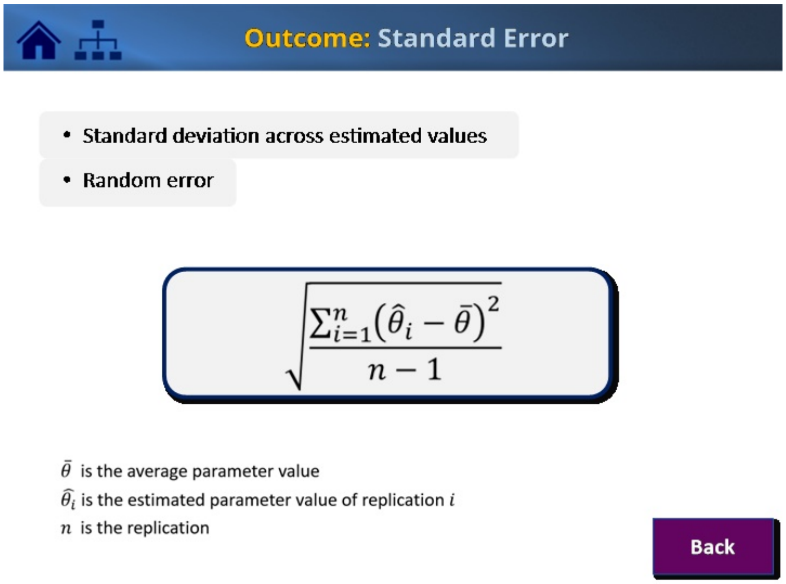
2.25 Experimental Design (V)



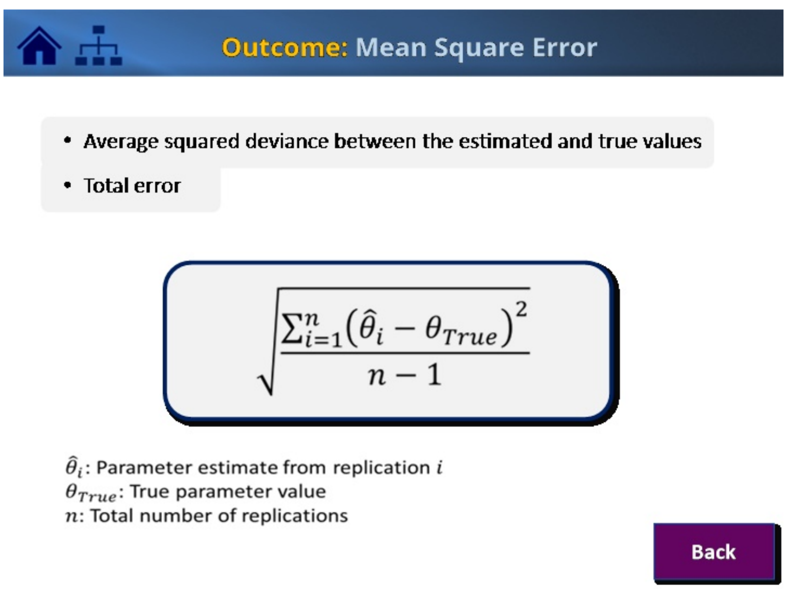
Bias (Slide Layer)



Standard Error (Slide Layer)

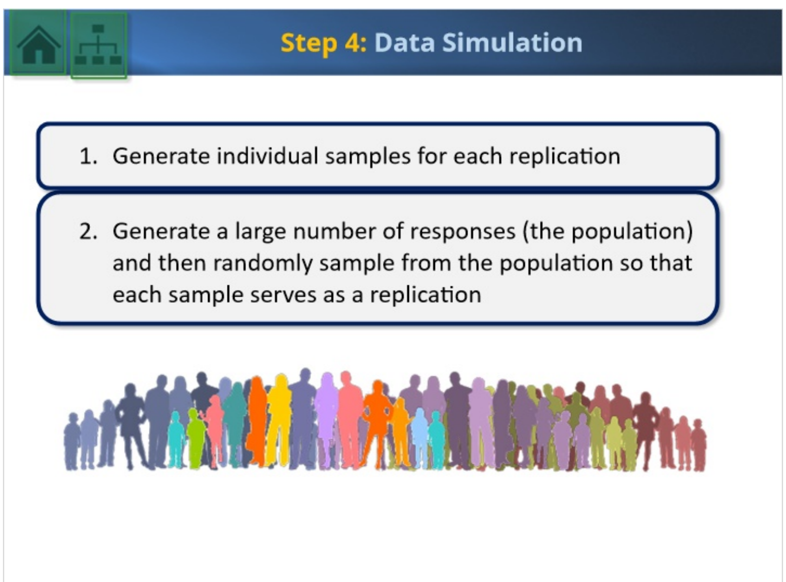


Mean Squared Error (Slide Layer)

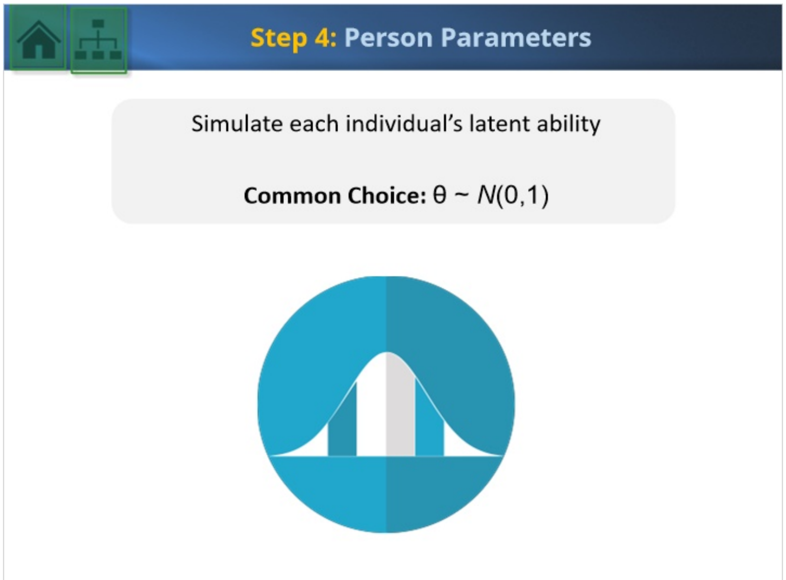


2.26 Bookend: Step 3

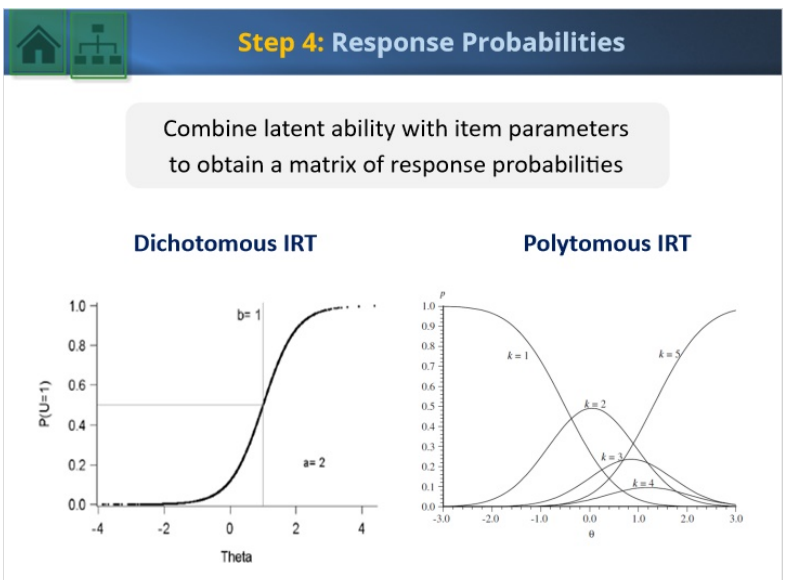
2.27 Simulating Data (I)



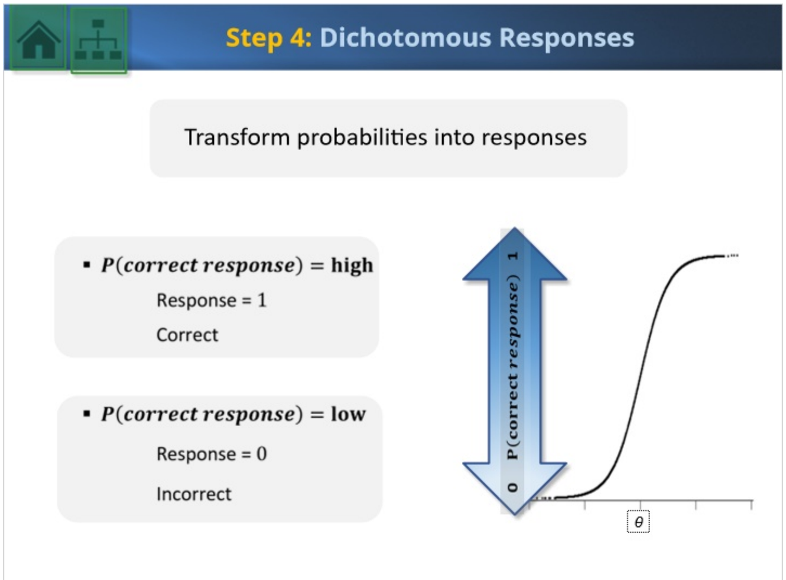
2.28 Simulating Data (II)



2.29 Simulating Data (III)



2.30 Simulating Data (IV)



2.31 Bookend: Step 4

2.32 Estimating Parameters (I)

Step 5: Parameter Estimation

- **Available tools:**
  - ✓ Commercially available programs (e.g., Winsteps or Bilog)
  - ✓ Open source packages (e.g., R package ltm or Stan)
  - ✓ Their own routine (e.g., Fortran, SAS, or Basic)
- **Adequacy** of the estimation algorithm must be documented
- **Validation evidence examples:**
  - ✓ Using the program to analyze a well-known dataset
  - ✓ Analyzing item responses with perfect fit to an item response function in which case the estimation routine should return the exact item parameters

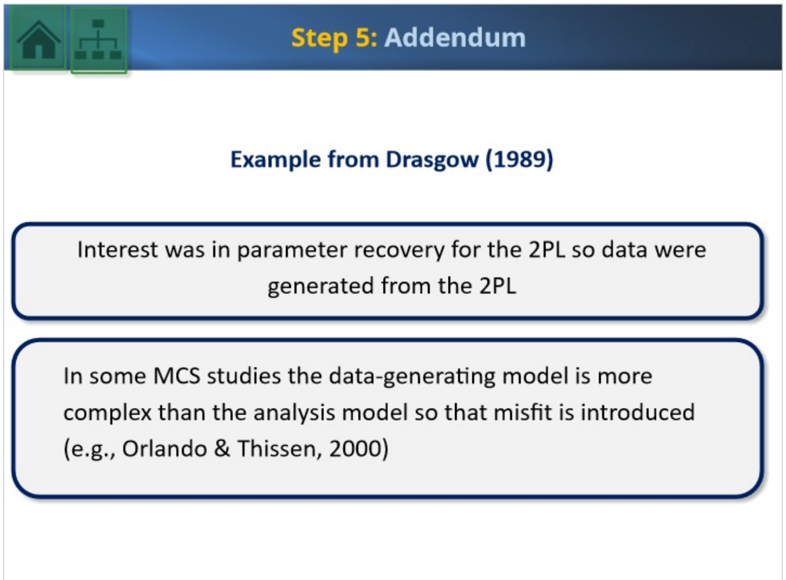
2.33 Estimating Parameters (II)

Step 5: Example

Example from Drasgow (1989)

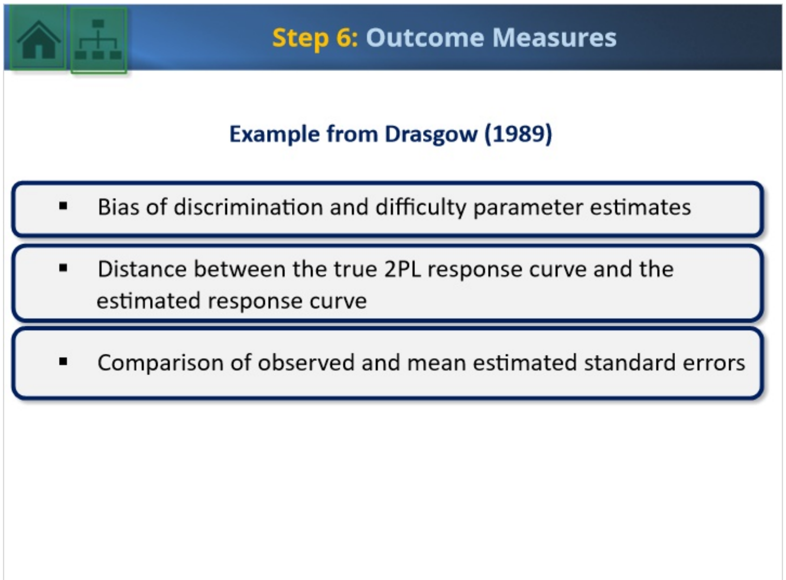
- A FORTRAN program was written to estimate the *a* (discrimination) and *b* (difficulty) parameters of the 2PL
- Validity evidence was not provided

2.34 Estimating Parameters (III)



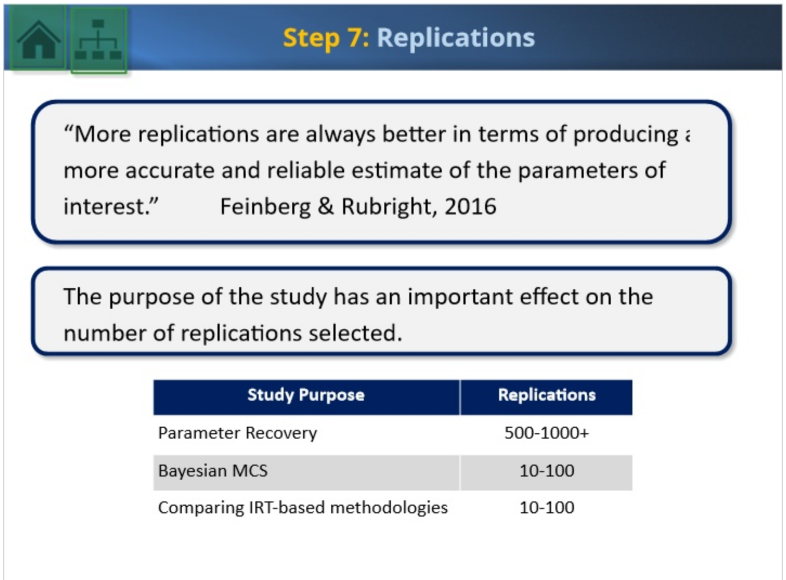
2.35 Bookend: Step 5

2.36 Outcome Measures



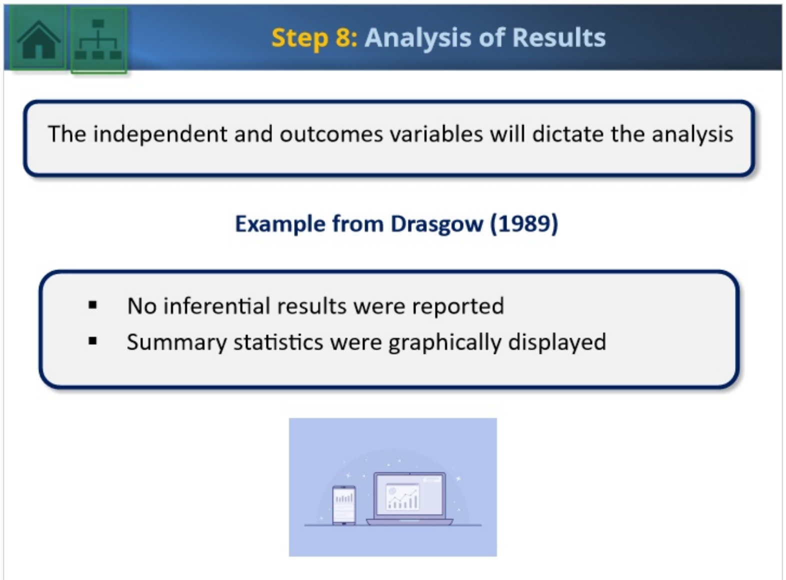
2.37 Bookend: Step 6

2.38 Replications



2.39 Bookend: Step 7

2.40 Analyzing Results



2.41 Bookend: Step 8

2.42 Simulation Standards

Standards for Monte Carlo Studies

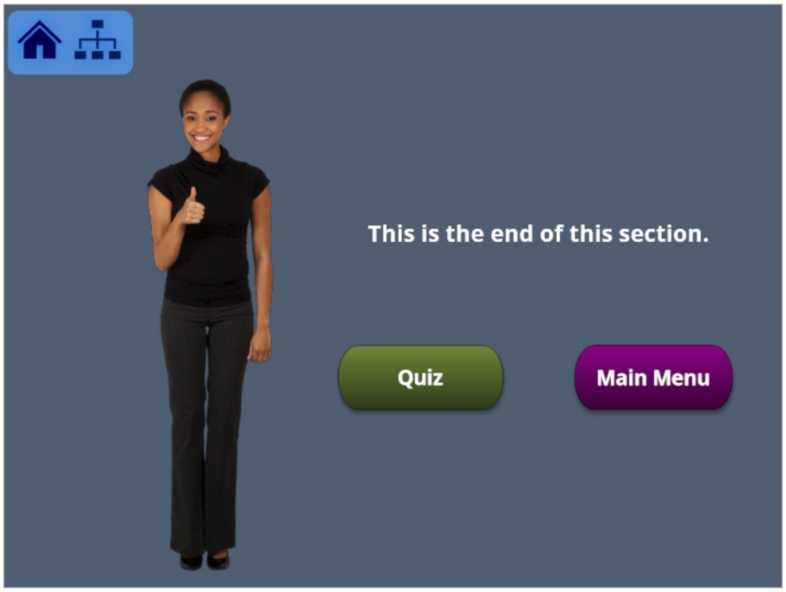
1. Can the problem could be solved analytically?
2. Is the study a minor extension of existing results?
3. Is an appropriate experimental design and analysis of MC results used?
4. Are locally-written software or modifications of public software properly documented?
5. Do the results depend on the starting values for iterative parameter estimation methods?
6. Are the choices of distributional assumptions and independent variables and their values realistic?

2.43 Summary

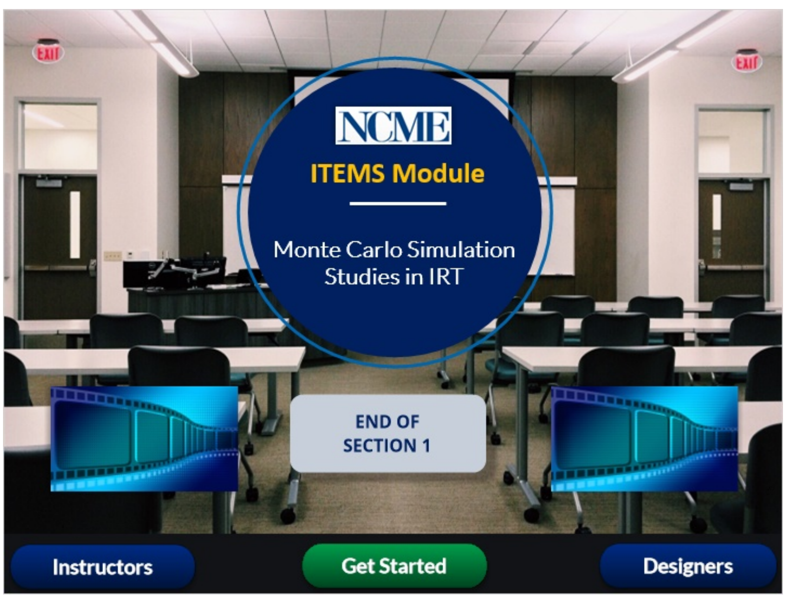
Summary

- Important steps of Monte Carlo simulation studies:
  - ✓ Always begins with a research question, and all aspects follow from the question
  - ✓ Simulations are not a substitution for empirical work as they are designed to answer methodological questions
- Demonstrated all steps from Drasgow's 1989 parameter recovery study
- Next steps: implement the software steps in SAS

