

ITEMS DM13: Simulation Studies in Item Response Theory

This document contains only the core content slides from the module. In the digital module all slides can be accessed individually.

Module Organization

The module starts with an introductory section that leads to the main menu from which learners can select individual sections; this slide deck contains the slides for the five content sections only as shown below; green images in this document are clickable hotspot regions:

Main Menu

Foundations	Applications
01 Conceptual Foundations [30 Minutes]	05 Bifactor Model [Advanced] [15 Minutes]
02 Total Score Distributions [10 Minutes]	06 Simulation Activity [30 Minutes]
03 Two-parameter IRT Model [30 Minutes]	07 Quizzes [20 Minutes]
04 Graded Response IRT Model [20 Minutes]	

DM13 VIDEO (Section 5, Version 1.0)

1. Module Overview

1.1 Module Cover (START)



1.2 Content Team

A screenshot of a video player interface showing two instructors. The top bar says "Meet the instructors:". Below are two video frames. The left frame shows a man with glasses and a beard, labeled "Brian Leventhal" and "James Madison University". The right frame shows a woman, labeled "Allison Ames" and "University of Arkansas". Each video frame has a red play button icon at the bottom right. A purple "Back" button is at the bottom center.

1.3 Design Team

Meet the designers:

The interface shows two video player cards side-by-side. Each card features a portrait of a man, a play button icon at the bottom right, and a name and title below it. The background is dark blue.

Designer	Title
Jonathan Lehrfeld	ETS
André A. Rupp	Consultant

Back

1.4 Welcome

Welcome to the
ITEMS Module!

The woman to the left is Laura!

Along with the instructors
she will be guiding you
through the module content

Untitled Layer 1 (Slide Layer)

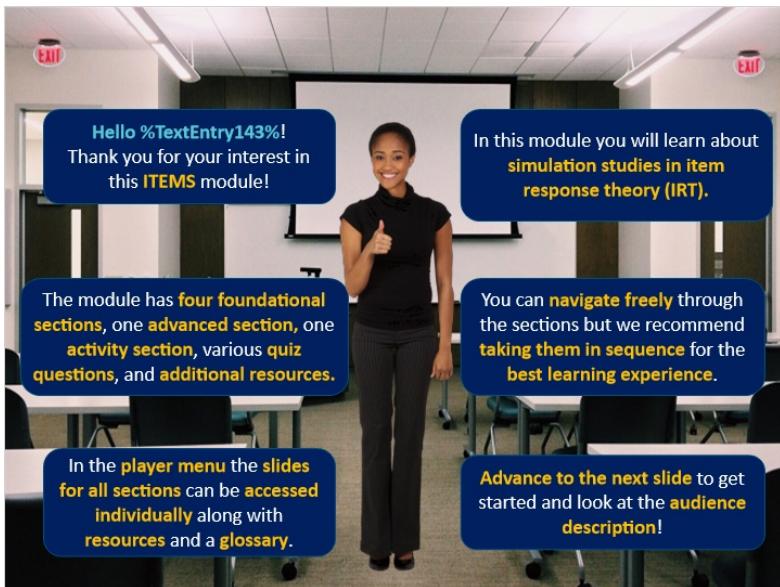


Welcome to the
ITEMS Module!

The woman to the left is Laura!

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through the module content

1.5 Overview



Hello %TextEntry143%!
Thank you for your interest in
this ITEMS module!

In this module you will learn about
**simulation studies in item
response theory (IRT).**

The module has **four foundational
sections**, one **advanced section**, one
activity section, various **quiz
questions**, and **additional resources**.

You can **navigate freely** through
the sections but we recommend
taking them in sequence for the
best learning experience.

In the **player menu** the **slides
for all sections** can be **accessed
individually** along with
resources and a **glossary**.

Advance to the next slide to get
started and look at the audience
description!

1.6 Target Audience

Target Audience

Anyone who would like a gentle statistical introduction to this topic:

- graduate students and faculty in Master's, Ph.D., or certificate programs
- psychometricians and other measurement professionals
- data scientists / analysts
- research assistants or research scientists
- technical project directors
- assessment developers