2.44 Bookend: Section 1

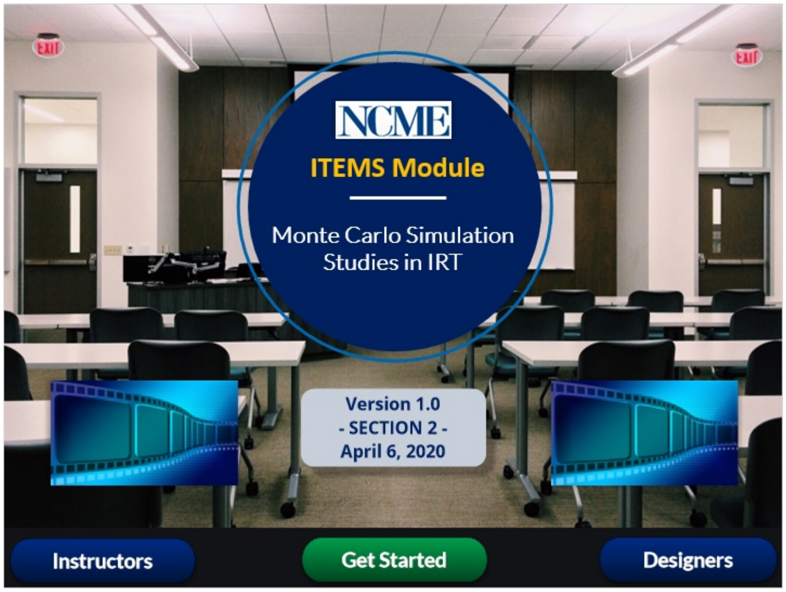


2.45 Module Cover (END)

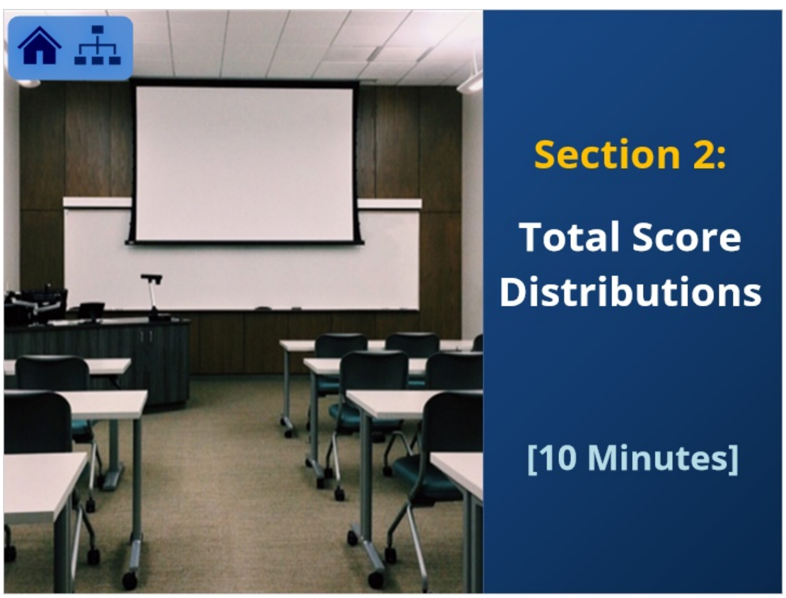


3. Section 2: Total Score Distributions

3.1 Module Cover (START)

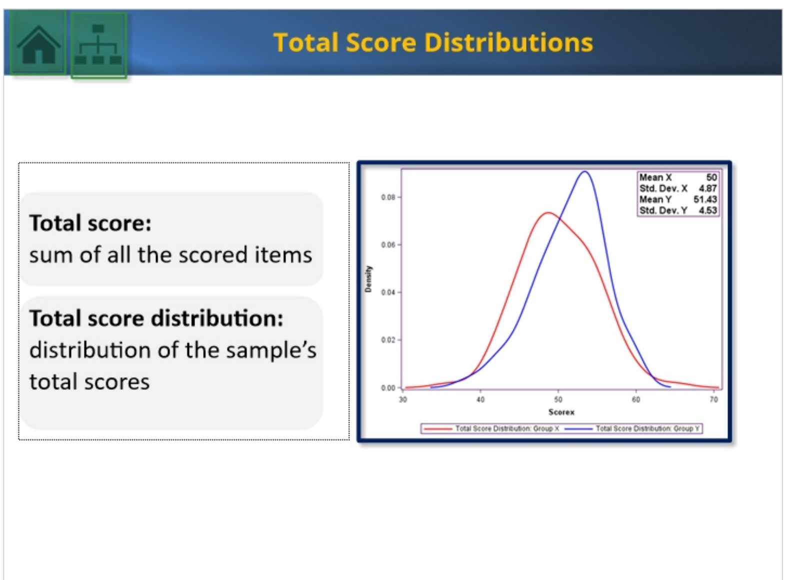


3.2 Cover: Section 2

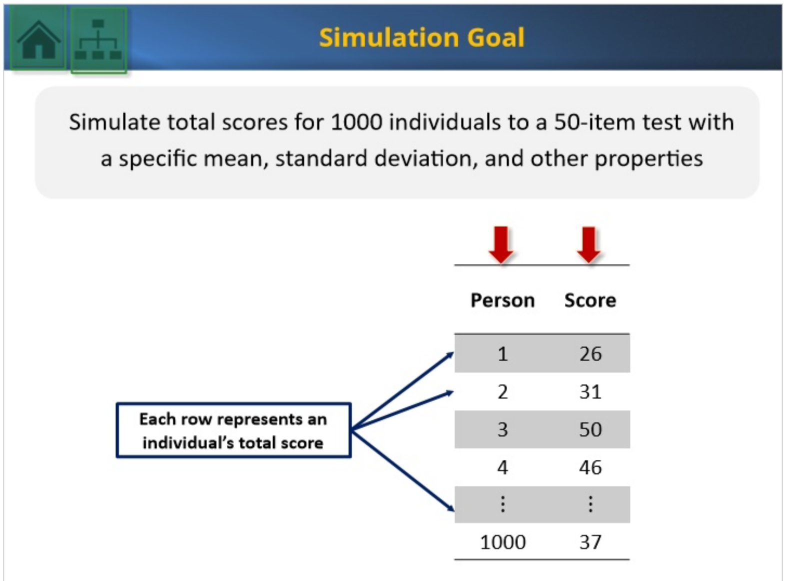


3.3 Objectives

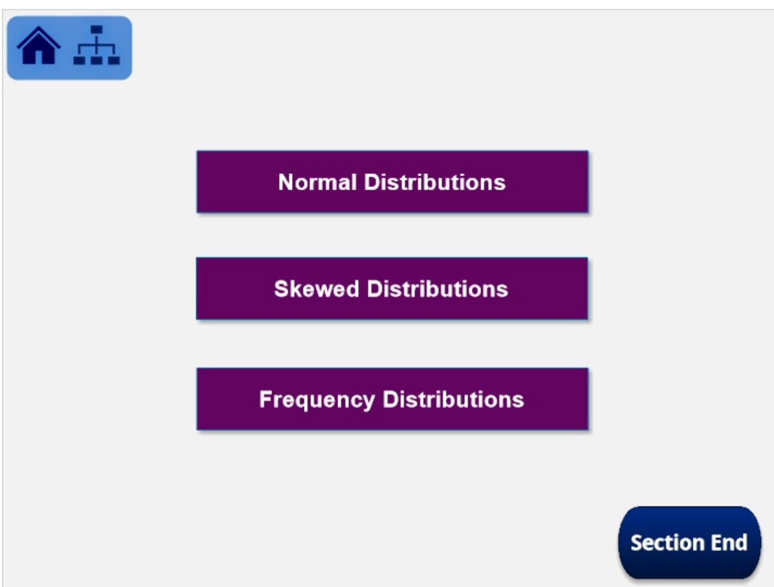
3.4 Total Score Distributions



3.5 Simulation Goal



3.6 Topic Selection



3.7 Bookmark: Normally Distributed Data

3.8 Standard Normal (I)

Generate Random Normal Values

```
x = rand("Normal",50,5);
```

Yields normally distributed data (mean = 50, standard deviation of = 5)

Result:

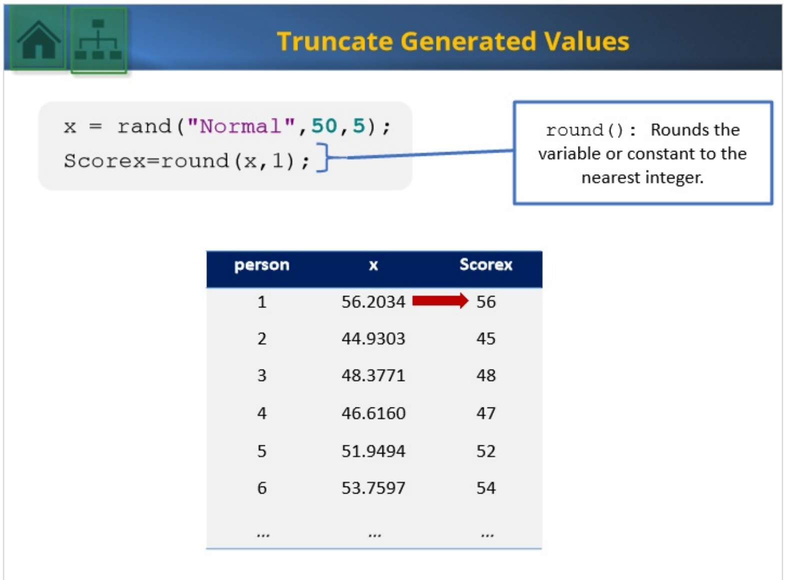
person	x
1	56.2034
2	44.9303
3	48.3771
4	46.6160
5	51.9494
6	53.7597
...	...

Test scores are typically recorded in whole numbers, not decimals.

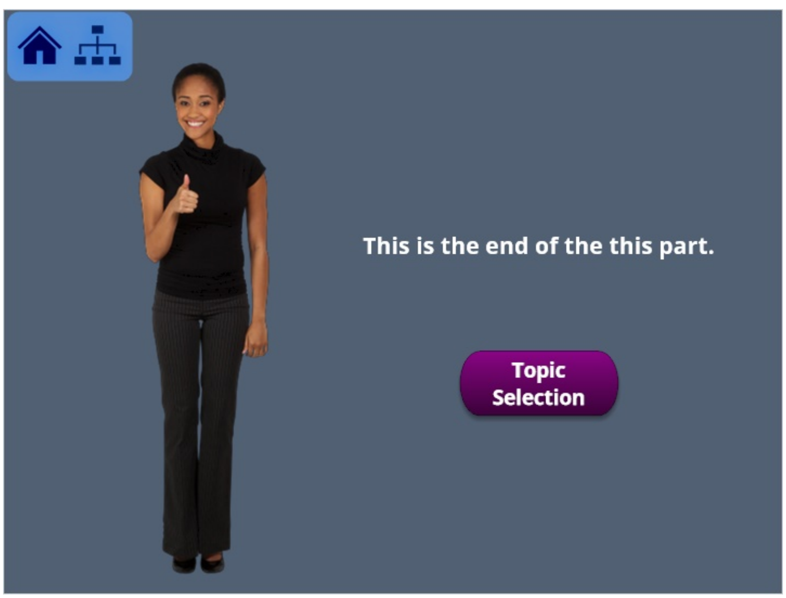
We want:

person	x
1	56
2	45
3	48
4	47
5	52
6	54
...	...

3.9 Standard Normal (II)

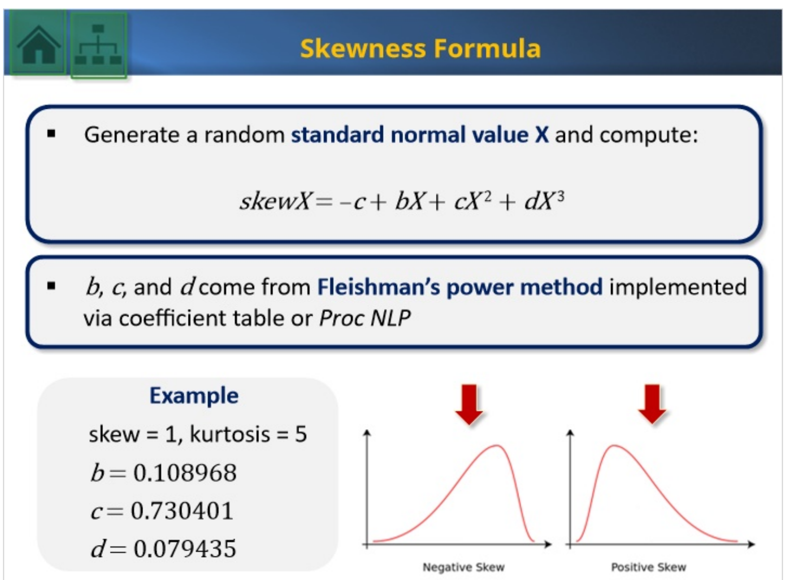


3.10 Bookend: Normal Distributions

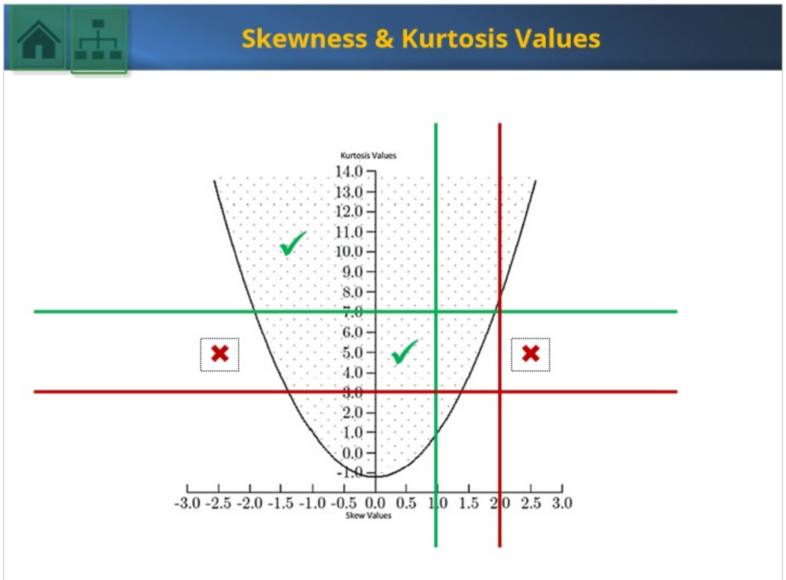


3.11 Bookmark: Skewed Data

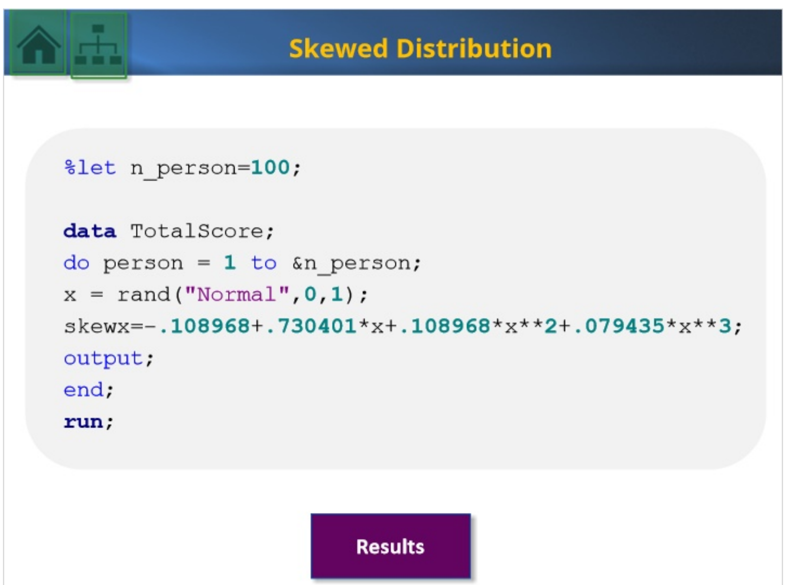
3.12 Skewed Distribution (I)



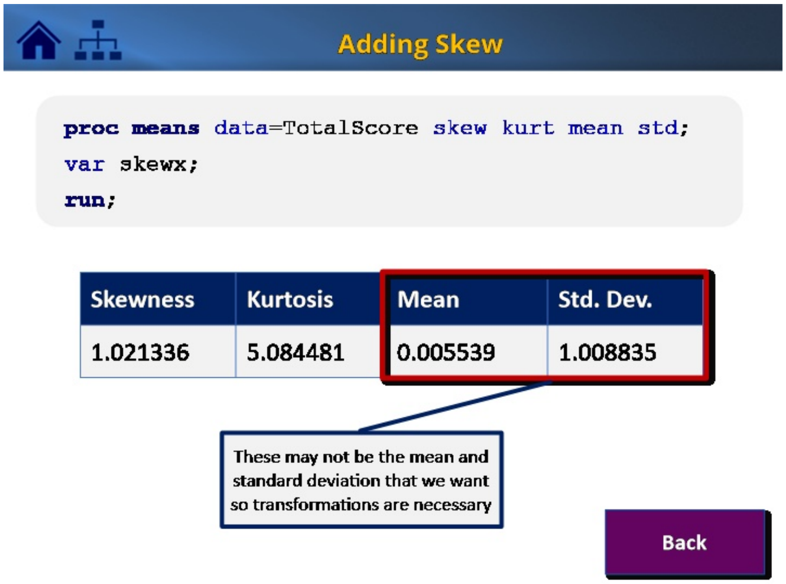
3.13 Skewed Distribution (II)



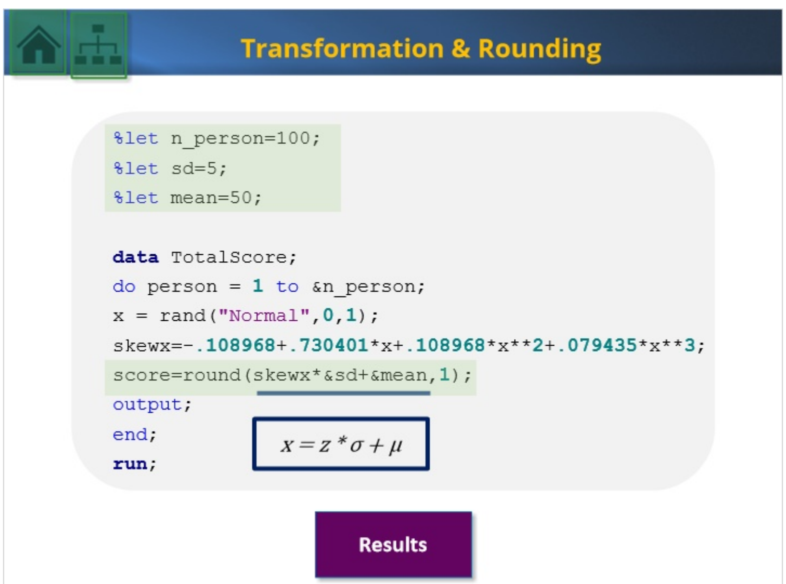
3.14 Skewed Distribution (III)



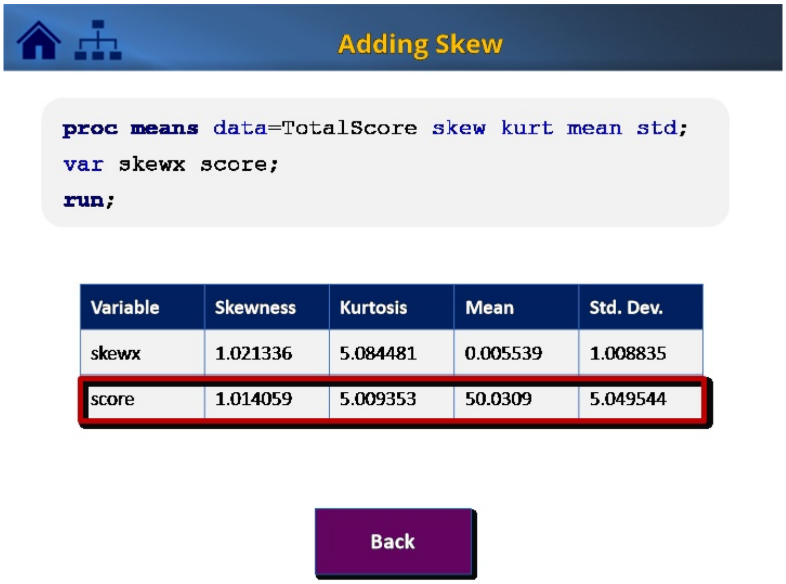
Results (Slide Layer)



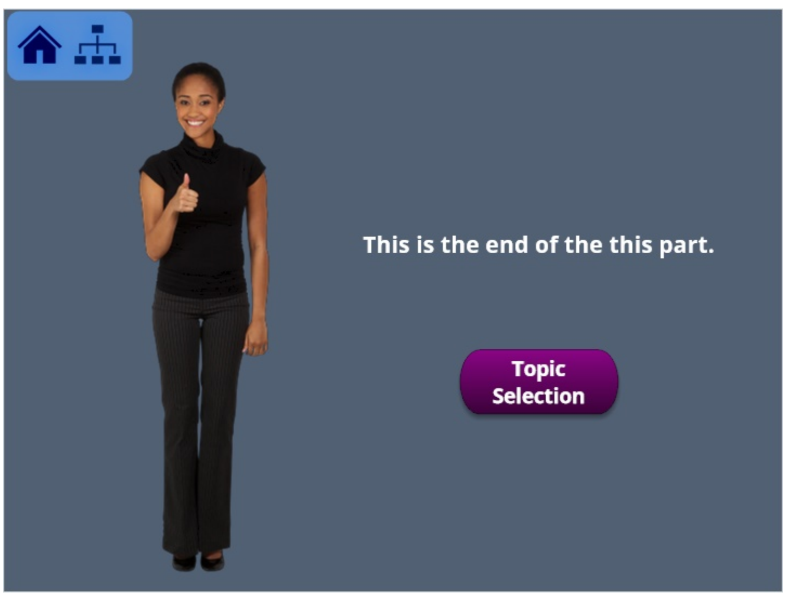
3.15 Skewed Distribution (IV)



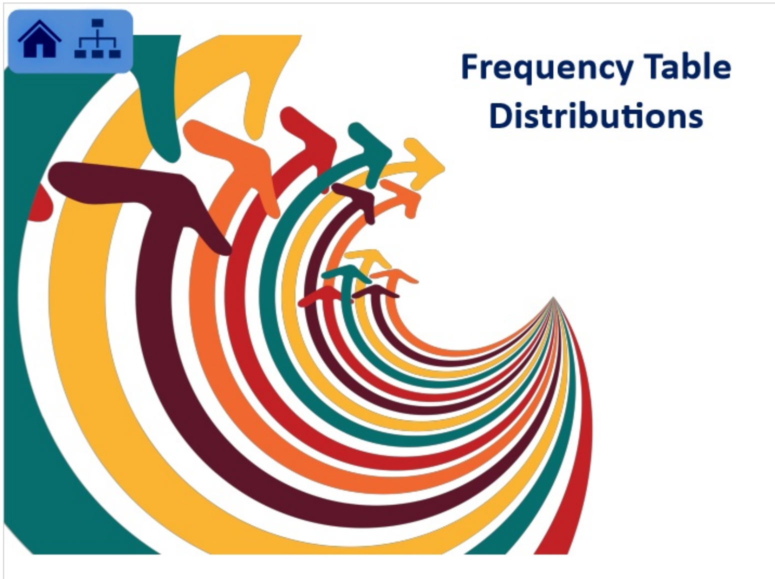
Results (Slide Layer)



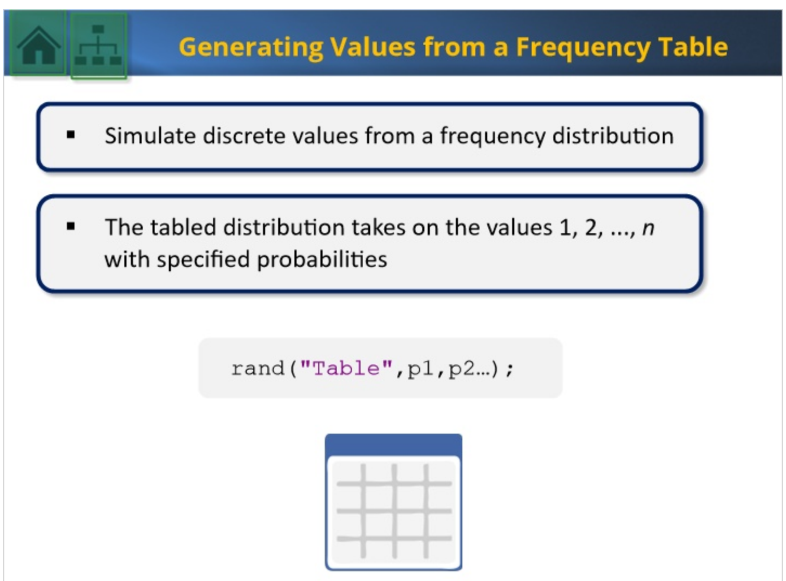
3.16 Bookend: Skewed Distributions



3.17 Bookmark: Data from Freq Distribution



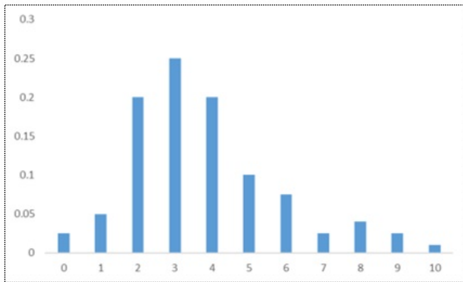
3.18 Frequency Table (I)



3.19 Frequency Table (II)

Example

Score	p
0	0.025
1	0.050
2	0.200
3	0.250
4	0.200
5	0.100
6	0.075
7	0.025
8	0.040
9	0.025
10	0.010