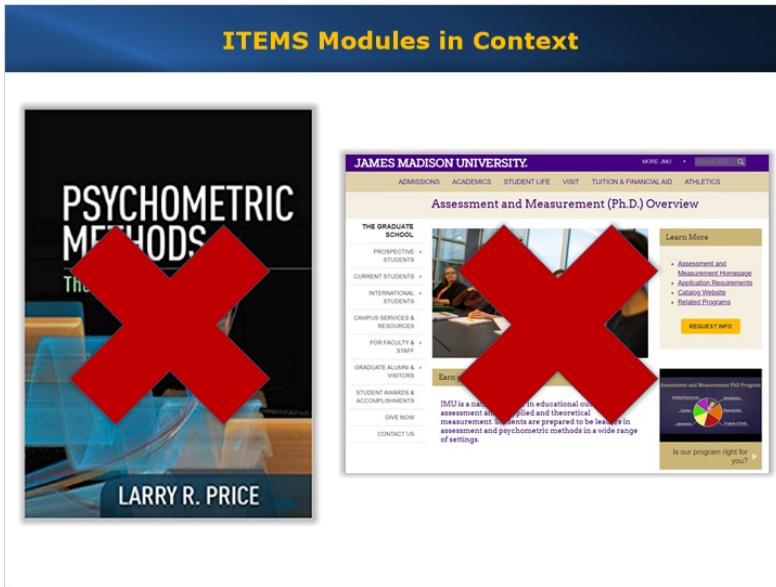
However, we hope that you find the information in this module useful no matter what your official title or role in an organization is!

1.7 Expectations (I)



Let's discuss expectations....

1.8 Expectations (II)



1.9 Learning Objectives

The slide is titled "Learning Objectives". It features a decorative background image of wooden blocks spelling out "PLAN" and a numbered list of learning objectives.

Learning Objectives:

1. Identify the major considerations for a Monte Carlo simulation study
2. Learn important SAS procedures and techniques for data simulation
3. Adapt basic simulation techniques to IRT-specific examples
4. Apply principles from examples to more complex models and scenarios

1.10 Software Note

Software Note

The instructors use the **commercial suite SAS** for all parts of the module



If you do not own SAS and work in **freeware suites like R or Python** you can still **learn about the principles** and **adopt code relatively easily**

1.11 Prerequisites

Prerequisites

- **Working knowledge of foundational measurement concepts:**
 - ✓ Unidimensional IRT models
 - ✓ Person parameters / latent trait parameters
 - ✓ Item parameters / thresholds
 - ✓ Response probabilities and observed scores
 - ✓ Visualizations of key relationships

- **Working knowledge of foundational simulation concepts:**
 - ✓ Bias, standard error, and mean squared error
 - ✓ Replications
 - ✓ Estimation, parameter recovery, and model fit

1.12 Resources

Resources

Module Citation

Leventhal, B., & Ames, A. (2020). Simulation studies in item response theory (Digital ITEMS Module 13). *Educational Measurement: Issues and Practice*, 39(2), XX-XX.

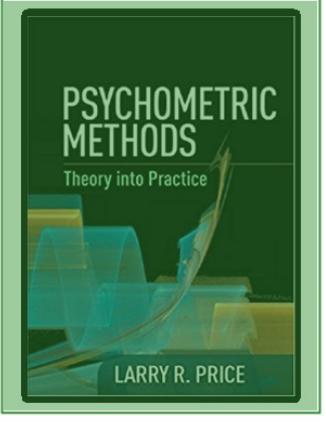


Additional Resources

References (Slide Layer)

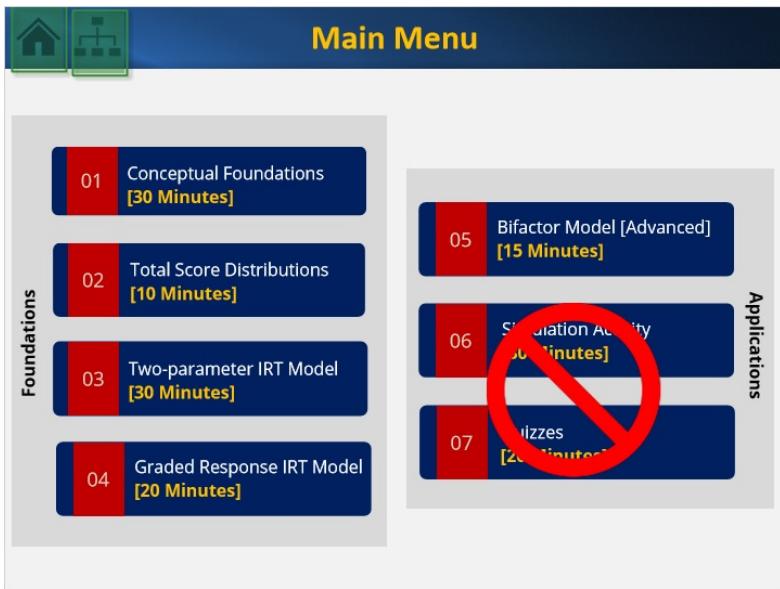
Resources



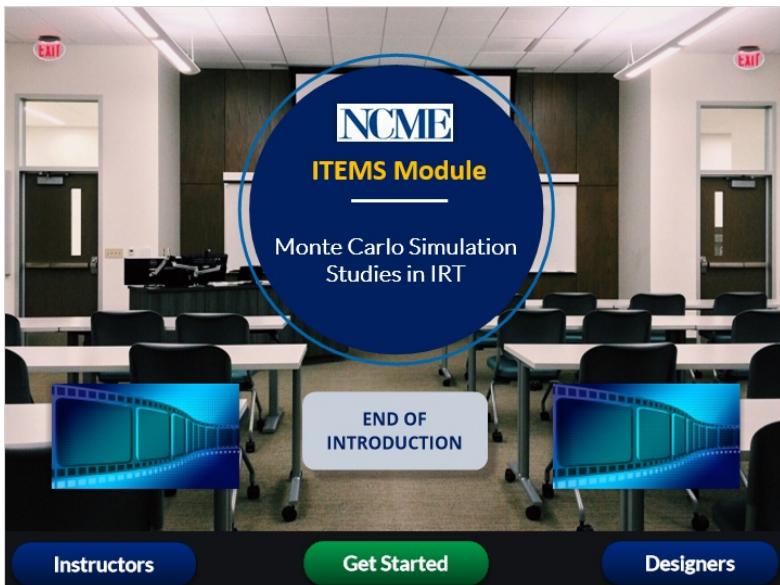


Back

1.13 Main Menu

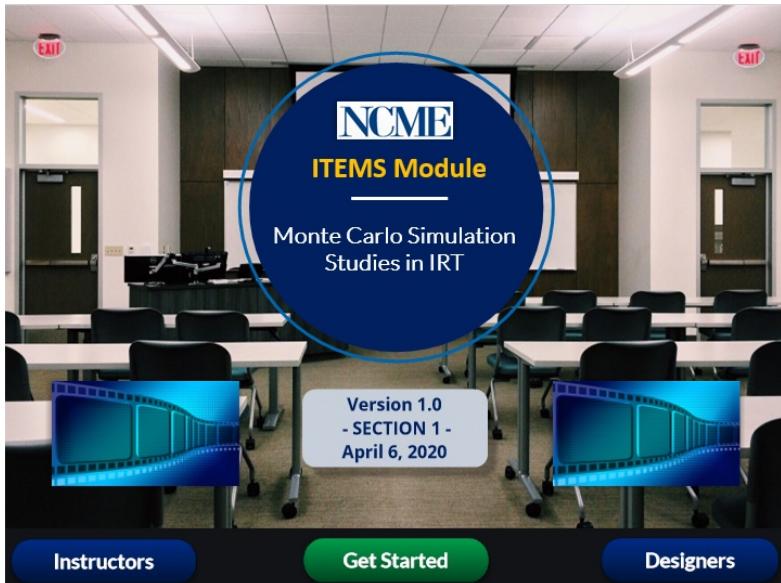


1.14 Module Cover (END)

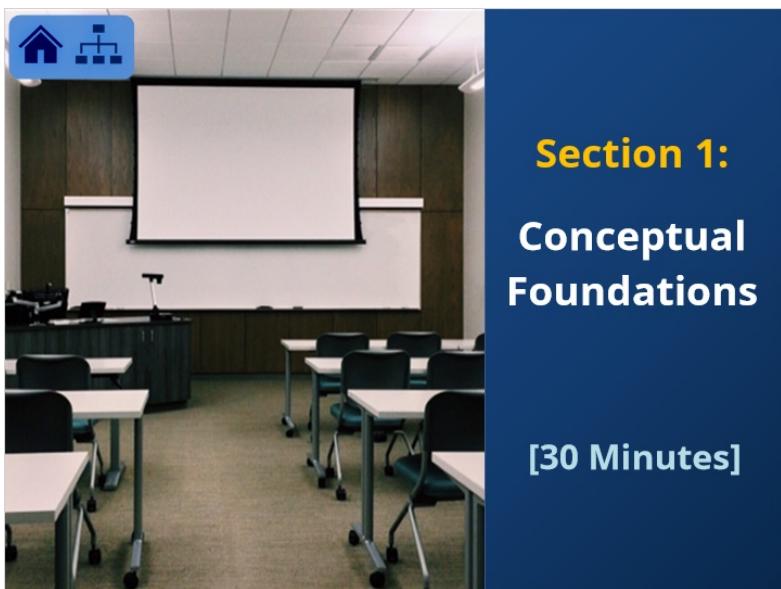


2. Section 1: Conceptual Foundations

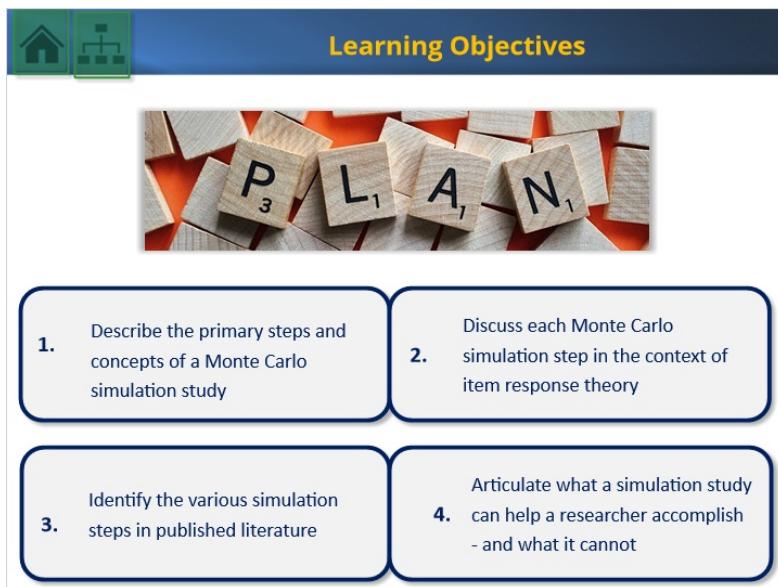
2.1 Module Cover (START)



2.2 Cover: Section 1



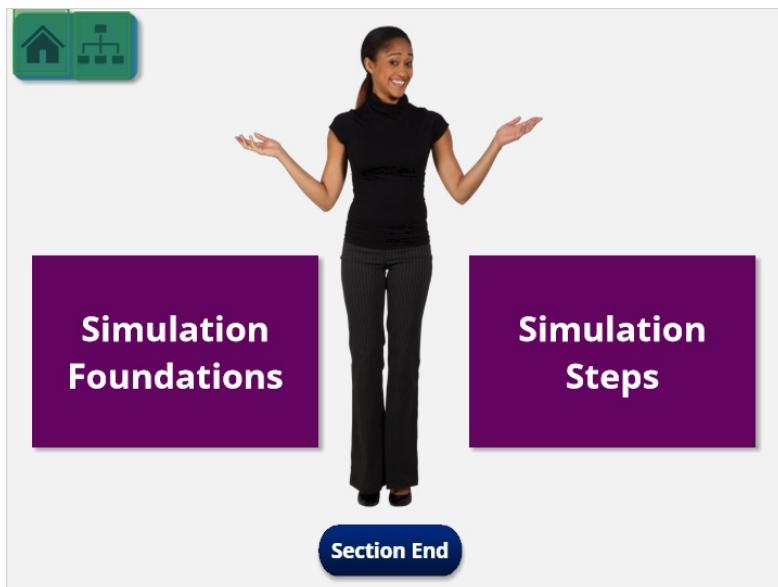
2.3 Objectives



The slide features a blue header bar with the text "Learning Objectives" in yellow. Below the header is a photograph of wooden blocks spelling out "PLAN" against a red background. The main content area is divided into four numbered boxes:

- 1. Describe the primary steps and concepts of a Monte Carlo simulation study
- 2. Discuss each Monte Carlo simulation step in the context of item response theory
- 3. Identify the various simulation steps in published literature
- 4. Articulate what a simulation study can help a researcher accomplish - and what it cannot

2.4 Topic Selection



The slide features a woman in a black top and pants standing with her arms outstretched. She is positioned between two purple rectangular boxes: "Simulation Foundations" on the left and "Simulation Steps" on the right. In the bottom center is a dark blue button labeled "Section End". The top left corner contains a green icon bar with a house and a tree.