3.20 Frequency Table (III)

Implementation

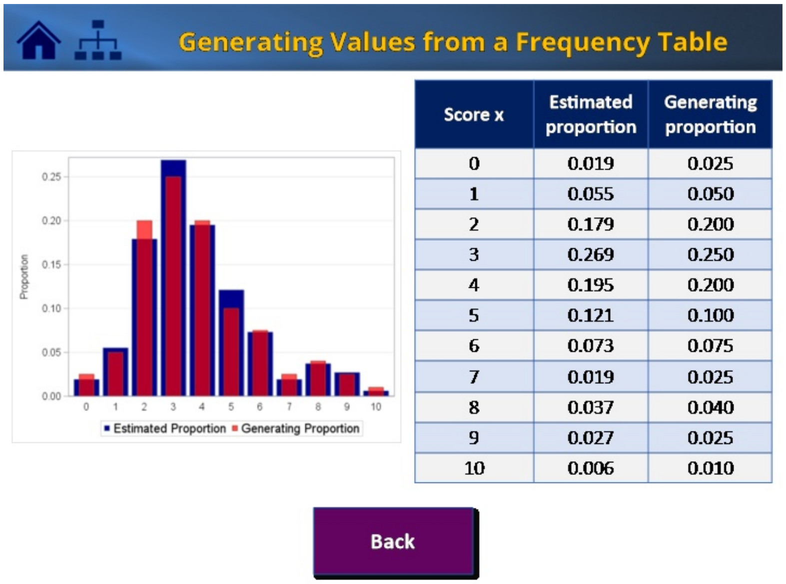
```
data CatScore;
do person = 1 to &n_person;
Score = rand("Table",
0.025,0.05,0.2,0.25,0.2,0.1,0.075,0.025,0.04,0.025,0.01);

Scorex=Score-1;

output;
end;
run;
```

Vector of the probability of scores from 0 to 1

SAS simulates scores from 1 to 11 but we want them from 0 to 10 so we need to subtract 1

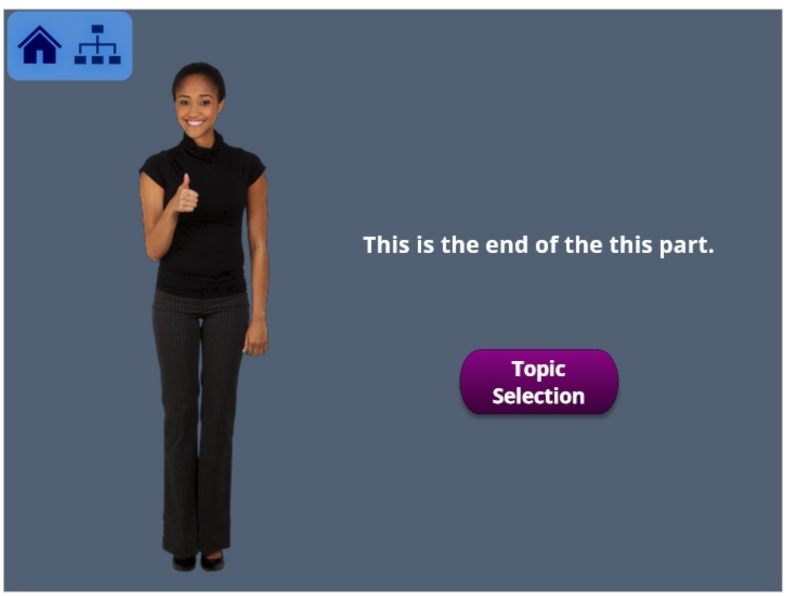
Results

Results (Slide Layer)

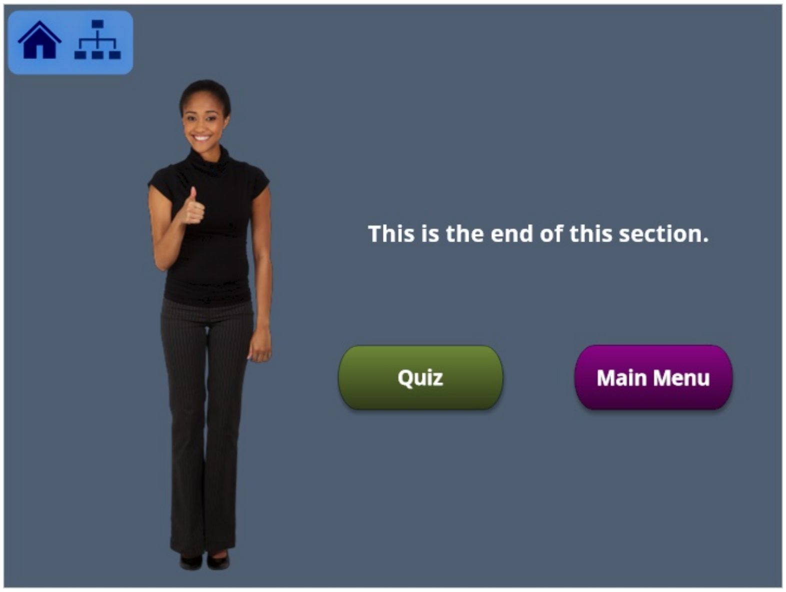


Score x	Estimated proportion	Generating proportion
0	0.019	0.025
1	0.055	0.050
2	0.179	0.200
3	0.269	0.250
4	0.195	0.200
5	0.121	0.100
6	0.073	0.075
7	0.019	0.025
8	0.037	0.040
9	0.027	0.025
10	0.006	0.010

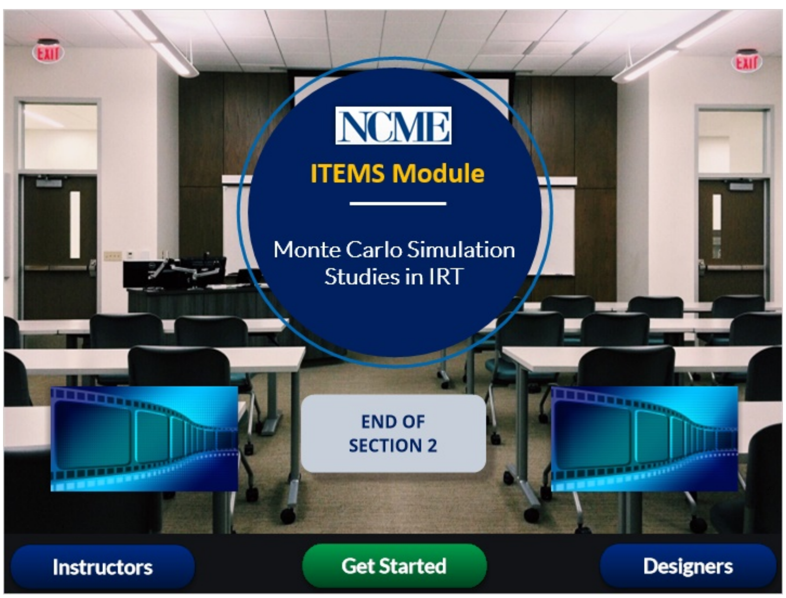
3.21 Bookend: Frequency Table Distributions



3.22 Bookend: Section 2

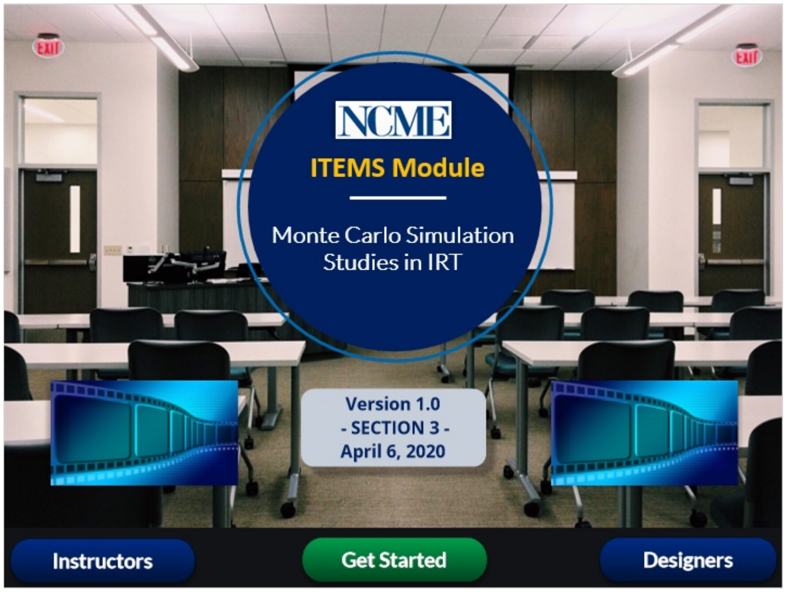


3.23 Module Cover (END)

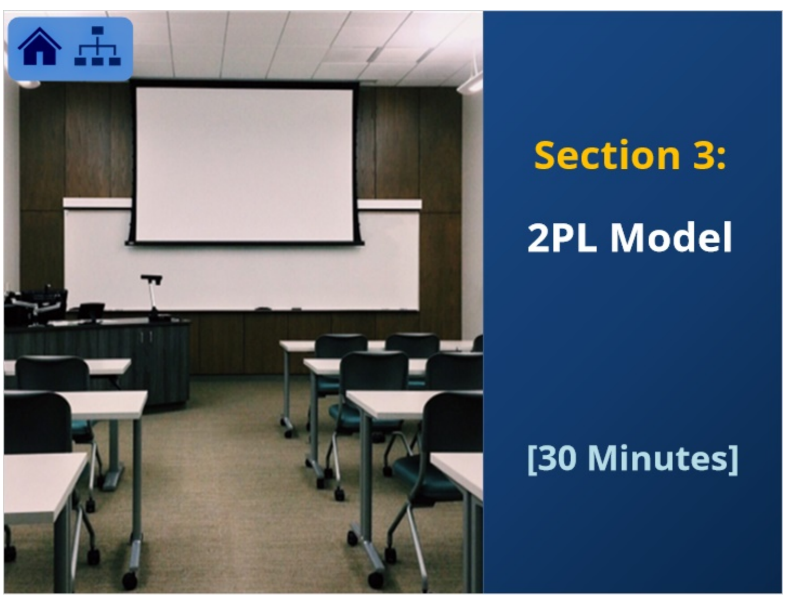


4. Section 3: Two-parameter Model

4.1 Module Cover (START)



4.2 Cover: Section 3



4.3 Objectives

Learning Objectives

1. Identify key parameters in the 2 PL model for simulation

2. Describe methods of how to chose generating parameters

3. Articulate the process for item response simulation using the 2PL

4. Modify example code for specific research scenarios

4.4 Simulation Goal

Simulation Goal

- Simulate item responses for 100 individuals to a 14-item test

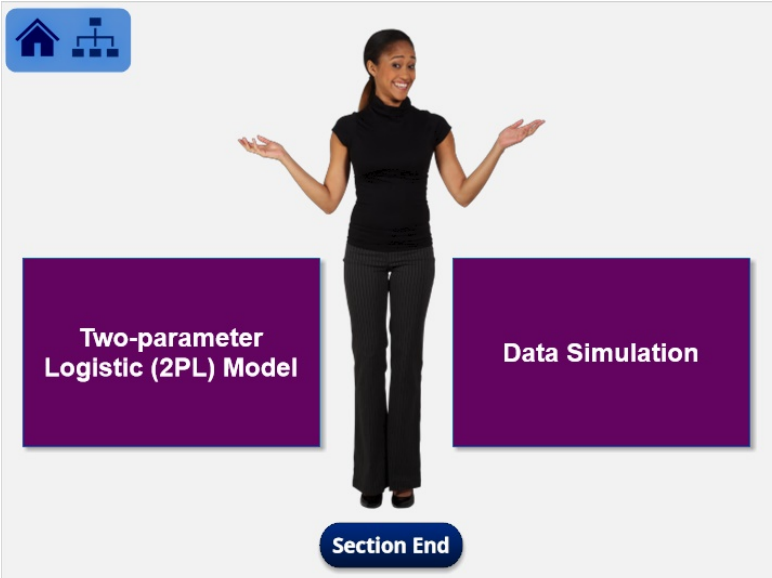
- Each item is going to be dichotomously scored as
  - ✓ 1 = Correct
  - ✓ 0 = Incorrect

- Goal dataset:

Item 1	Item 2	Item 3	...	Item 14
1	1	0	...	1
0	1	1	...	0
⋮	⋮	⋮	⋱	⋮
0	1	1	...	0

Each row represents an individual's response set

4.5 Topic Selection



4.6 Bookmark: Two-paramter Model

4.7 Two-parameter Model (I)

Two-Parameter Logistic (2PL) Model (I)

$$P_j(U_j = 1|\boldsymbol{\omega}_j, \theta) = \frac{1}{1 + e^{-Da_j(\theta - b_j)}}$$

- Probability an individual with a given ability θ correctly answers item *j* that has item parameters $\boldsymbol{\omega}_j$
- U_j: response on item *j* (1 = correct, 0 = incorrect)
- $\boldsymbol{\omega}_j$ is the vector of all item parameters
- θ is an ability value on the trait of interest

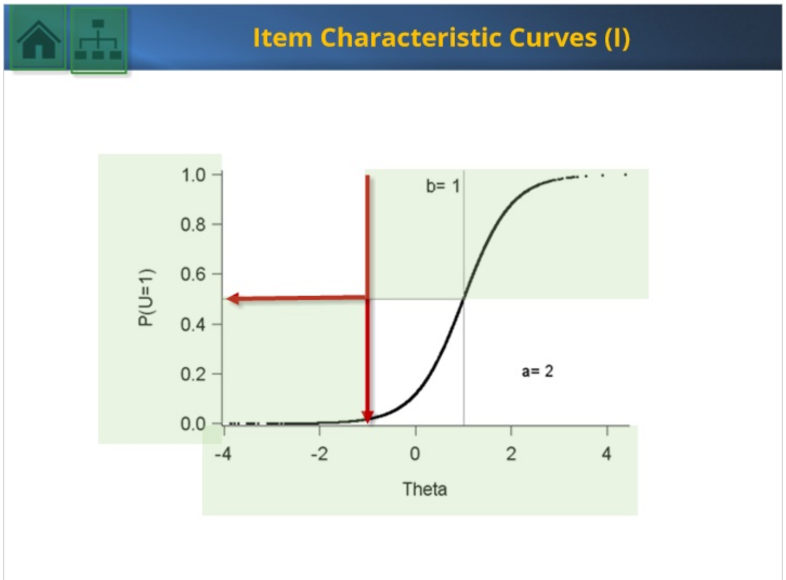
4.8 Two-parameter Model (II)

Two-Parameter Logistic (2PL) Model (II)

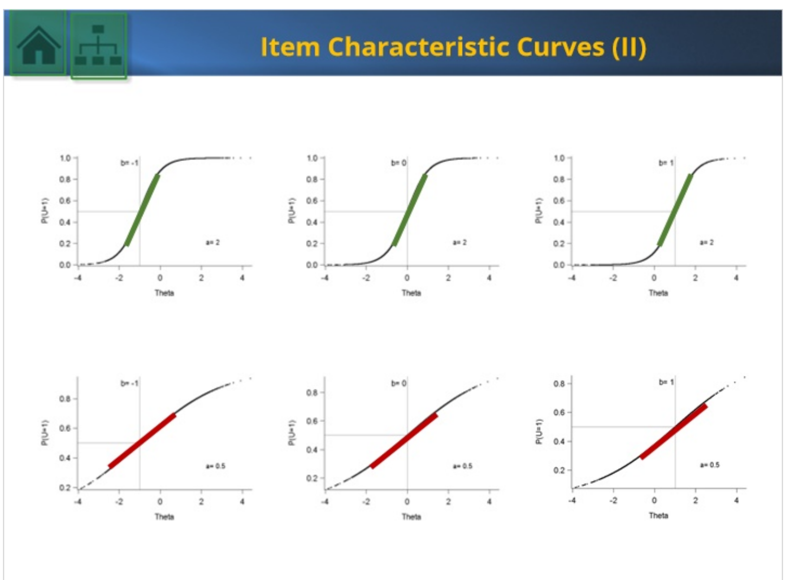
$$P_j(U_j = 1|\boldsymbol{\omega}_j, \theta) = \frac{1}{1 + e^{-Da_j(\theta - b_j)}}$$

- *e* is Euler's number (approx. 2.718)
- *D* is the scaling parameter
  - ✓ Here we set *D* = 1
  - ✓ Other common value is *D* = 1.702
- a_j is the discrimination parameter for item *j*
- b_j is the difficulty/threshold parameter for item *j*

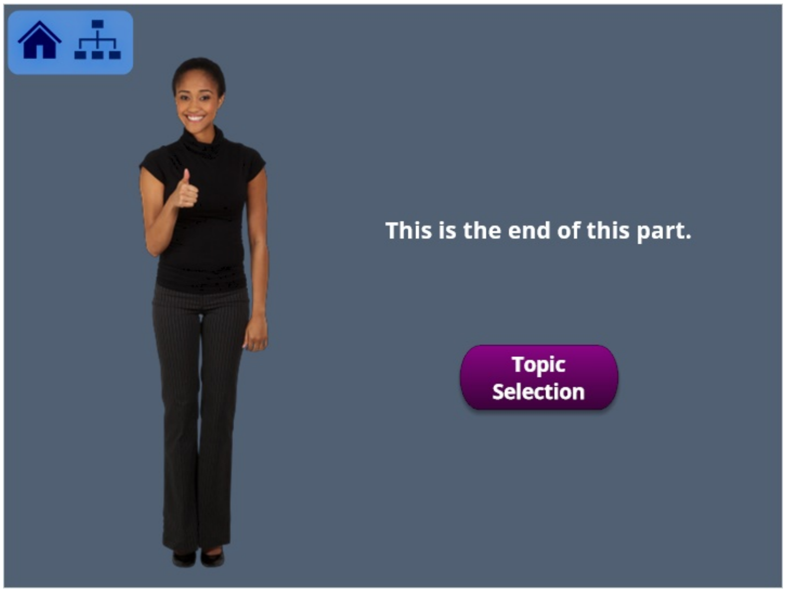
4.9 Item Characteristic Curves (I)



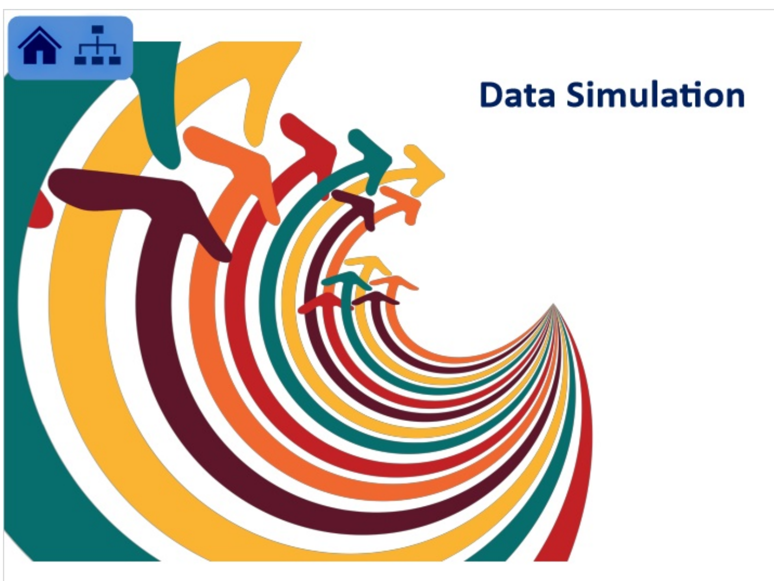
4.10 Item Characteristic Curves (II)



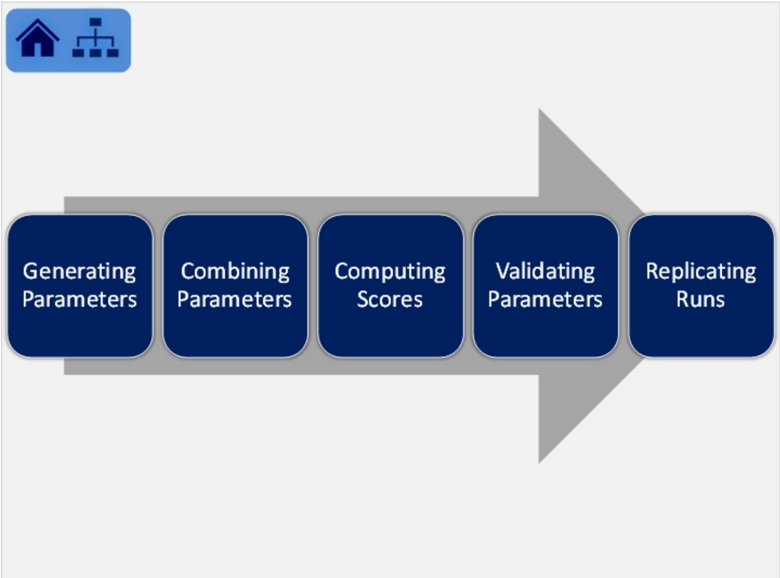
4.11 Bookend: Two-parameter Model



4.12 Bookmark: Generating Parameters with 2PLM



4.13 Step Selection

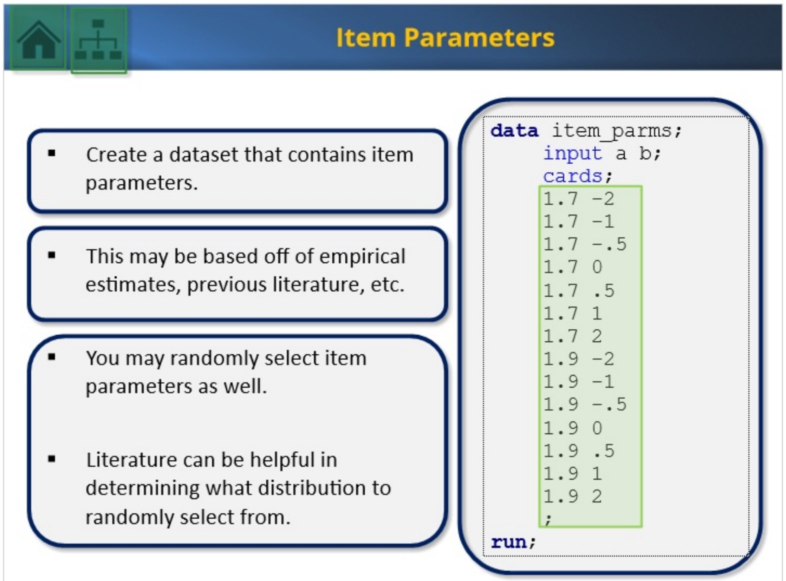


4.14 Person Parameters

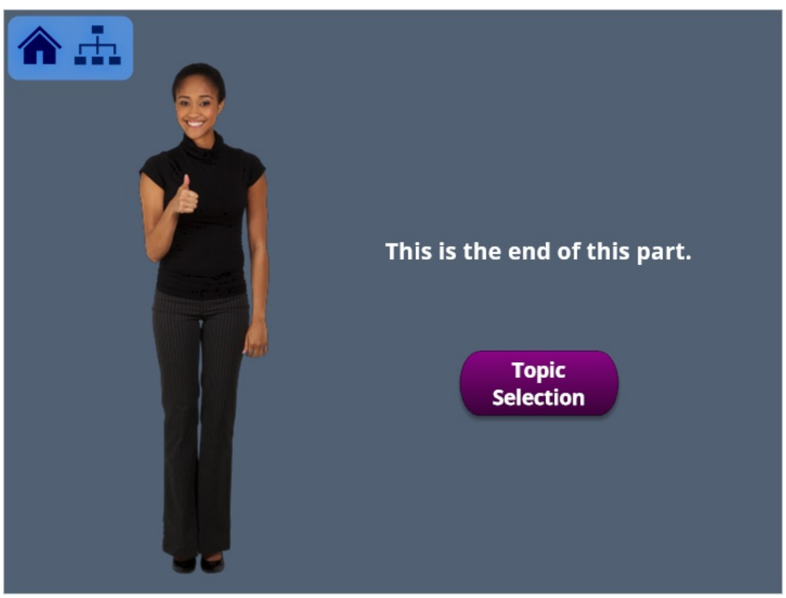
Person Parameters

```
%let n_person = 100;
data person_parm;
    do i = 1 to &n_person;
        theta=rand('normal',0,1);
        output;
    end;
    keep theta;
run;
```