2.5 Bookmark: Foundations



2.6 Important Resources

The image is a screenshot of a slide titled "Important Resources". The title is in yellow text on a dark blue header bar. Below the title is a bulleted list of resources:

- General resources:
 - [Feinberg, R.R., & Rubright, J.D. \(2016\). Conducting simulation studies in psychometrics. *Educational Measurement: Issues and Practice*, 35\(2\), 36–49.](#)
 - [Harwell, M., Stone, C.A., Hsu, T., & Kirisci, L. \(1996\). Monte Carlo studies in item response theory. *Applied Psychological Measurement*, 20\(2\), 101–125.](#)
- Application of a simulation:
 - [Drasgow, F. \(1989\). An evaluation of marginal maximum likelihood estimation for the two-parameter logistic model. *Applied Psychological Measurement*, 13, 77–90.](#)

Below the list is a thumbnail image of a journal article titled "Conducting Simulation Studies in Psychometrics" by Richard A. Feinberg and Jonathan D. Rubright. The thumbnail includes the journal title "Educational Measurement: Issues and Practice", the volume and issue information "Volume 2006, Vol. 35, No. 2 pp. 36-49", and the abstract text of the article.

2.7 Standards

Six standards:

1. Can the problem could be solved analytically?
2. Is the study a minor extension of existing results?
3. Is an appropriate experimental design and analysis of MC results used?
4. Are locally-written software or modifications of public software properly documented?
5. Do the results depend on the starting values for iterative parameter estimation methods?
6. Are the choices of distributional assumptions and independent variables and their values realistic?

Reference

Reference (Slide Layer)

Applied Psychological Measurement

Monte Carlo Studies in Item Response Theory
Michael Harrel, Clement A. Stone, Tee-Chih Hsu more...
First Published June 1, 1996 - Research Article
<https://doi.org/10.1177/01462699020200201>

Abstract

Monte carlo studies are being used in item response theory (IRT) to provide information about how validly these methods can be applied to realistic datasets (e.g., small numbers of examinees and multidimensional data). This paper describes the conditions under which monte carlo studies are appropriate in IRT-based research, the kinds of problems these techniques have been applied to, available computer programs for generating item responses and estimating item and examinee parameters, and the importance of conceptualizing these studies as statistical sampling experiments that should be subject to the same principles of experimental design and data analysis that pertain to empirical studies. The number of replications that should be used in these studies is also addressed.

Back

2.8 Introduction (I)

The slide has a blue header bar with the title "Introduction to Simulation Studies". Below the header, there is a list of three bullet points, each containing a quote from a research paper. The quotes are as follows:

- ❖ "As few as 200 persons and five items were required for 'essentially' unbiased estimates [for the 2PL]." (Drasgow, 1989; discussed in de Ayala, 2009)
- ❖ "The 2PL equating function is robust to the violation of local item independence." (de Ayala, 2014)
- ❖ "A 2:1 or larger ratio of examinees to items produced stable item parameter estimates [for the partial credit model]." (de Ayala, 1990; discussed in de Ayala, 2009)

2.9 Introduction (II)

The slide has a blue header bar with the title "Introduction to Simulation Studies". Below the header, there are two main sections, each enclosed in a rounded rectangle:

- **How were these sample size heuristics and claims determined?**
 - ✓ Monte Carlo simulation (MCS) techniques were used.
 - ✓ In MCS, data are created by researchers based on a model and used to answer a methodological research question.
- **Applications of MCS techniques in IRT:**
 - ✓ Evaluating estimation procedures or parameter recovery,
 - ✓ Evaluating the statistical properties of an IRT-based statistic, or
 - ✓ Comparing methodologies used in conjunction with IRT.

2.10 Example: Bias

Example: Estimation Bias

Evaluating estimation procedures or parameter recovery

Research Question:

Are the item parameter estimates derived from this new estimation procedure, on average, close to their true values at varying sample sizes?

Illustration:
Bias vs. Unbiased

Introduction to Simulation Studies (Slide Layer)

Introduction to Simulation Studies

If the center of the target is the “true” population parameter. When estimates are clustered around the “true” parameter, the estimator is said to be unbiased.

Biased 	Unbiased 
--	--

Back

2.11 Example: Type-I Error

Example: Properties of a Statistic

Evaluating the statistical properties of an IRT-based statistic
For example, a goodness-of-fit statistic is evaluated

Research Question:
What is the type-I error rate of this statistic?

Illustration:
Type-I Error

Type-I Error (Slide Layer)

Type-I Error

	Finding: No Misfit	Finding: Misfit
True Model: No Misfit	Correct Decision! ✓	Type I Error! ⚡
True Model: Misfit	Type II Error! ⚡	Correct Decision! ✓ (Power)

Back

2.12 Caveats

The screenshot shows a software window with a blue header bar. On the left, there are two icons: a house and a tree-like structure. The header bar contains the word "Caveats". Below the header is a white box with a black border containing the text: "MCS are not a replacement for empirical studies that use real data!". Underneath this box, the text "They cannot answer questions such as:" is followed by a list of four items, each preceded by a checkmark and enclosed in a blue-bordered box:

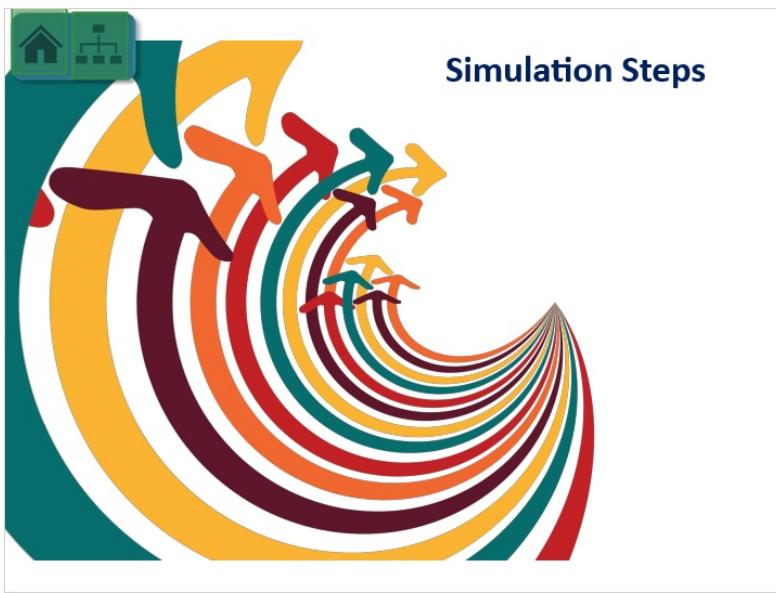
- ✓ "Is differential item functioning (DIF) present in a set of items?"
- ✓ "Is the intervention effective for deterring cheating on assessments?"
- ✓ "How motivated were students in responding to survey items?"
- ✓ "Is this test form easier or more difficult than the previous one?"

Below the list is a yellow warning sign icon with an exclamation mark.

2.13 Bookend: Foundations

The screenshot shows a software window with a dark blue background. In the top left corner, there are two icons: a house and a tree-like structure. A woman in a black top and dark pants is standing in the center, smiling and giving a thumbs-up. To her right, the text "This is the end of this part." is displayed. In the bottom right corner, there is a purple rounded rectangle button with the text "Topic Selection" in white.

2.14 Bookmark: Simulation Steps



2.15 Step Selection

An interface titled "Overview" showing a list of 8 steps for simulation. A large red arrow points downwards towards the list. The steps are numbered 1 through 8 and have corresponding icons: a book icon for step 2, a person at a computer icon for step 5, and a person at a desk icon for step 7.

1. Specifying the IRT research question(s)	
2. Defining and justifying conditions	
3. Specifying the experimental design and outcome(s) of interest	
4. Simulating data under the specified conditions	
5. Estimating parameters	
6. Comparing true and estimated parameters	
7. Replicating the procedure a specified number of times	
8. Analyzing results based on the design and research questions	

Click on each row to learn more

Topic Selection

2.16 Research Questions (I)

 Step 1: Research Question(s)

The research question will guide all other aspects of the simulation

→ “Articulating a clear research question ... forces the researcher to ensure that the exact design choices made align with the question being asked.”
Feinberg & Rubright, 2016

→ “... relies heavily on knowledge of a literature.”
Harwell, Stone, Hsu, & Kirisci, 1996



2.17 Research Questions (II)

 Step 1: Example

Drasgow (1989) provides background from a literature review:

- Large sample sizes and long tests
- Common situations in which these are unrealistic or untenable

Example research question from Drasgow (1989):

What is the range of conditions that allow accurate calibration of two-parameter logistic items by marginal maximum likelihood estimation?

Reference

Reference (Slide Layer)

The screenshot shows a slide titled "Reference" with a blue header bar containing icons for home and search. The main content area displays an abstract from a journal article:

An evaluation of marginal maximum likelihood estimation for the two-parameter logistic model.