4.15 Item Parameters



4.16 Bookend: Step 1



4.17 Parameter Matrix (I)

Parameter Matrix (I)

- We need in the same data set / parameter matrix:
  - ✓ an individual's ability (θ)
  - ✓ item parameters (a_j and b_j)
- We want our data set / parameter matrix to look like this:

Theta	a_1	b_1	…	a_{14}	b_{14}
-1.43	1.7	-2	…	1.9	2
2.81	1.7	-2	…	1.9	2
⋮	⋮	⋮	⋱	⋮	⋮
0.09	1.7	-2	…	1.9	2

4.18 Parameter Matrix (II)

Parameter Matrix (II)

	a	b
1	1.7	-2
2	1.7	-1
3	1.7	-0.5
4	1.7	0
5	1.7	0.5
6	1.7	1
7	1.7	2
8	1.9	-2
9	1.9	-1
10	1.9	-0.5
11	1.9	0
12	1.9	0.5
13	1.9	1
14	1.9	2

```
proc transpose data=item_parms out=a_wide prefi
      var a;
run;
```

	NAME OF FORMER VARIABLE	a1	a2	a3	a4	a5	a6	a7
1	a	1.7	1.7	1.7	1.7	1.7	1.7	1.7

```
proc transpose data=item_parms out=b_wide prefi
      var b;
run;
```

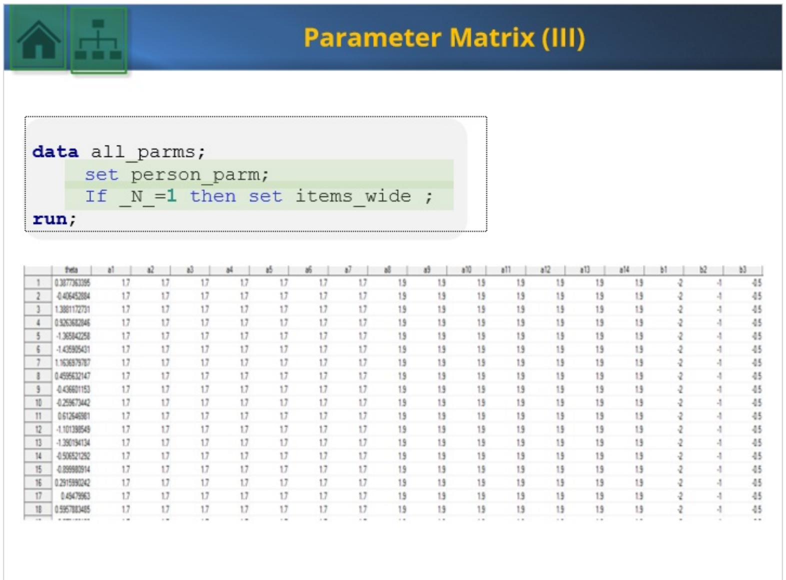
	NAME OF FORMER VARIABLE	b1	b2	b3	b4	b5	b6	b7	b8
1	b	-2	-1	-0.5	0	0.5	1	2	-2

```
data items_wide;
      merge a_wide b_wide;
      drop _name_;
run;
```

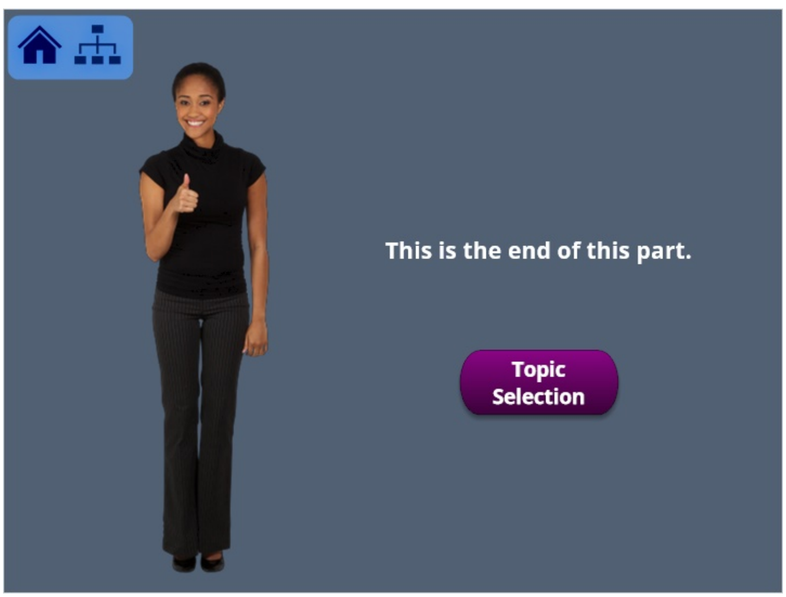
Goal:

a1	a2	a3	a4	a5	a6	a7	a8	a9	a10	a11	a12	a13	a14	b1	b2	b3
1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.9	1.9	1.9	1.9	1.9	1.9	1.9	-2	-1	-0.

4.19 Parameter Matrix (III)



4.20 Bookend: Step 2



4.21 Score Computation (I)

Response Probabilities

We want a data file that looks like:

Item 1	Item 2	Item 3	...	Item 14
1	1	0	...	1
0	1	1	...	0
⋮	⋮	⋮	⋱	⋮
0	1	1	...	0

Let's consider one individual:

Theta	a_1	b_1	...	a_{14}	b_{14}
-1.43	1.7	-2	...	1.9	2

$$P_j(U_j = 1|\boldsymbol{\omega}_j, \theta) = \frac{1}{1 + e^{-a_j(\theta - b_j)}}$$

For item 1: $P(U_1 = 1) = \frac{1}{1+e^{-1.7(-1.43-(-2))}} = .725$

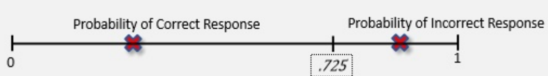
⋮

For item 14: $P(U_{14} = 1) = \frac{1}{1+e^{-1.9(-1.43-(2))}} = .0015$

4.22 Score Computation (II)

Response Computation (I)

- Incorporate **randomness** into the process
- Choose a **random number** between 0 and 1 with **equal probability** from a **uniform distribution**
  - ✓ If the randomly selected value is **less than or equal to .725**, assign a score of **1**
  - ✓ If the randomly selected value is **greater than .725**, assign a score of **0**



For item 1: $P(U_1 = 1) = \frac{1}{1+e^{-1.7(-1.43-(-2))}} = .725$

4.23 Score Computation (III)

Response Computation (II)

```
data responses;
    set all_parms;
    array a{&n_item} a1-a&n_item;
    array b{&n_item} b1-b&n_item;
    array p{&n_item} p1-p&n_item;
    array item{&n_item} item1 - item&n_item;
```

4.24 Score Computation (IV)

Response Computation (III)

```
data responses;
    set all_parms;
    array a{&n_item} a1-a&n_item;
    array b{&n_item} b1-b&n_item;
    array p{&n_item} p1-p&n_item;
    array item{&n_item} item1 - item&n_item;
    do j = 1 to &n_item;
        p[j]=1/(1+exp(-(a[j]*(theta-b[j]))));
        y=rand('uniform');
        if y<=p[j] then item[j]=1;
        else if y>p[j] then item[j]=0;
    end;
run;
```

Results

Results (Slide Layer)

Results

Obs	p1	p2	p3	p4	p5	p6	p7	item1	item2	item3	item4	item5	item6	item7
20	0.96695	0.84241	0.69557	0.49407	0.29448	0.15139	0.03156	1	1	1	0	1	1	0
21	0.90752	0.64193	0.43382	0.24671	0.12279	0.05645	0.01081	0	1	0	0	0	0	0
22	0.74623	0.34946	0.18673	0.08937	0.04026	0.01761	0.00326	1	1	0	1	1	0	0
23	0.99891	0.99404	0.98616	0.96821	0.92866	0.84764	0.50406	1	1	1	1	1	1	1
24	0.98711	0.93327	0.85669	0.71870	0.52200	0.31822	0.07857	1	1	1	1	0	0	1
25	0.90371	0.63161	0.42290	0.23851	0.11807	0.05412	0.01034	1	1	0	1	0	0	0

Back

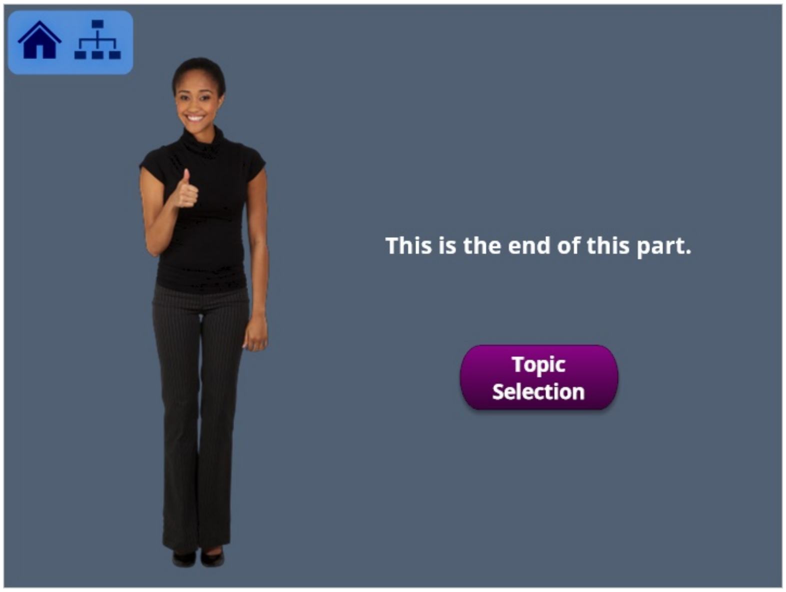
4.25 Score Computation (V)

Data Set Cleanup

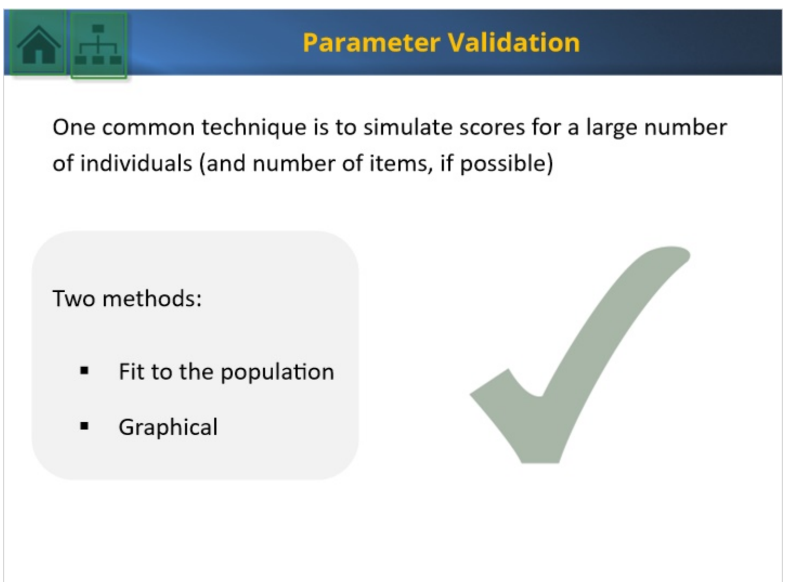
```
data responses_only;
     set responses;
     keep item:;
run;
```

	item1	item2	item3	item4	item5	item6	item7	item8	item9	item10
1	1	1	1	1	0	0	0	1	1	1
2	1	0	1	0	1	0	0	1	0	1
3	1	1	1	1	0	0	0	1	1	0
4	1	1	1	1	0	0	0	1	1	1
5	1	1	1	1	0	0	0	1	1	1
6	1	0	0	0	0	0	0	1	1	0
7	1	1	1	1	1	1	0	1	0	1
8	1	1	1	1	1	0	0	1	1	1
9	1	1	1	1	1	0	0	1	0	1
10	1	1	1	1	1	1	0	1	1	0
11	0	0	0	0	0	0	0	0	1	0
12	1	1	0	1	0	1	0	1	1	0
13	1	1	1	1	1	1	0	1	1	1
14	1	1	1	1	1	0	0	1	1	1
15	1	1	1	1	1	0	0	1	1	1
16	1	1	1	1	0	0	0	1	0	1

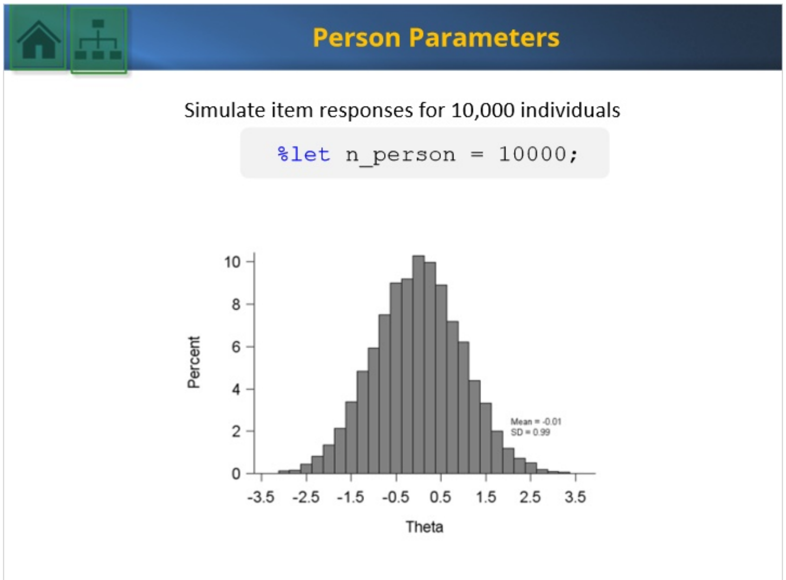
4.26 Bookend: Step 3



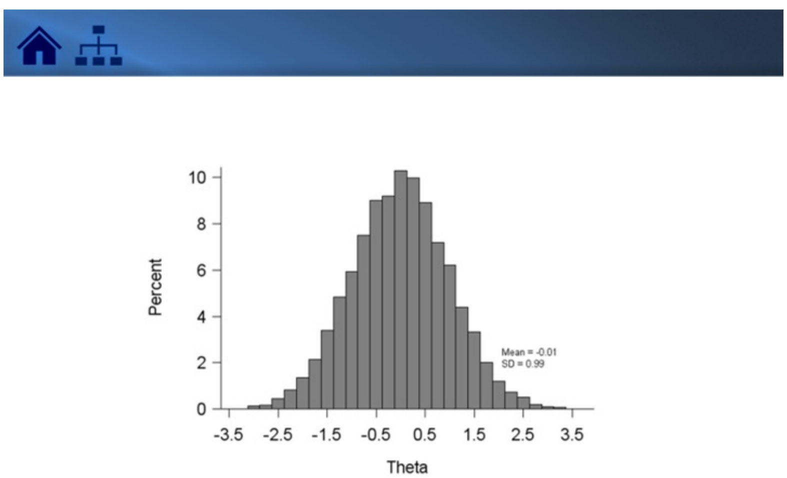
4.27 Parameter Validation (I)



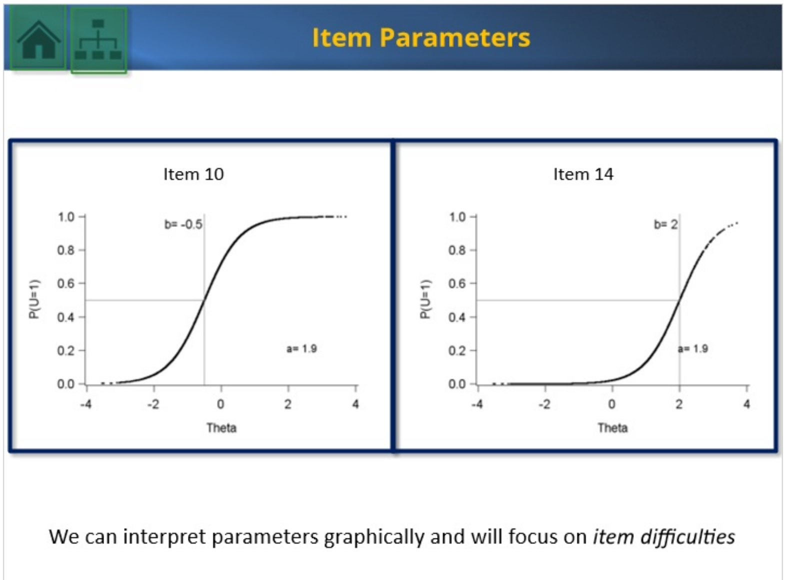
4.28 Parameter Validation (II)



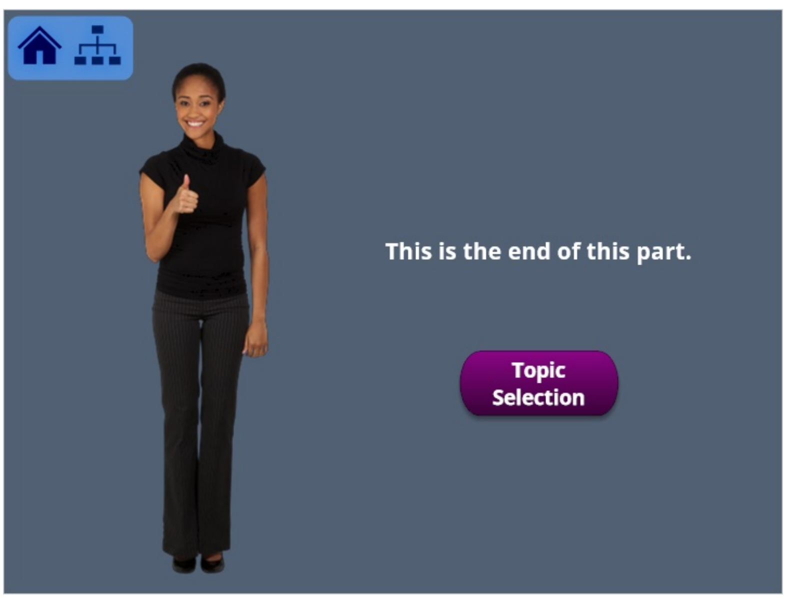
Results (Slide Layer)



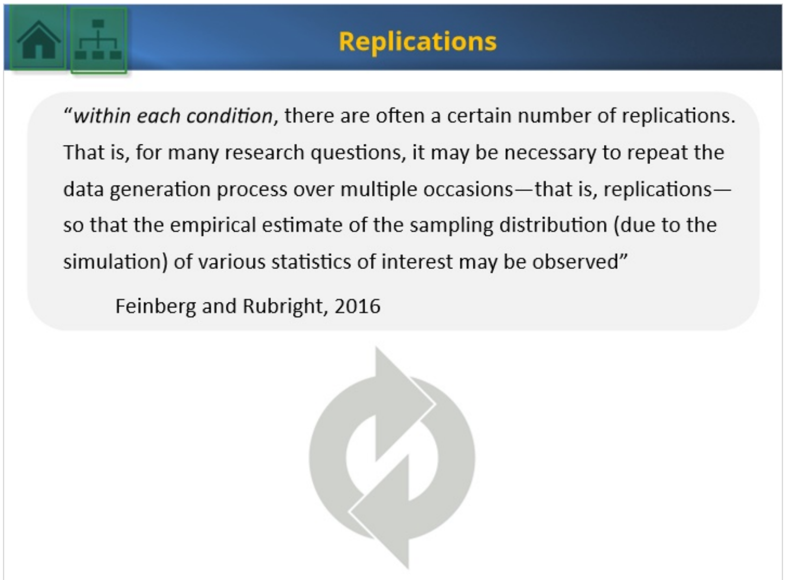
4.29 Parameter Validation (III)



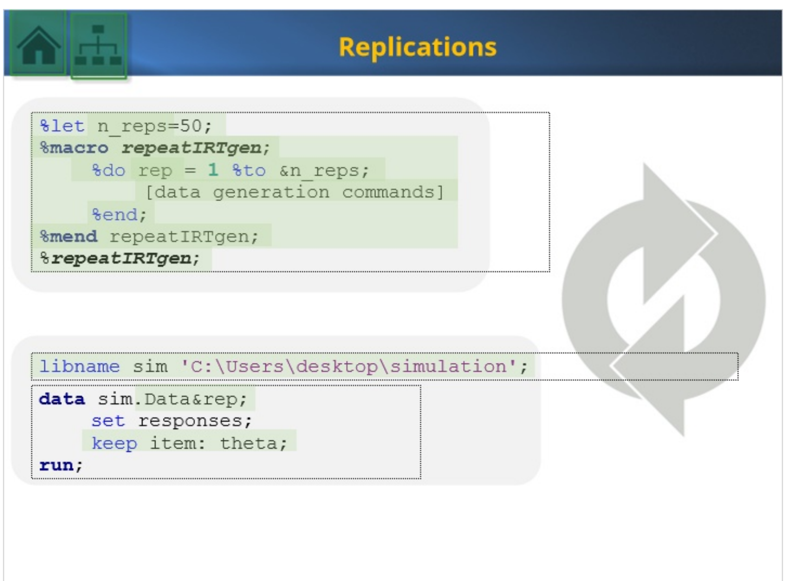
4.30 Bookend: Step 4



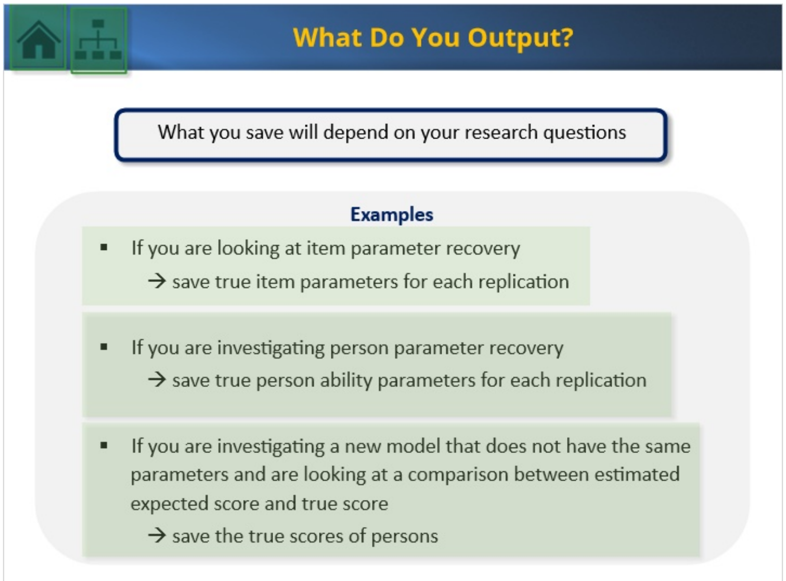
4.31 Replications (I)