Dragg, F. (1989). An evaluation of marginal maximum likelihood estimation for the two-parameter logistic model. *Applied Psychological Measurement*, 13(1), 77–90. <https://doi.org/10.1177/014662168901300108>

Citation

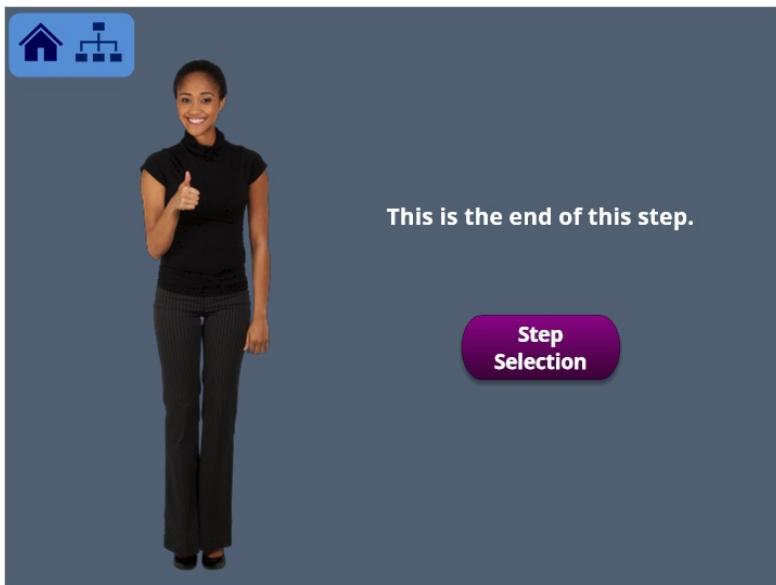
Dragg, F. (1989). An evaluation of marginal maximum likelihood estimation for the two-parameter logistic model. *Applied Psychological Measurement*, 13(1), 77–90. <https://doi.org/10.1177/014662168901300108>

Abstract

Investigated the accuracy of marginal maximum likelihood estimates of the item parameters of the 2-parameter logistic model. Estimates were obtained for 4 sample sizes (200, 300, 500, and 1,000) and 4 test lengths (5-, 10-, 15-, and 25-items); joint maximum likelihood estimates were also computed for the 2 longer test lengths. Each condition was replicated 10 times, which allowed evaluation of the accuracy of estimated item characteristic curves, item parameter estimates, and estimated standard errors of item parameter estimates for individual items. Results show that items typical for widely used job selection scale and moderately easy tests had satisfactory marginal estimation for all sample sizes and test lengths. Larger samples were required for items with extreme difficulty or discrimination parameters. Marginal estimation was substantially better than joint maximum likelihood estimation. (PsycINFO Database Record (c) 2016 APA, all rights reserved)

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2.18 Bookend: Step 1



2.19 Justifying Conditions



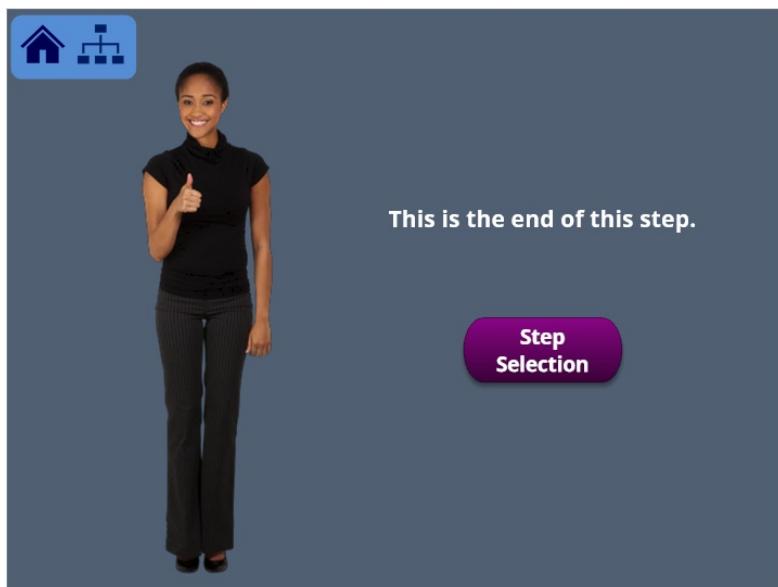
Step 2: Justification

- The research questions dictate the independent variables and resulting conditions to be included in the simulation
- Conditions must be justified and clearly delineated

Example from Drasgow (1989):

- Test length: 5 to 25 items
- Sample size: 200 to 1,000 simulated examinees
- Justification: These ranges encompass the values seen in many applied studies at the time

2.20 Bookend: Step 2



2.21 Experimental Design (I)

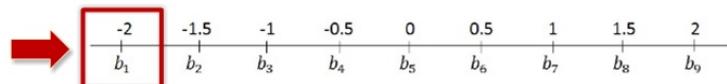
 

Step 3: Experimental Conditions (I)