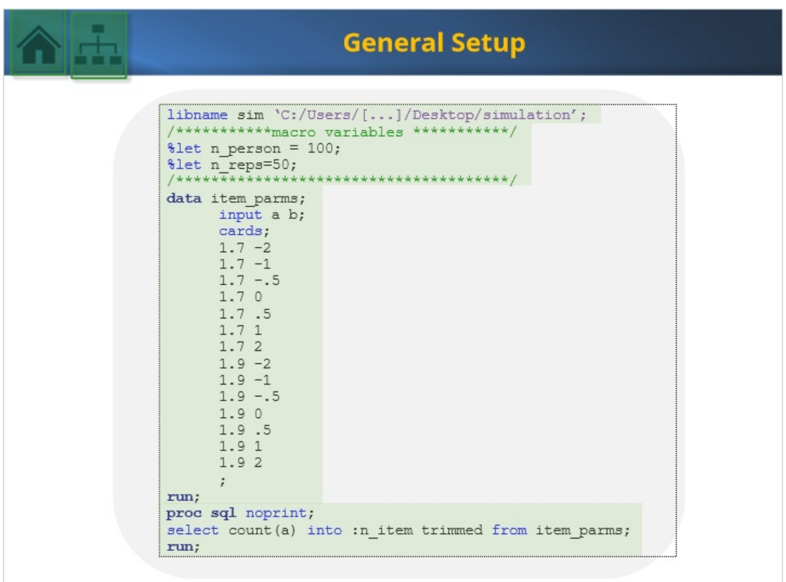
4.32 Replications (II)



4.33 Replications (III)



4.34 Replications (IV)



4.35 Replications (V)

Generating Person Parameters

```
%macro repeatIRTgen;
%do rep = 1 %to &n_reps;
proc datasets library=work noprint;
     save item_parms;
run;
quit;

data person_parm;
     do i = 1 to &n_person;
          theta=rand('normal',0,1);
          output;
     end;
     keep theta;
run;
```

Full code is available in the "Resources" of the module

4.36 Replications (VI)

Merging Item and Person Parameters

```
proc transpose data=item_parms out=a_wide prefix=a;
     var a;
run;

proc transpose data=item_parms out=b_wide prefix=b;
     var b;
run;

data items_wide;
     merge a_wide b_wide;
     drop _name_;
run;

data all_parms;
     set person_parm;
     If _N_=1 then set items_wide ;
run;
```

4.37 Replications (VII)

Simulating Responses

```
data responses;
     set all_parms;
     array a{&n_item} a1-a&n_item;
     array b{&n_item} b1-b&n_item;
     array p{&n_item} p1-p&n_item;
     array item{&n_item} item1 - item&n_item;
     do j = 1 to &n_item;
         p[j]=1/(1+exp(-(a[j]*(theta-b[j]))));
         y=rand('uniform');
         if y<=p[j] then item[j]=1;
         else if y>p[j] then item[j]=0;
     end;
run;
```

4.38 Replications (VIII)

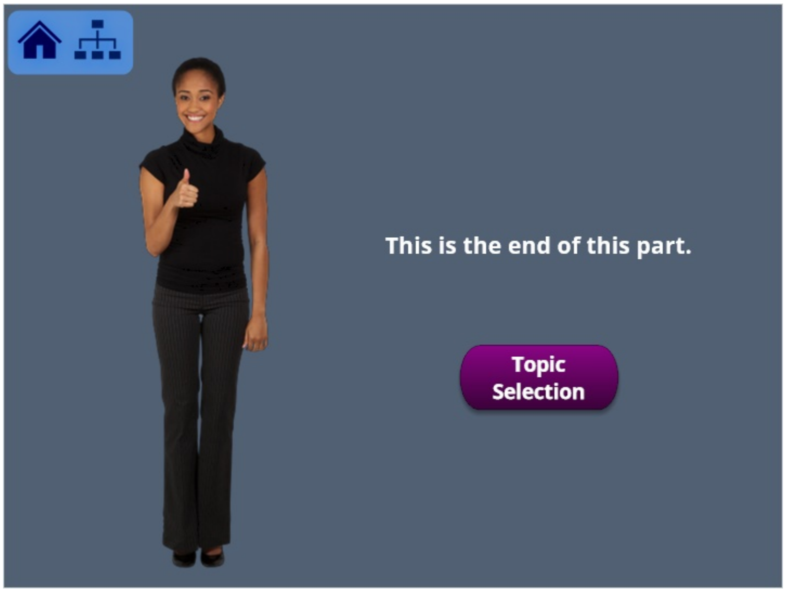
Saving Generated Data Sets

```
data sim.Data&rep;
     set responses;
     keep item: theta;
run;

%end;
%mend repeatIRTgen;


%repeatIRTgen;
```

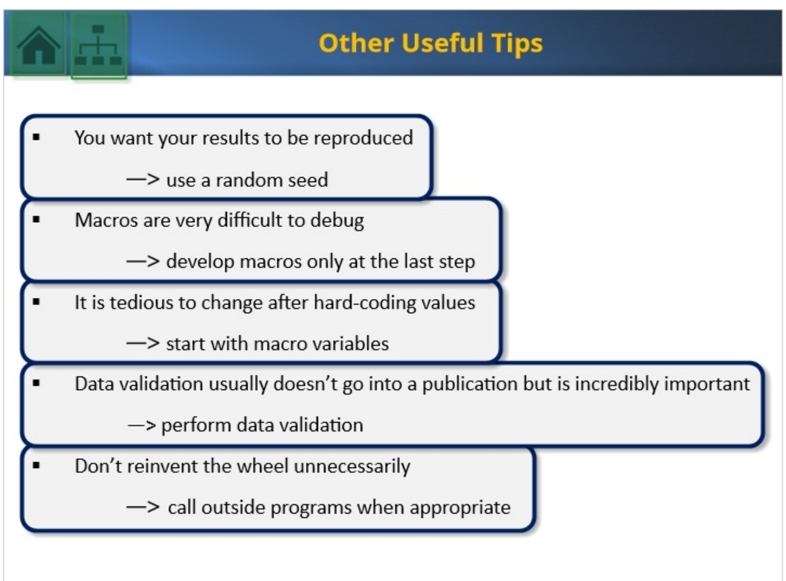
Name

- data1.sas7bdat
- data2.sas7bdat
- data3.sas7bdat
- data4.sas7bdat
- data5.sas7bdat

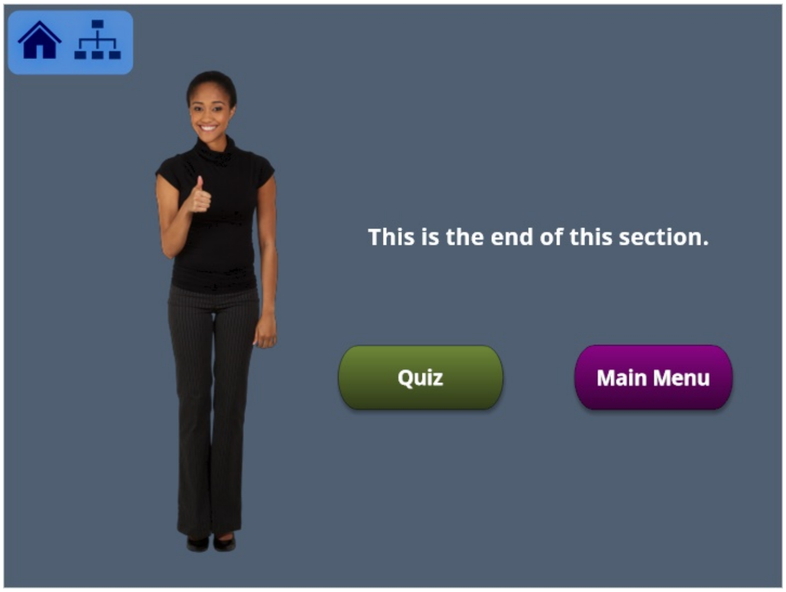
4.39 Bookend: Step 5



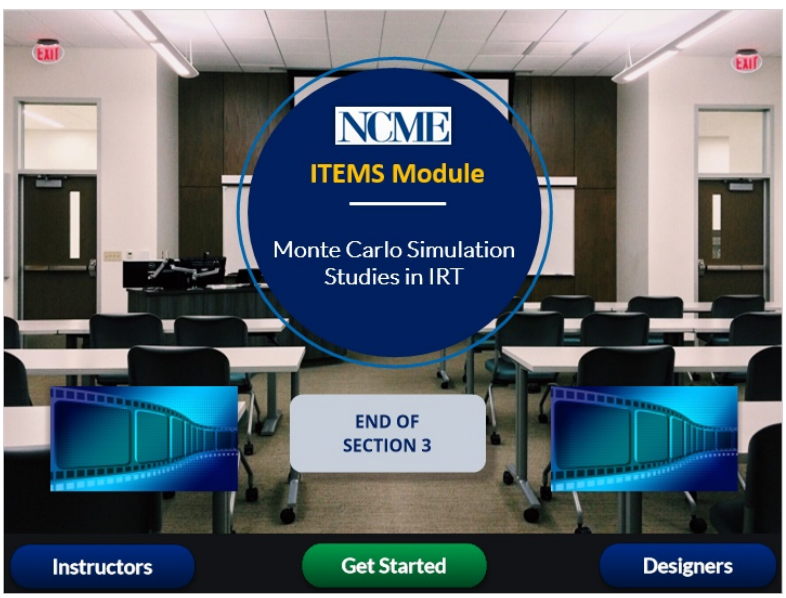
4.40 Other Useful Tips



4.41 Bookend: Section 3

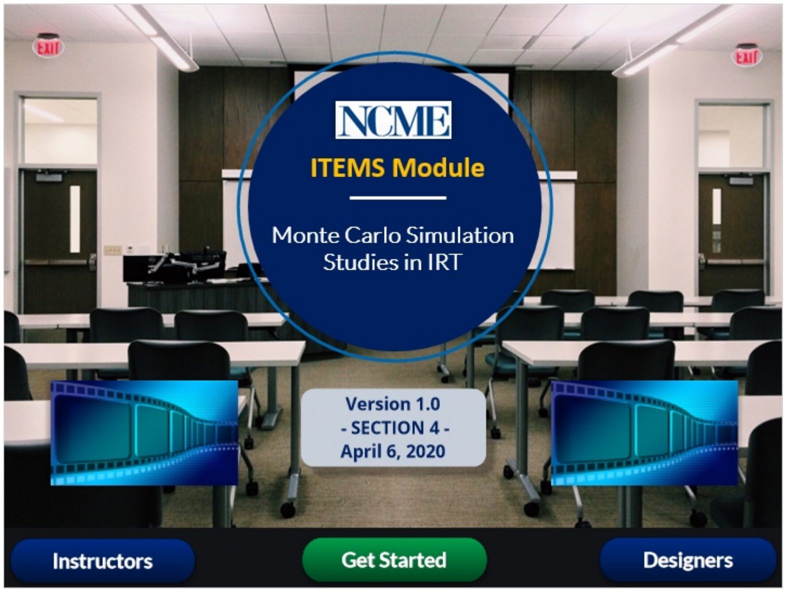


4.42 Module Cover (END)

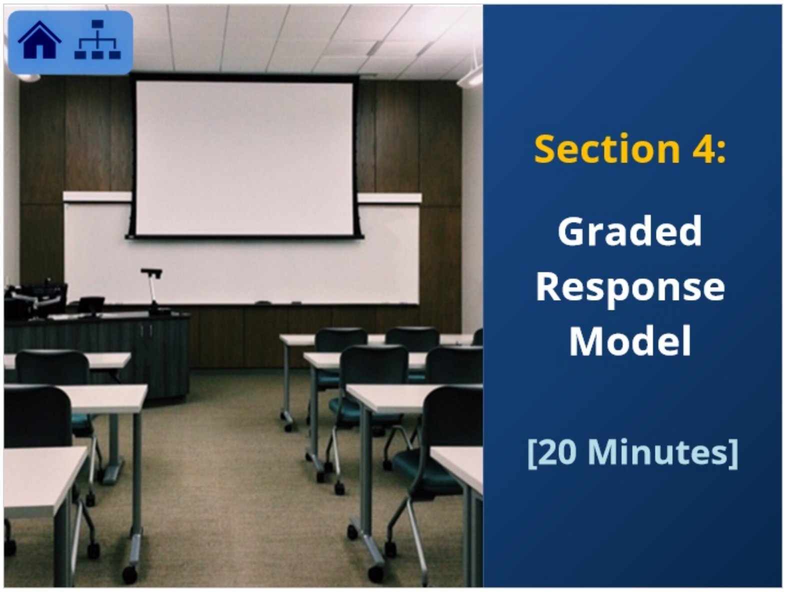


5. Section 4: Graded Response Model

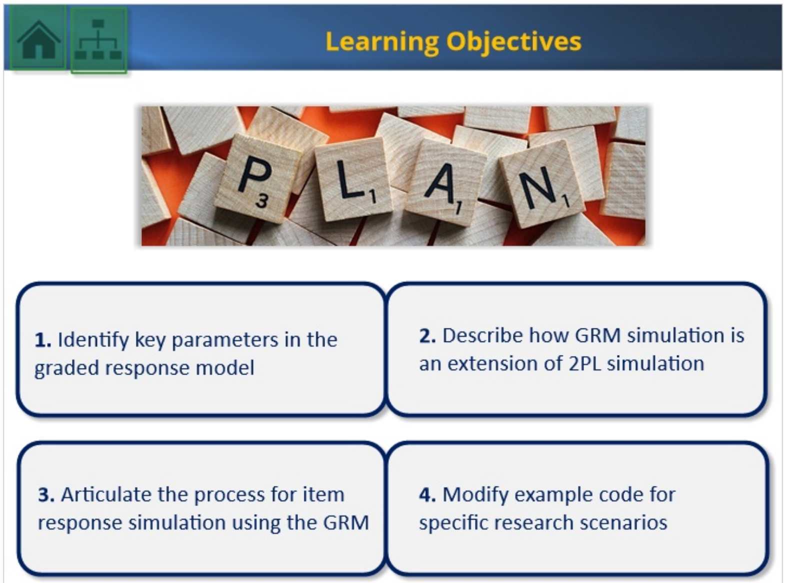
5.1 Module Cover (START)



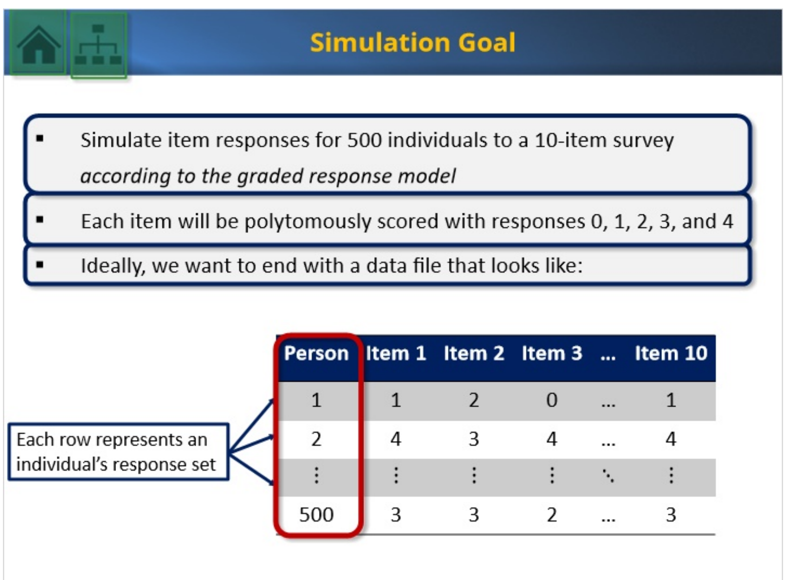
5.2 Cover: Section 4



5.3 Objectives

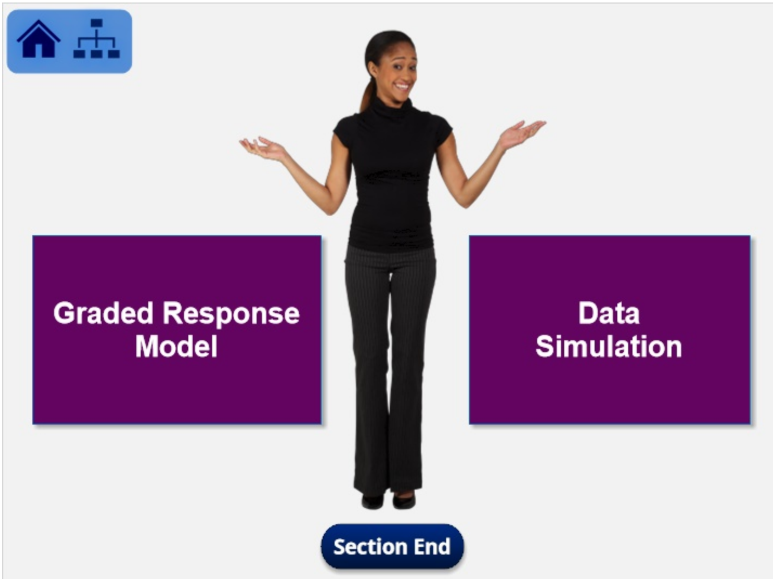


5.4 Simulation Goal



Person	Item 1	Item 2	Item 3	...	Item 10
1	1	2	0	...	1
2	4	3	4	...	4
⋮	⋮	⋮	⋮	⋱	⋮
500	3	3	2	...	3

5.5 Topic Selection



5.6 Bookmark: GRM

5.7 GRM (I)

Overview

Probabilities (p_{ijk}) for individual score categories are defined as differences between cumulative probabilities (p^*_{ijk})

Example

$$p^*_{ijk=3} - p^*_{ijk=4} = p_{ijk=3}$$

1 2 3 4 5 – 1 2 3 4 5 = 1 2 3 4 5

Each p^*_{ijk} is modeled as a 2PL

5.8 GRM (II)

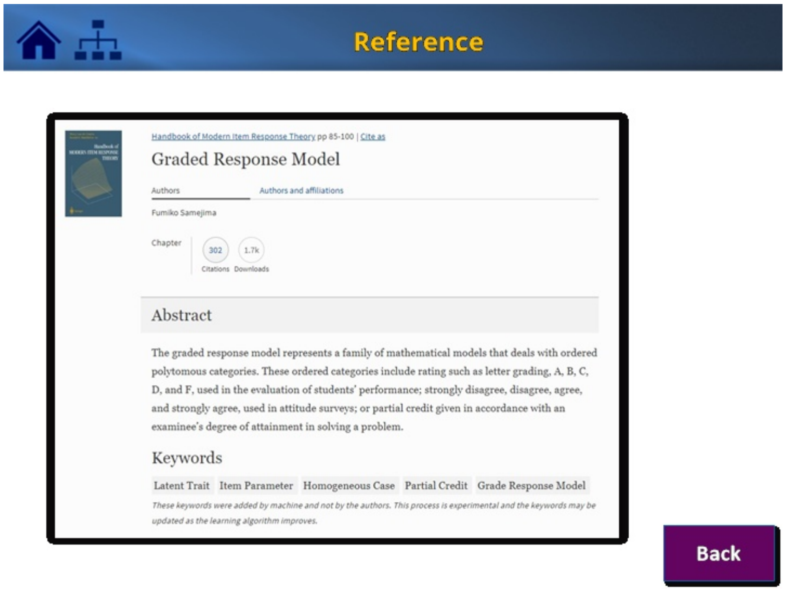
Core Formula

$$P^*_j(U_j \geq k|\theta) = \frac{1}{1 + \mathrm{e}^{-Da_j(\theta - b_{jk})}}$$

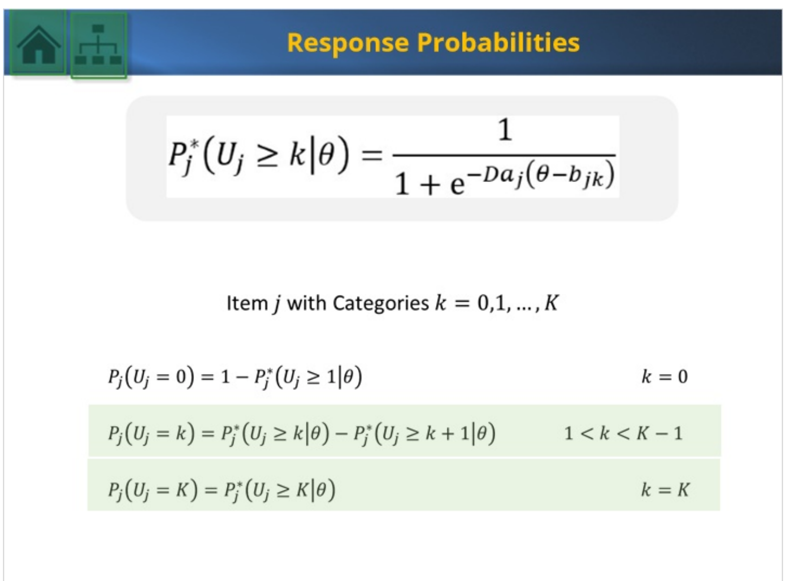
- Cumulative probability an individual with a given ability or trait θ selects or receives a category score of k or higher for item j
- U_j: response on item j
- θ is an ability value on the trait of interest
- a_j is the slope/discrimination parameter for item j
- b_{jk} is the threshold parameter for item j and category k
- D is the scaling parameter
  - ✓ Here we set D = 1
  - ✓ Other common value is D = 1.702

Reference

References (Slide Layer)



5.9 GRM (III)

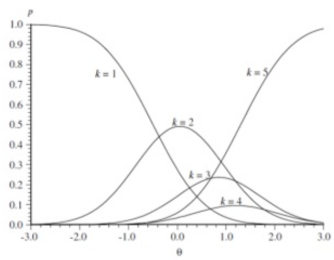


5.10 GRM (IV)

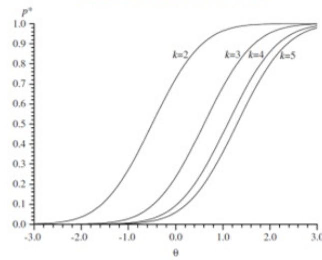
Example: $\theta = 1$

k	p_{ijk}	p^{*}_{ijk}
1	.036	1
2	.280	.964
3	.232	.684
4	.093	.452
5	.359	.359

Category Probabilities



Cumulative Probabilities



5.11 GRM (V)

Thresholds

- b_{jk} is the threshold parameter for item j and category k
- b_{jk} is the point on the θ scale in which a respondent has a 0.5 probability of selecting that category or greater
- For an item with K categories, there are $\mathrm{K}-1$ thresholds

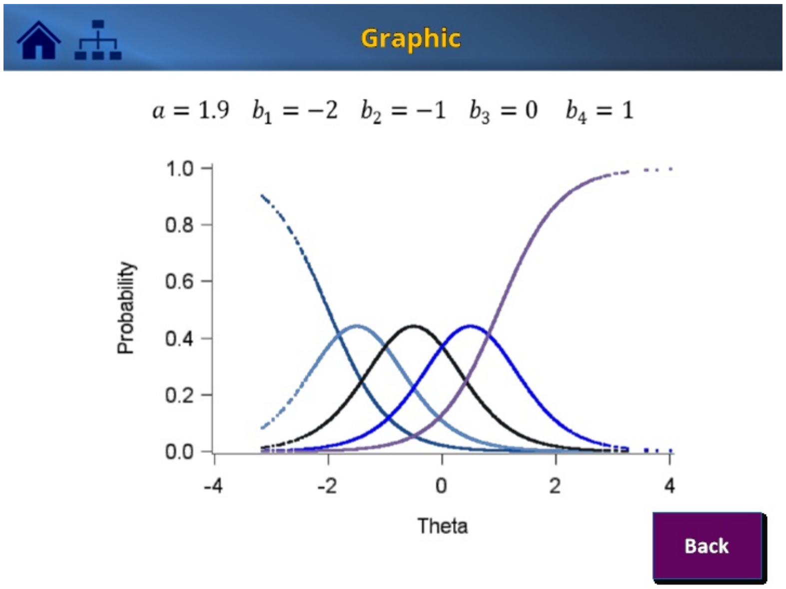
Example

For an item with 5 categories scored 0, 1, 2, 3, and 4, there are four thresholds such that:

$$b_{j1} \le b_{j2} \le b_{j3} \le b_{j4}$$

Graphic

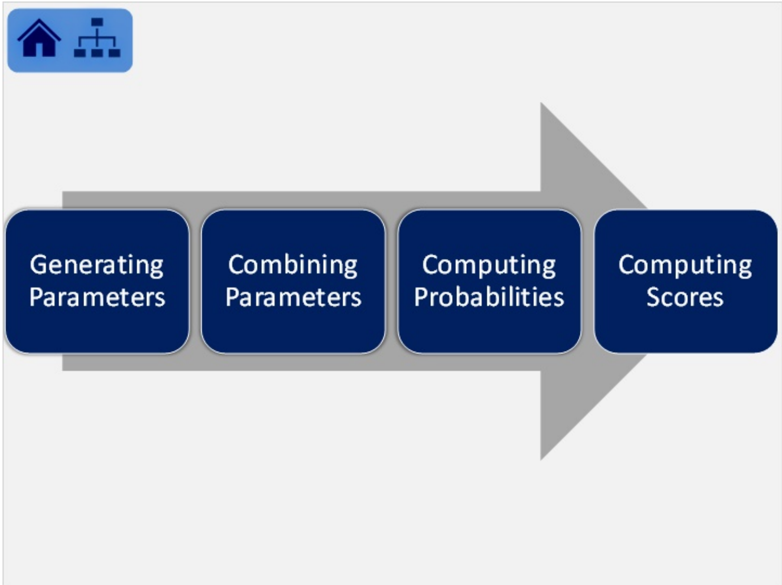
Graphic (Slide Layer)



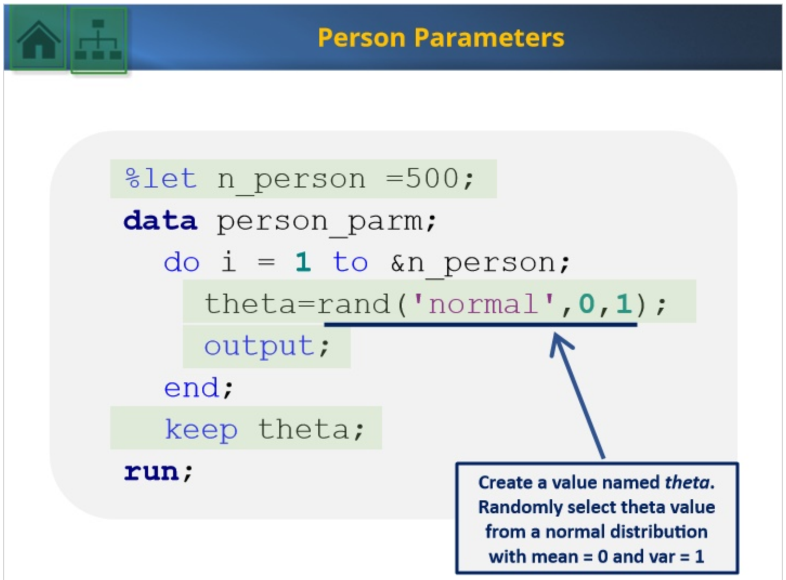
5.12 Bookend: GRM

5.13 Bookmark: Data Simulation

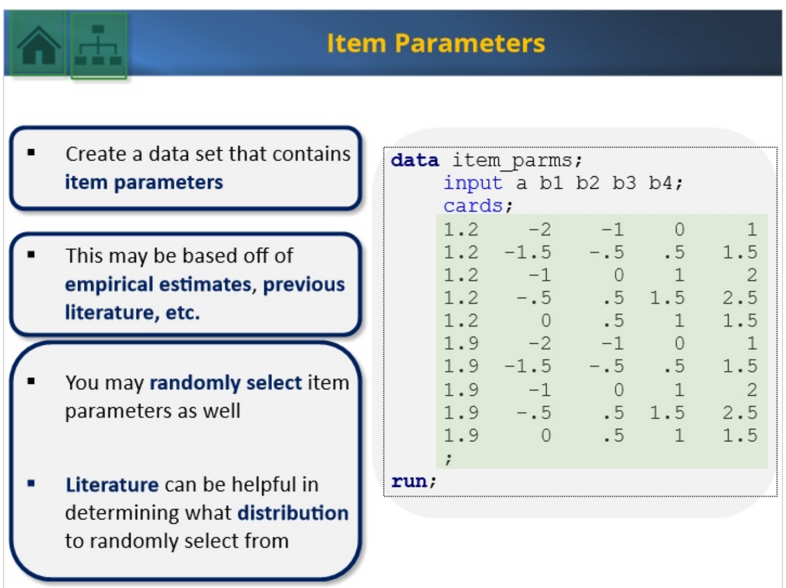
5.14 Step Selection



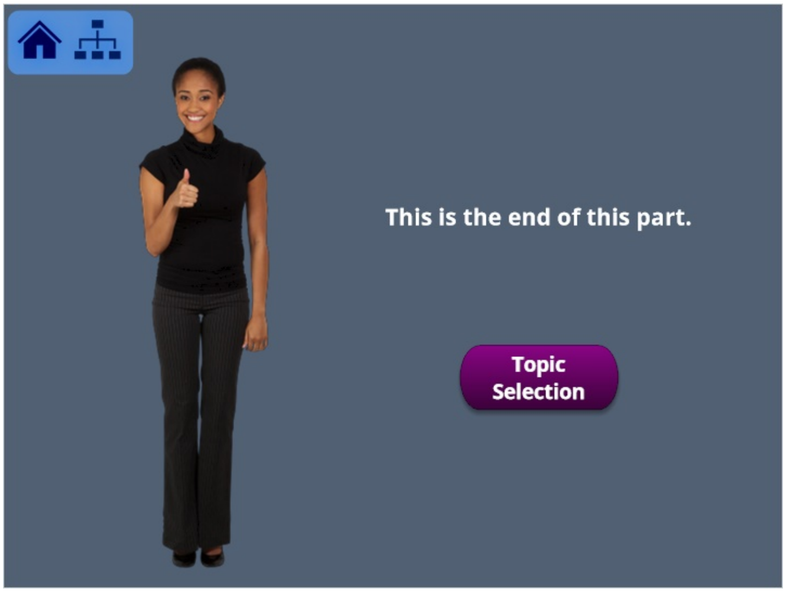
5.15 Person Parameters



5.16 Item Parameters



5.17 Bookend: Step 1



5.18 Parameter Matrix (I)

Parameter Matrix (I)

- We need in the same data set / parameter matrix:
  - ✓ an individual's ability parameter θ
  - ✓ item parameters a_j and b_{jk}
- We want our data set / parameter matrix to look like this:

Theta	a_1	$b_{1,1}$	$b_{1,2}$	$b_{1,3}$	$b_{1,4}$	…	a_{10}	$b_{10,1}$	$b_{10,2}$	$b_{10,3}$	$b_{10,4}$
1.43	1.2	-2	-1	0	1	…	1.9	0	.5	1	1.5
2.81	1.2	-2	-1	0	1	…	1.9	0	.5	1	1.5
⋮	⋮	⋮	⋮	⋮	⋮	⋱	⋮	⋮	⋮	⋮	⋮
0.09	1.2	-2	-1	0	1	…	1.9	0	.5	1	1.5

5.19 Parameter Matrix (II)

Parameter Matrix (II)

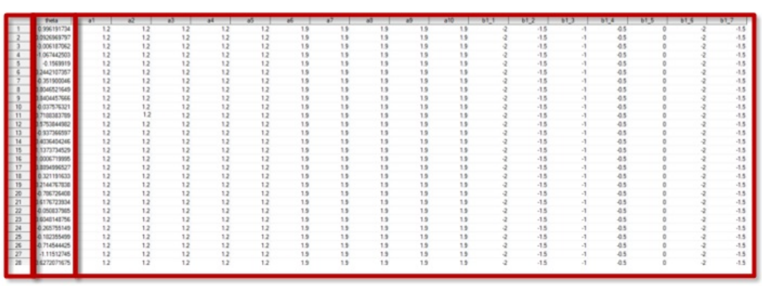
```
proc transpose data=item_parms out=a_wide prefix=a;
var a;
run;

proc transpose data=item_parms out=b1_wide prefix=b1_;
var b1;
run;

proc transpose data=item_parms out=b2_wide prefix=b2_;
var b2;
run;

proc transpose data=item_parms out=b3_wide prefix=b3_;
var b3;
run;

proc transpose data=item_parms out=b4_wide prefix=b4_;
var b4;
run;

data items_wide;
     merge a_wide b1_wide b2_wide b3_wide b4_wide;
     drop _name_;
run;
```

Turn parameters into **wide form**

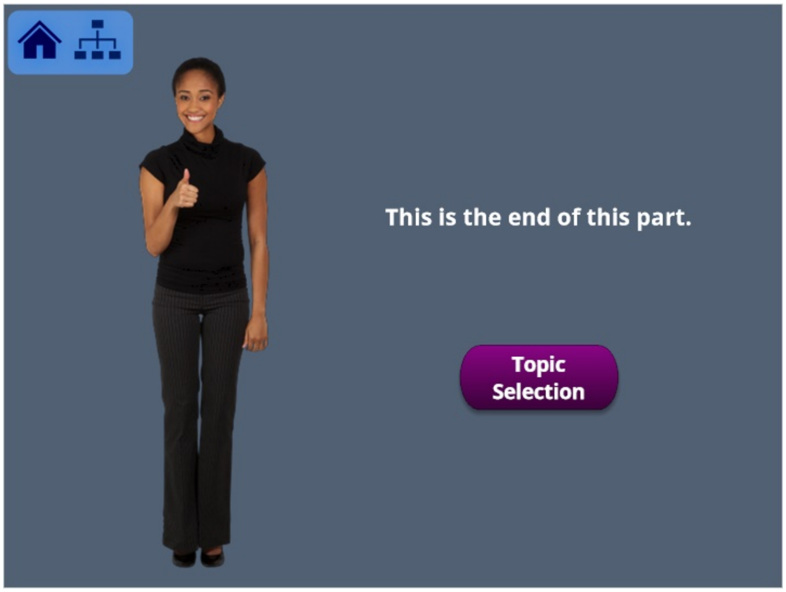
5.20 Parameter Matrix (III)

Parameter Matrix (III)

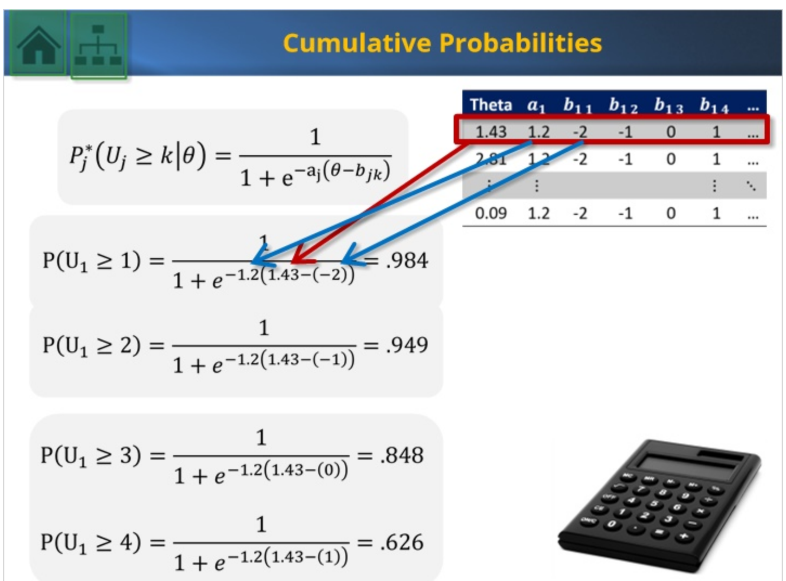
```
data all_parms;
     set person_parm;
     if _N_=1 then set items_wide ;
run;
```



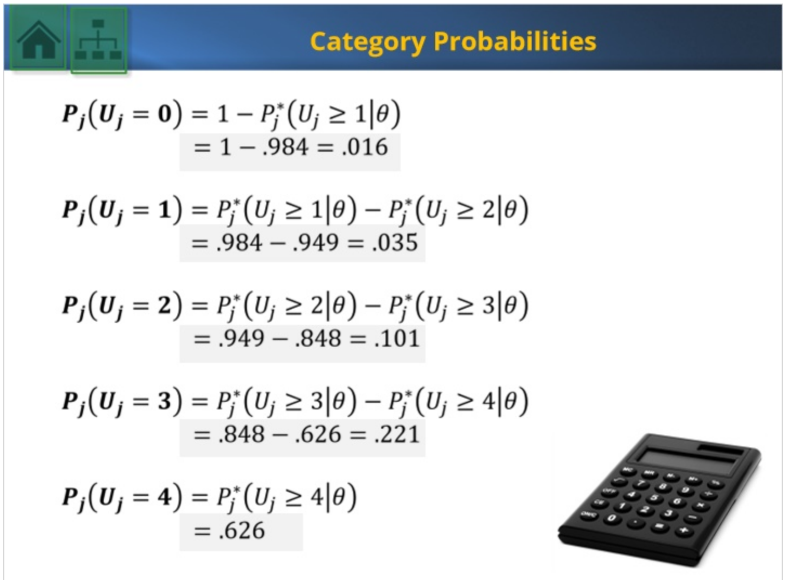
5.21 Bookend: Step 2



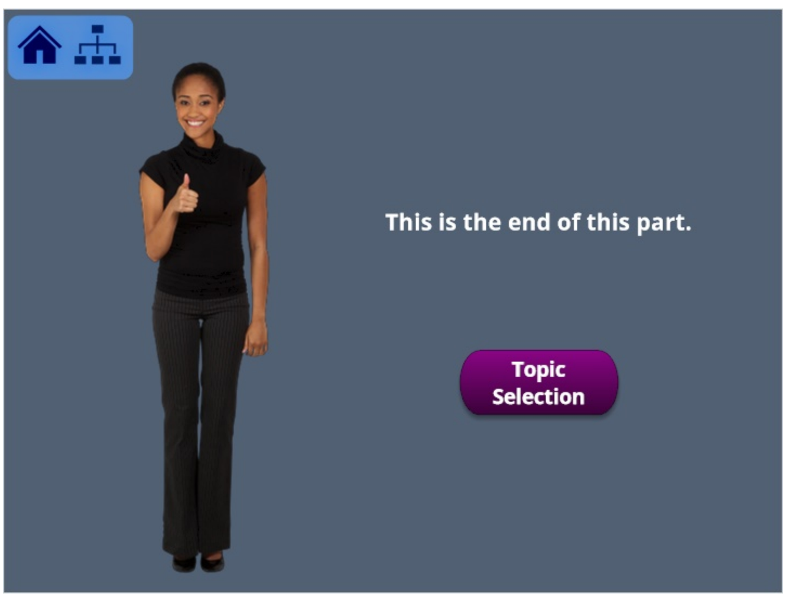
5.22 Cumulative Probabilities



5.23 Category Probabilities



5.24 Bookend: Step 3



5.25 Response Computation (I)

Response Computation (I)

- Choose a **random number** between 0 and 1 with **equal probability** from a **uniform distribution**
- If the random number falls between response categories ***k*–1** and ***k*** then response category ***k*** is assigned to the item

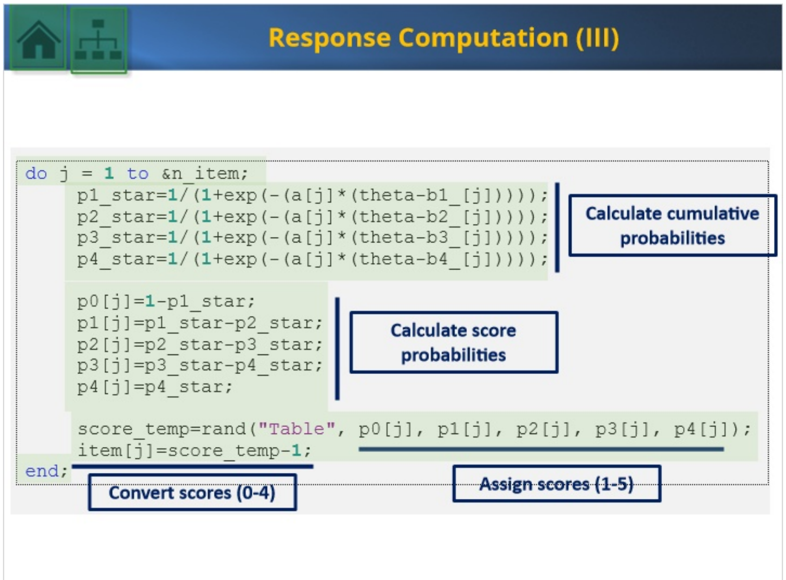
$$P_j(U_j = 0) = .016$$
$$P_j(U_j = 1) = .035$$
$$P_j(U_j = 2) = .101$$
$$P_j(U_j = 3) = .221$$
$$P_j(U_j = 4) = .626$$

5.26 Response Computation (II)

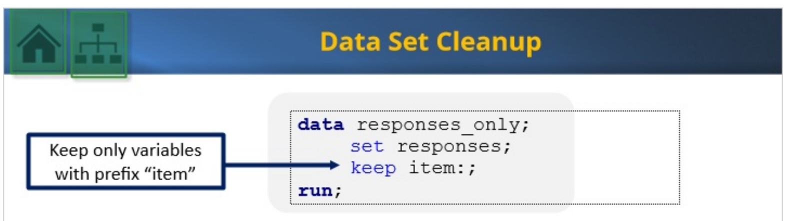
Response Computation (II)

```
data responses;
    set all_parms;
    array a{&n_item} a1-a&n_item;
    array b1_{&n_item} b1_1-b1_&n_item;
    array b2_{&n_item} b2_1-b2_&n_item;
    array b3_{&n_item} b3_1-b3_&n_item;
    array b4_{&n_item} b4_1-b4_&n_item;

    array p0{&n_item} p0_1-p0_&n_item;
    array p1{&n_item} p1_1-p1_&n_item;
    array p2{&n_item} p2_1-p2_&n_item;
    array p3{&n_item} p3_1-p3_&n_item;
    array p4{&n_item} p4_1-p4_&n_item;

    array item{&n_item} item1 - item&n_item;
```

5.27 Response Computation (III)

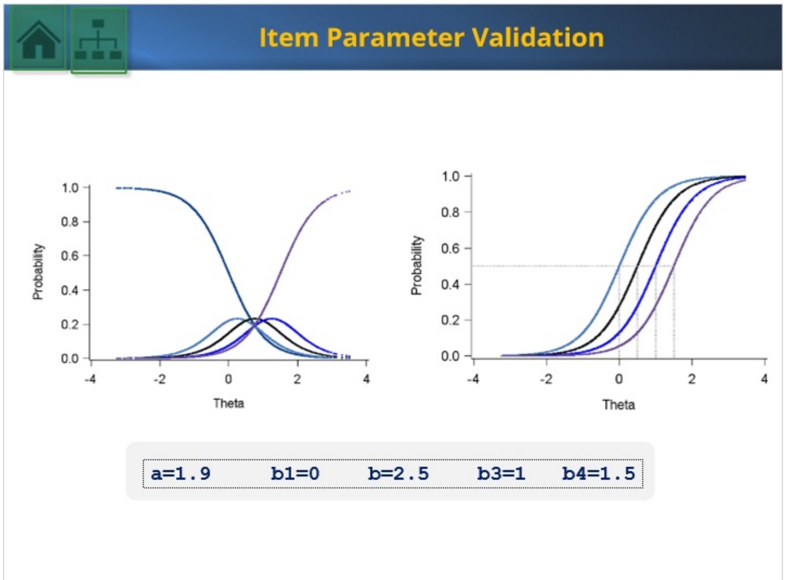


5.28 Data Set Cleanup

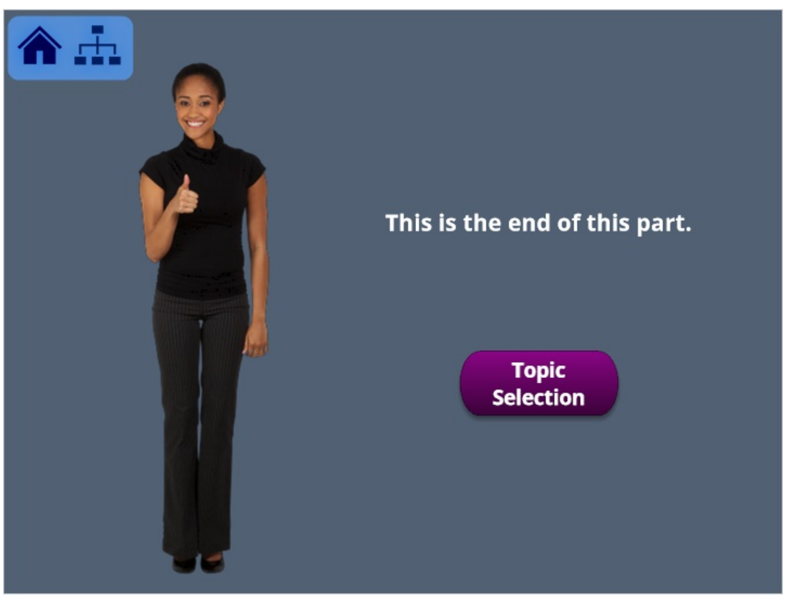


	item1	item2	item3	item4	item5	item6	item7	item8	item9	item10
1	4	3	3	4	4	4	4	3	2	3
2	4	3	2	2	4	4	4	1	3	3
3	4	3	2	2	4	4	3	4	3	4
4	3	2	2	0	0	3	1	1	0	0
5	4	3	3	1	0	3	3	2	3	0
6	3	3	2	1	0	4	3	2	1	4
7	3	1	2	1	1	2	3	3	2	1
8	3	2	0	0	4	2	3	0	0	0
9	3	1	1	3	0	1	0	1	2	0
10	1	2	0	1	4	3	1	2	2	0
11	2	4	4	4	4	4	4	1	4	1
12	3	2	2	2	0	3	3	3	1	3
13	1	3	0	0	2	0	0	1	0	0
14	2	4	0	1	1	1	2	3	1	1
15	3	0	0	0	0	3	0	0	0	0
16	1	0	0	0	0	3	0	0	0	0
17	3	1	3	0	0	2	2	0	2	0
18	2	4	1	0	3	1	3	4	1	2
19	1	2	0	0	1	0	0	0	0	0
20	0	3	0	0	0	0	0	0	0	0
21	2	2	1	3	3	2	3	2	3	3
22	2	1	1	0	0	3	2	0	1	4
23	2	3	2	0	1	1	2	0	1	4
24	4	4	3	1	0	3	4	2	2	3

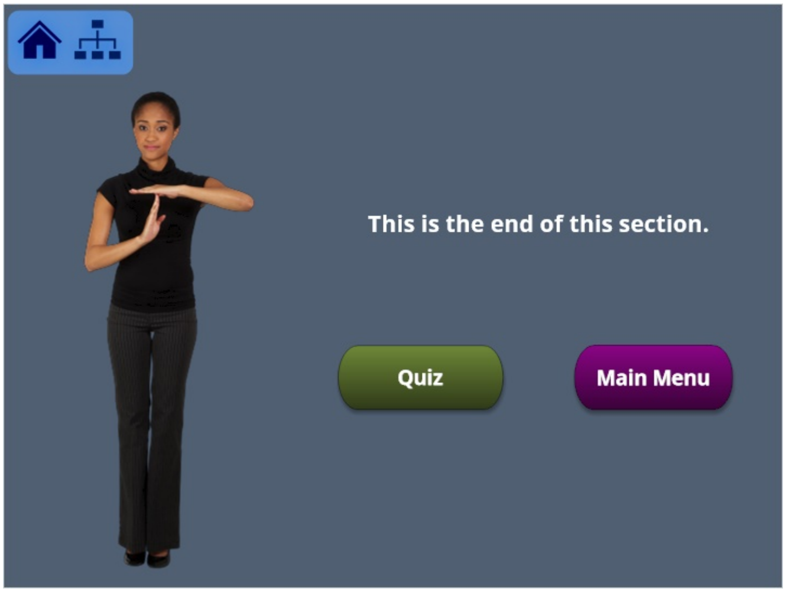
5.29 Item Parameter Validation



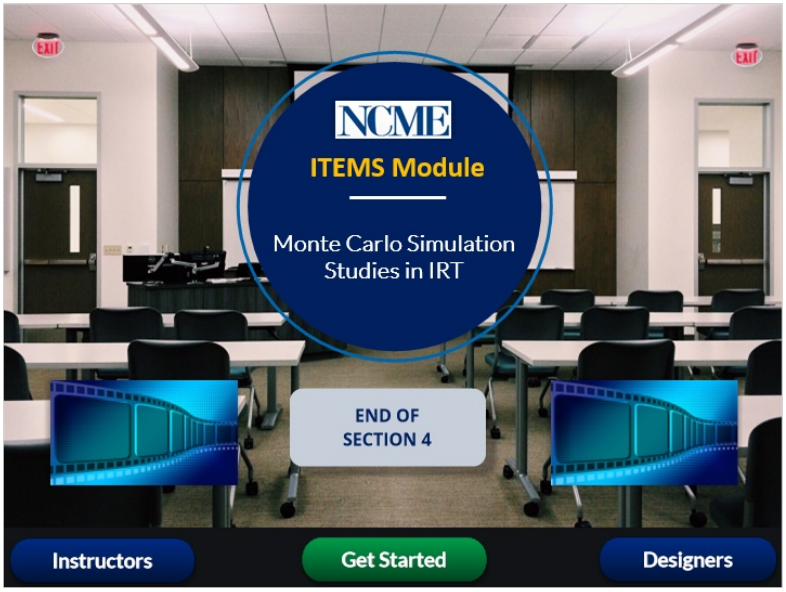
5.30 Bookend: Step 4



5.31 Bookend: Section 4

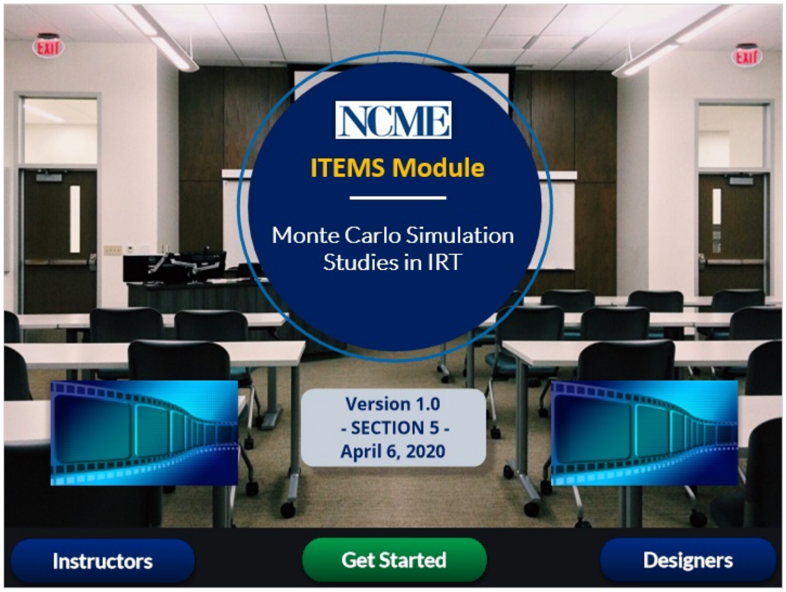


5.32 Module Cover (END)



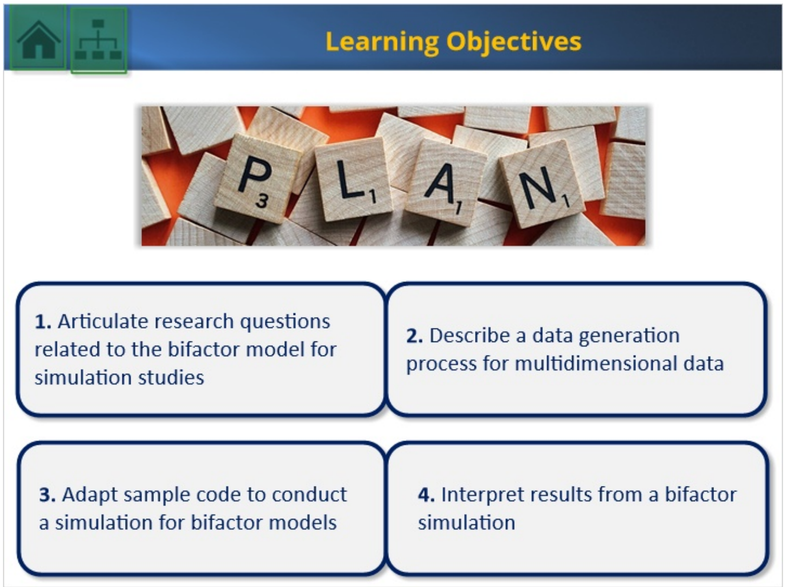
6. Section 5: Bifactor Model [Advanced]

6.1 Module Cover (START)

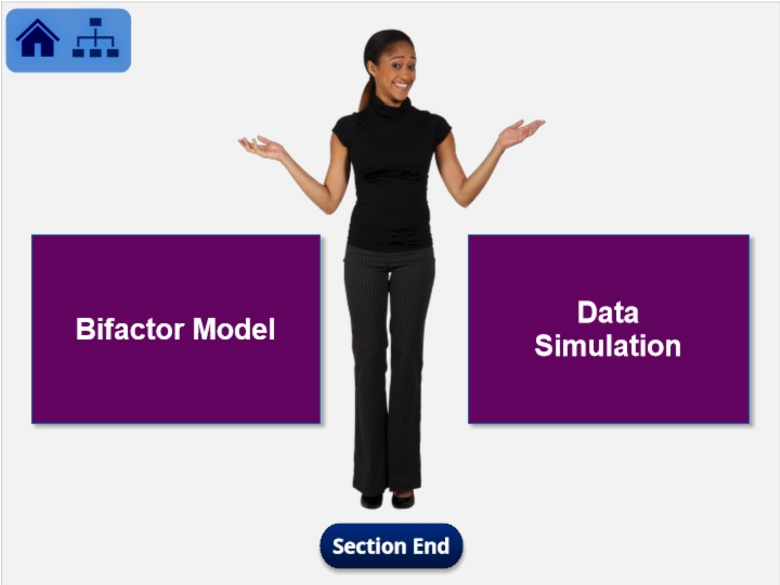


6.2 Cover: Section 4

6.3 Objectives

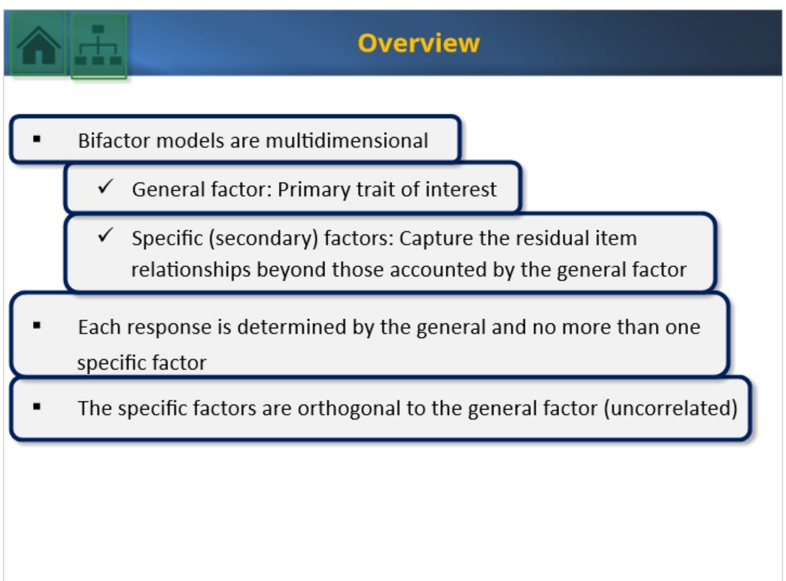


6.4 Topic Selection

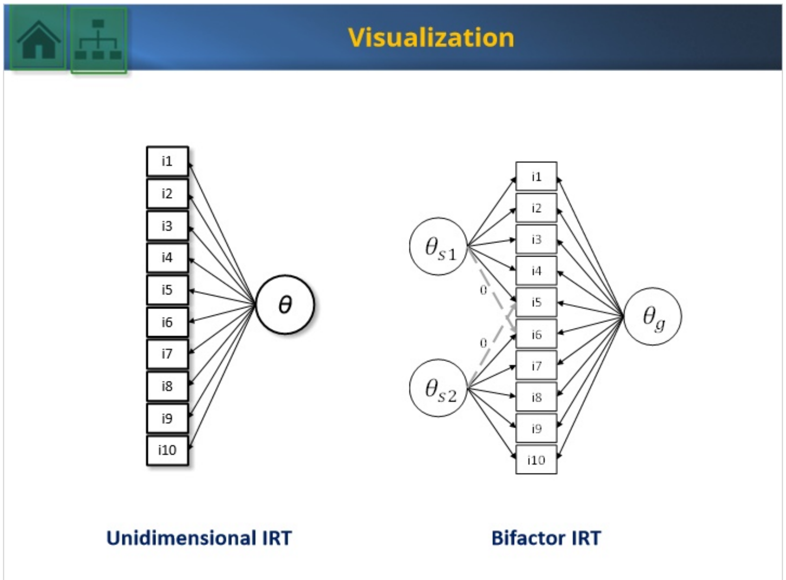


6.5 Bookmark: Bifactor Model

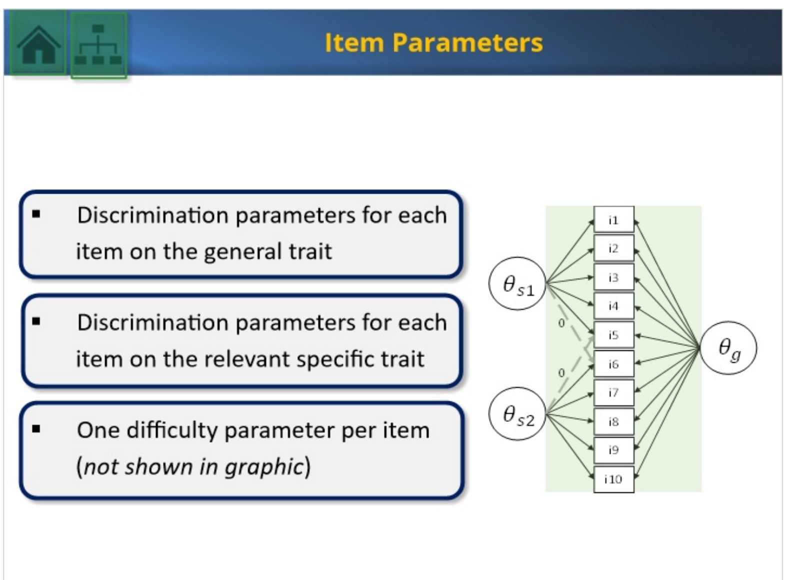
6.6 Bifactor Model (I)



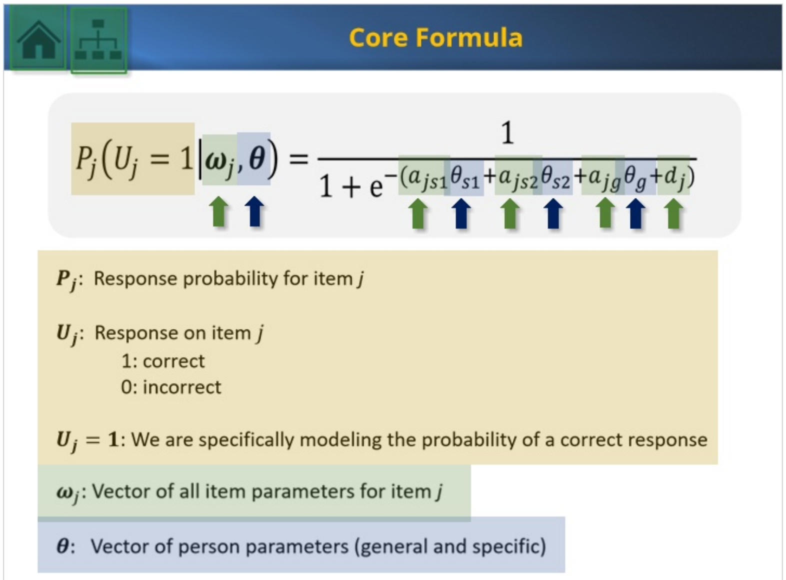
6.7 Bifactor Model (II)



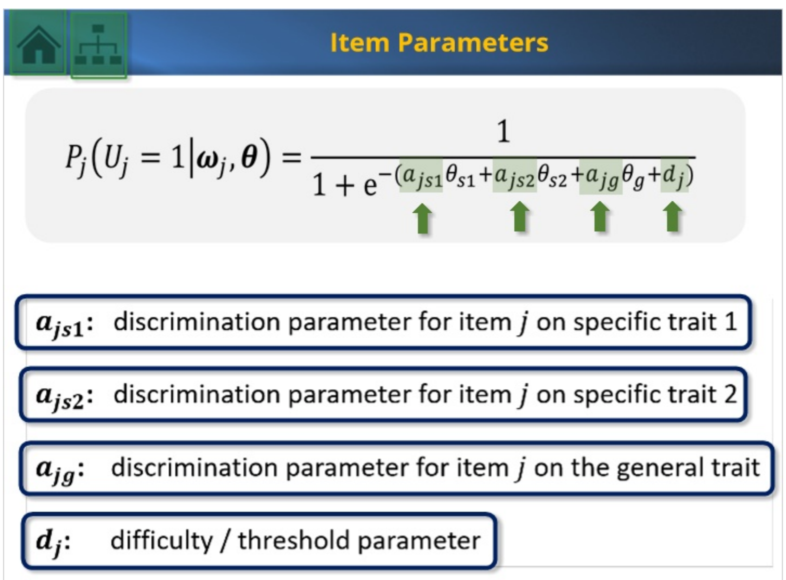
6.8 Bifactor Model (III)



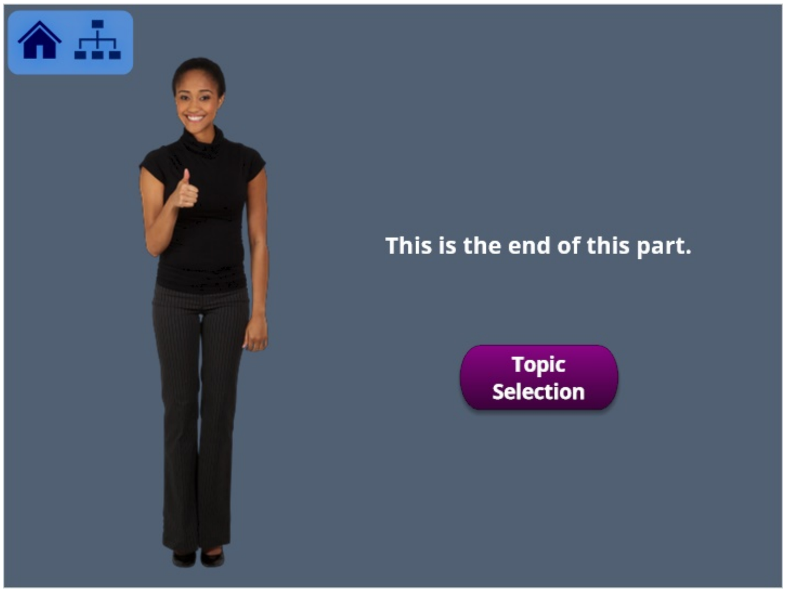
6.9 Bifactor Model (IV)



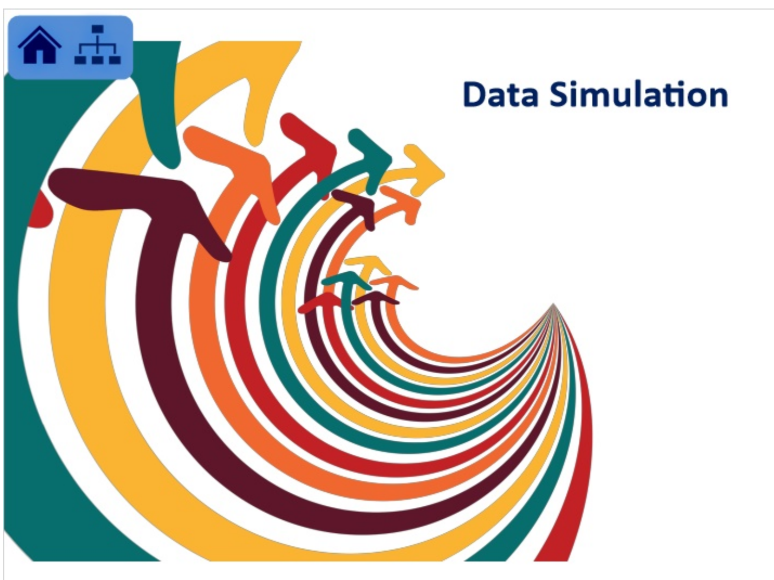
6.10 Bifactor Model (V)



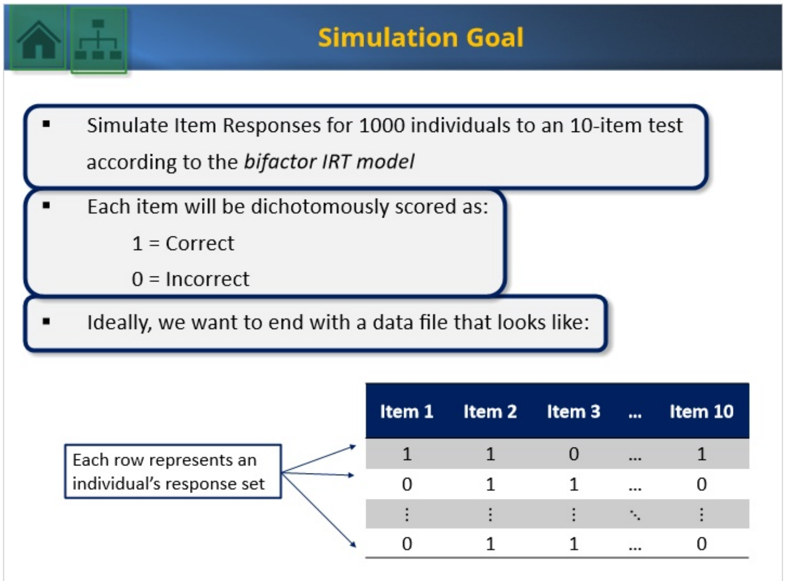
6.11 Bookend: Bifactor Model



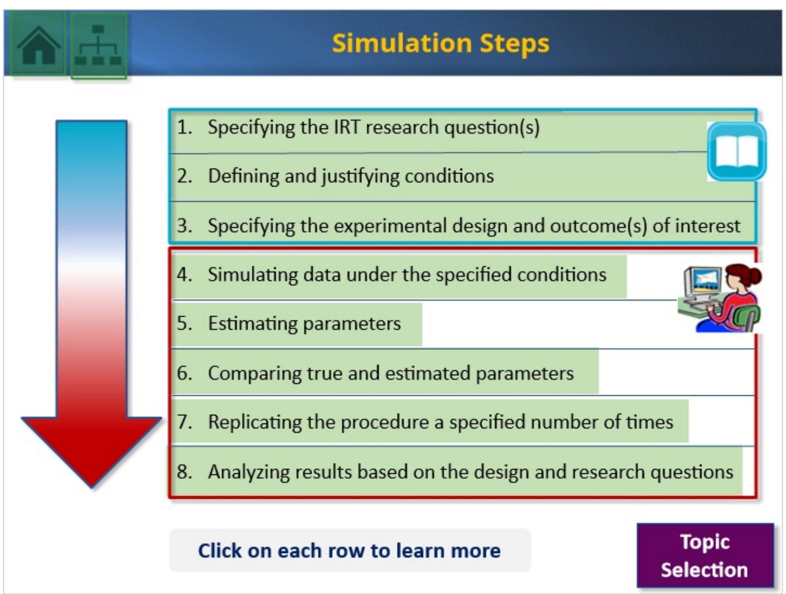
6.12 Bookmark: Data Simulation



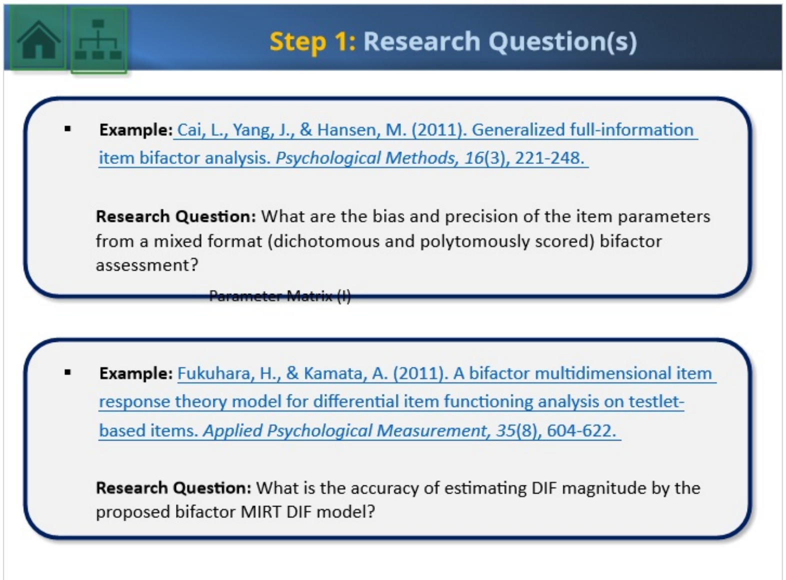
6.13 Simulation Goal



6.14 Step Selection

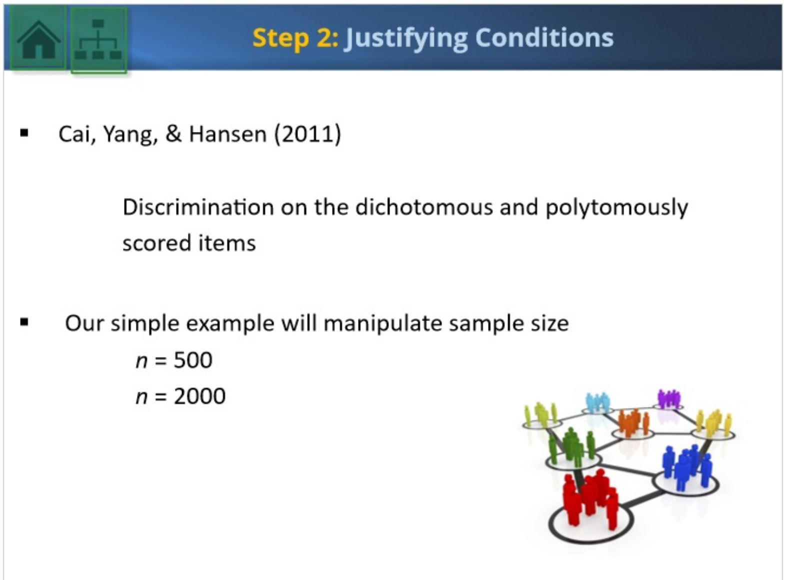


6.15 Research Questions



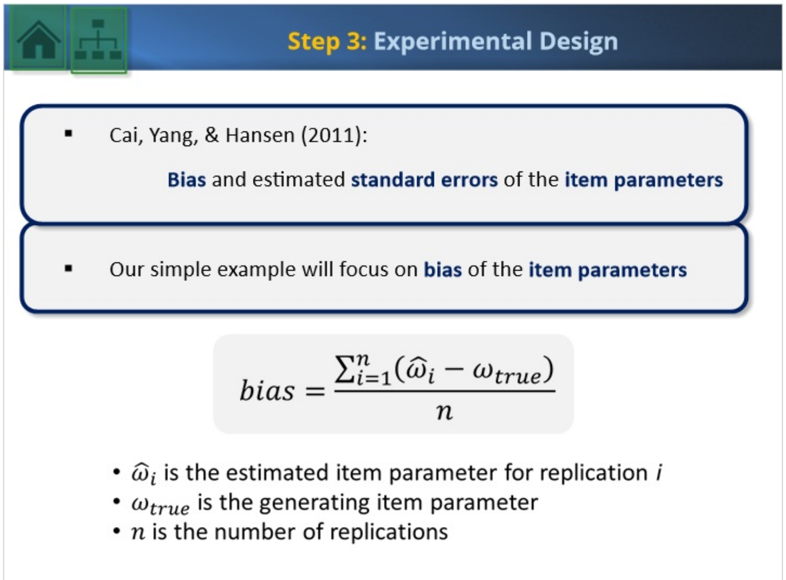
6.16 Bookend: Step 1

6.17 Justifying Conditions



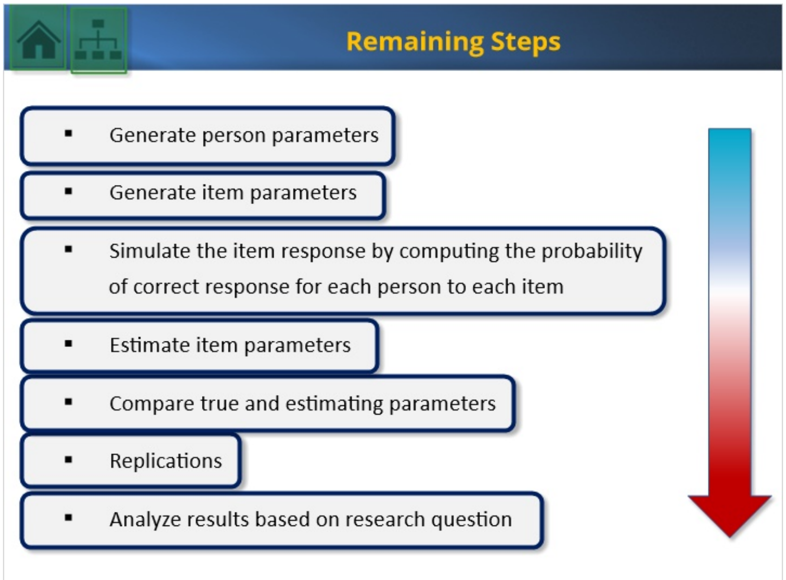
6.18 Bookend: Step 2

6.19 Experimental Design

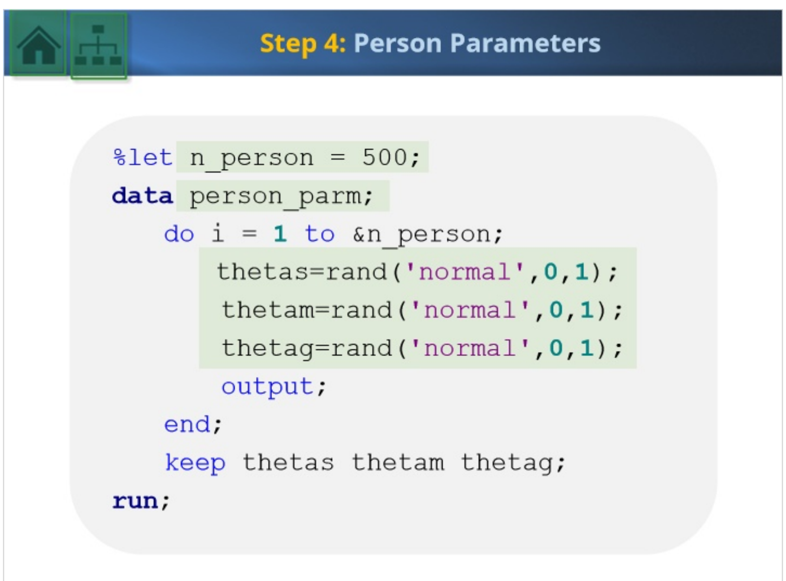


6.20 Bookend: Step 3

6.21 Remaining Steps



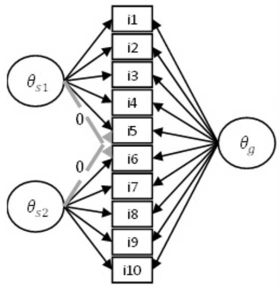
6.22 Person Parameters



6.23 Item Parameters

Step 4: Item Parameters

- Borrow item parameters from DeMars (2013)
- Loadings not in the model are fixed to 0



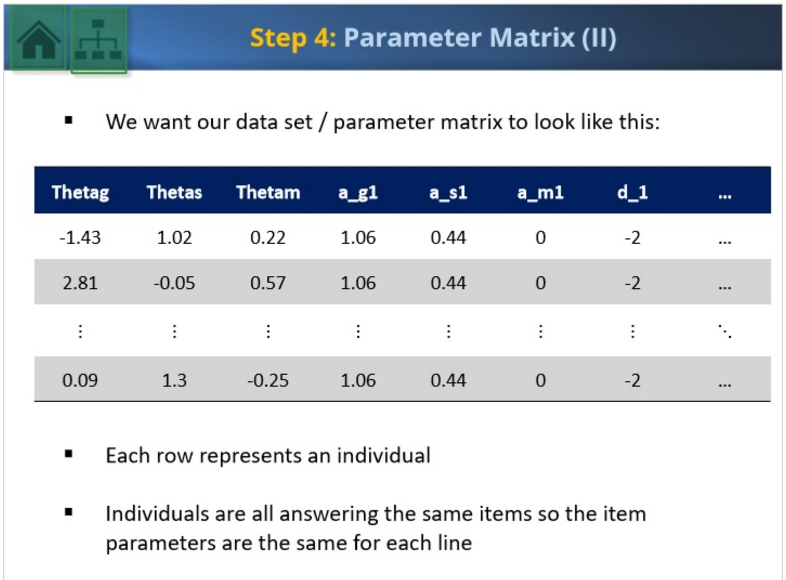
Item	a_{jg}	a_{js}	a_{jm}	d_j
1	1.06	0.44	0	-0.01
2	1.59	0.45	0	-0.13
3	0.98	0.36	0	0.44
4	0.88	0.48	0	-0.34
5	0.27	0.33	0	1.26
6	1.04	0	0.51	-0.77
7	0.61	0	0.23	-0.07
8	1.34	0	0.45	-0.14
9	0.96	0	0.46	2.22
10	1.20	0	0.42	0.18

6.24 Parameter Matrix (I)

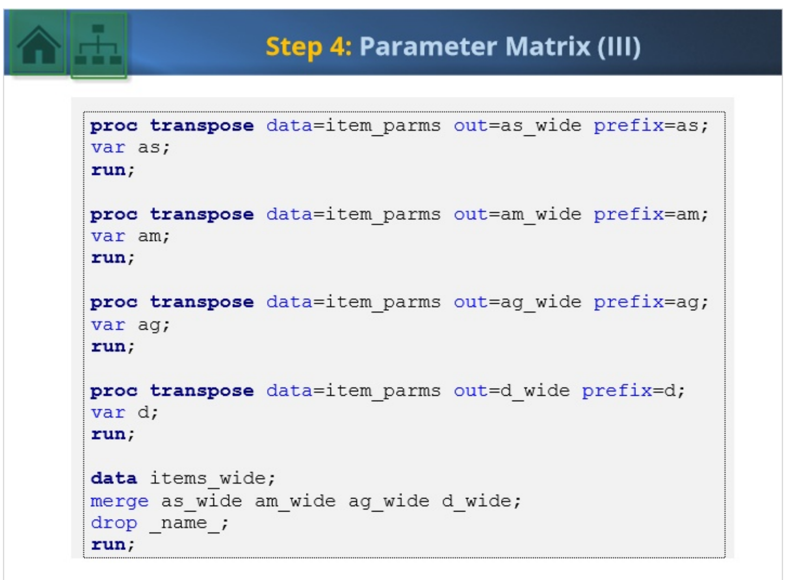
Step 4: Parameter Matrix (I)

```
data item_parms;
input ag as am d;
cards;
1.06 .44 0 -.01
1.59 .45 0 -.13
0.98 .36 0 0.44
0.88 .48 0 -.34
0.27 .33 0 1.26
1.04 0 .51 -.77
0.61 0 .23 -.07
1.34 0 .45 -.14
0.96 0 .46 2.22
1.20 0 .42 0.18
;
run;
```

6.25 Parameter Matrix (II)



6.26 Parameter Matrix (III)



6.27 Parameter Matrix (IV)

Step 4: Parameter Matrix (IV)

- Let's consider one individual and the first item:

thetas	thetam	thetag	as1	am1	ag1	d1
1.54	-1.30	-0.81	0.44	0	1.06	-0.01

- We want a data file that looks like this:

Item 1	Item 2	Item 3	...	Item 10
1	1	0	...	1
0	1	1	...	0
⋮	⋮	⋮	⋱	⋮
0	1	1	...	0

6.28 Parameter Matrix (V)

Step 4: Parameter Matrix (V)

```
data responses;
set all_parms;
   array as{&n_item} as1-as&n_item;
   array am{&n_item} am1-am&n_item;
   array ag{&n_item} ag1-ag&n_item;
   array d{&n_item} d1-d&n_item;
   array p{&n_item} p1-p&n_item;
   array item{&n_item} item1 - item&n_item;
   do j = 1 to &n_item;
      p[j]=1/(1+exp(-(as[j]*thetas+am[j]*thetam+ag[j]*thetag+d[j])
      y=rand('uniform');
     if y<=p[j] then item[j]=1;
     else if y>p[j] then item[j]=0;
   end;
run;
```

$$P_1(U_1 = 1|\boldsymbol{\omega}_j, \boldsymbol{\theta}) = \frac{1}{1 + e^{-(a_{s1}\theta_s + a_{m1}\theta_m + a_{g1}\theta_g + d_1)}}$$

6.29 Bookend: Step 4

6.30 Parameter Estimation

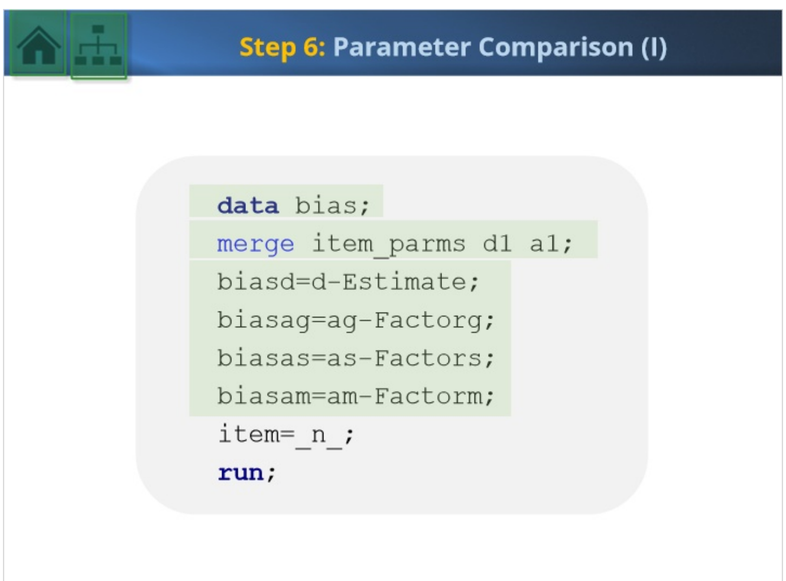
Step 5: Parameter Estimation

Estimate the bifactor model in *PROC IRT*

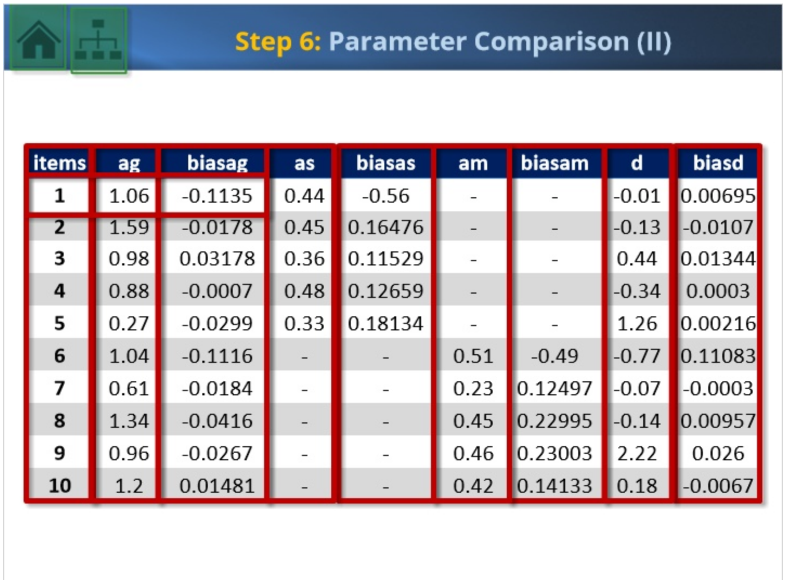
```
proc irt data=responses;
   var item1-item10;
   factor  Factorg===> item1-item10,
           Factors===> item1-item5= 1. load2 load3
                       load4 load5,
           Factorm===> item6-item10= 1. load7 load8
                       load9 load10;
   ods output ParameterEstimates=d1 slope=a1;
run;
quit;
```

6.31 Bookend: Step 5

6.32 Parameter Comparison (I)



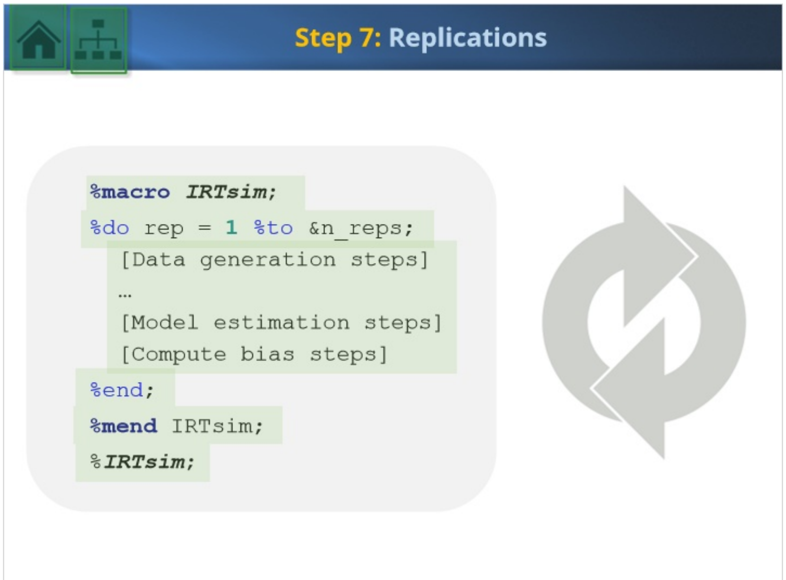
6.33 Parameter Comparison (II)



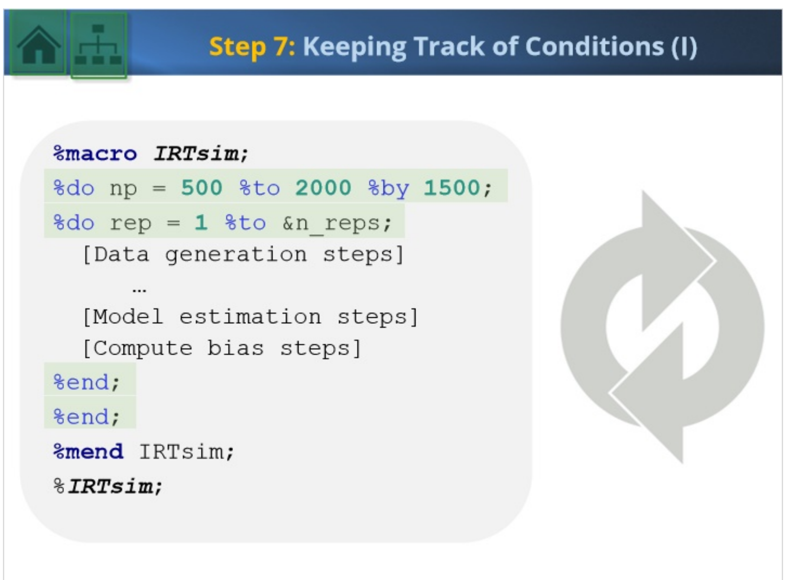
items	ag	biasag	as	biasas	am	biasam	d	biasd
1	1.06	-0.1135	0.44	-0.56	-	-	-0.01	0.00695
2	1.59	-0.0178	0.45	0.16476	-	-	-0.13	-0.0107
3	0.98	0.03178	0.36	0.11529	-	-	0.44	0.01344
4	0.88	-0.0007	0.48	0.12659	-	-	-0.34	0.0003
5	0.27	-0.0299	0.33	0.18134	-	-	1.26	0.00216
6	1.04	-0.1116	-	-	0.51	-0.49	-0.77	0.11083
7	0.61	-0.0184	-	-	0.23	0.12497	-0.07	-0.0003
8	1.34	-0.0416	-	-	0.45	0.22995	-0.14	0.00957
9	0.96	-0.0267	-	-	0.46	0.23003	2.22	0.026
10	1.2	0.01481	-	-	0.42	0.14133	0.18	-0.0067

6.34 Bookend: Step 6

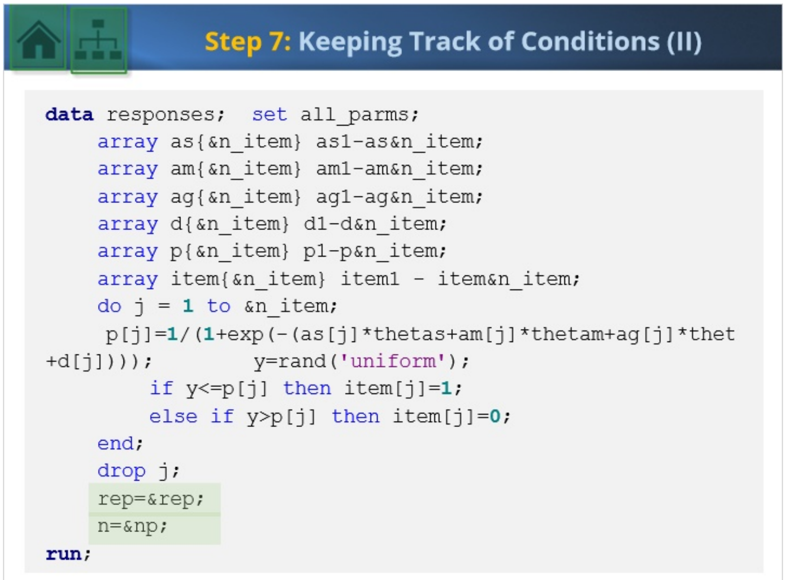
6.35 Replications (I)



6.36 Replications (II)

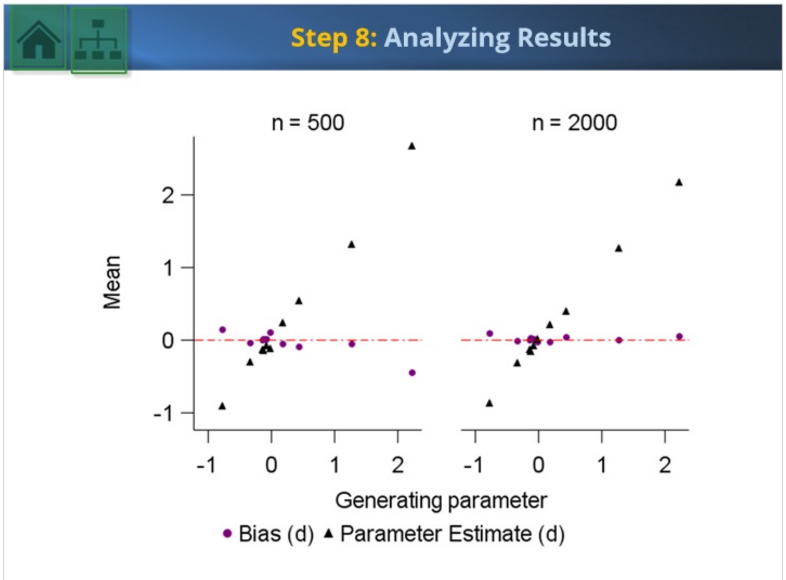


6.37 Replications (III)



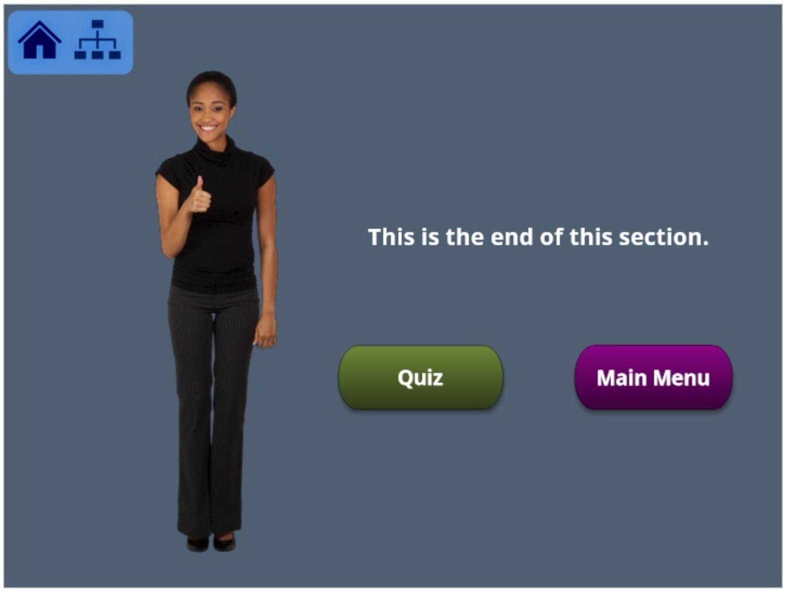
6.38 Bookend: Step 7

6.39 Analyzing Results



6.40 Bookend: Step 8

6.41 Bookend: Section 5



6.42 Module Cover (END)