"Model parameters also represent an independent variable in an MCS"
Harwell, Stone, Hsu, & Kirisci, 1996

Fixed effect: Parameters are represented as (often equally-spaced) values across a fixed range or as estimates from a previously calibrated test

- **Example:** 9 b parameters are evenly spaced from -2 to 2
- **Advantage:** Simple setup with known values
- **Disadvantage:** Limited generalizability



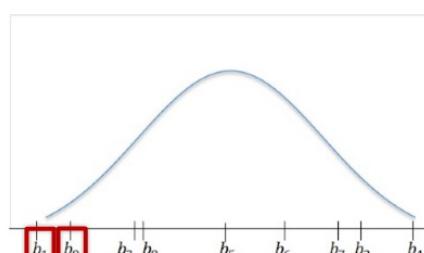
2.22 Experimental Design (II)

Step 3: Experimental Conditions (II)

Random effect: Parameter values are sampled from a specified distribution

- **Example:** $b \sim N(0, \sigma^2 = 4)$
- **Advantage:** Some generalizability is obtained
- **Disadvantage:** Unusual combinations of parameters might occur



2.23 Experimental Design (III)



Step 3: Experimental Conditions (III)

"Researchers also must consider the relationship between the number of independent variables, the efficiency of the study, and the interpretability of the results."

Harwell, Stone, Hsu, & Kirisci, 1996

Example from Drasgow (1989)

- Test length (4 levels: 5, 10, 15, 25)
- Sample size (4 levels: 200, 300, 500, 1000)
- Item parameters (2 levels: typical, extreme)
- $4 \times 4 \times 2$ fully crossed design = 32 total combinations

2.24 Experimental Design (IV)



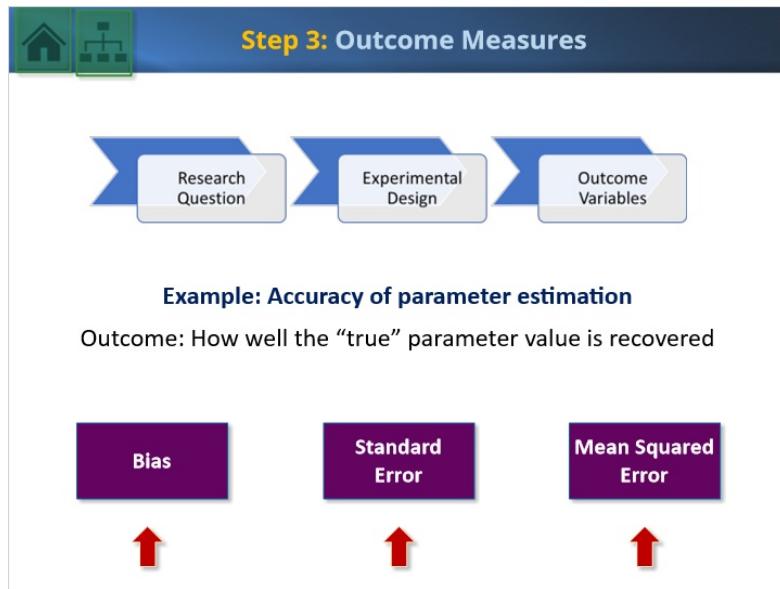
Step 3: Experimental Conditions (IV)

Type and number of independent variables guide selection of experimental design

Example from Drasgow (1989)

- **Factorial design:** Small number of independent variables with relatively few values
- **Investigative focus:** Each combination of the conditions is important and will be used in the simulation

2.25 Experimental Design (V)



Bias (Slide Layer)

Outcome: Bias

- Average deviance between the “true” and estimated values of the parameter
- Systematic error

$$\frac{\sum_{i=1}^n (\hat{\theta}_i - \theta_{True})}{n}$$

$\hat{\theta}_i$: Parameter estimate from replication i
 θ_{True} : True parameter value
 n : Total number of replications

Back

Standard Error (Slide Layer)

  **Outcome:** Standard Error

- Standard deviation across estimated values
- Random error

$$\sqrt{\frac{\sum_{i=1}^n (\hat{\theta}_i - \bar{\theta})^2}{n - 1}}$$

$\bar{\theta}$ is the average parameter value

$\hat{\theta}_i$ is the estimated parameter value of replication i

n is the replication

[Back](#)

Mean Squared Error (Slide Layer)

  **Outcome:** Mean Square Error

- Average squared deviance between the estimated and true values
- Total error

$$\sqrt{\frac{\sum_{i=1}^n (\hat{\theta}_i - \theta_{True})^2}{n - 1}}$$

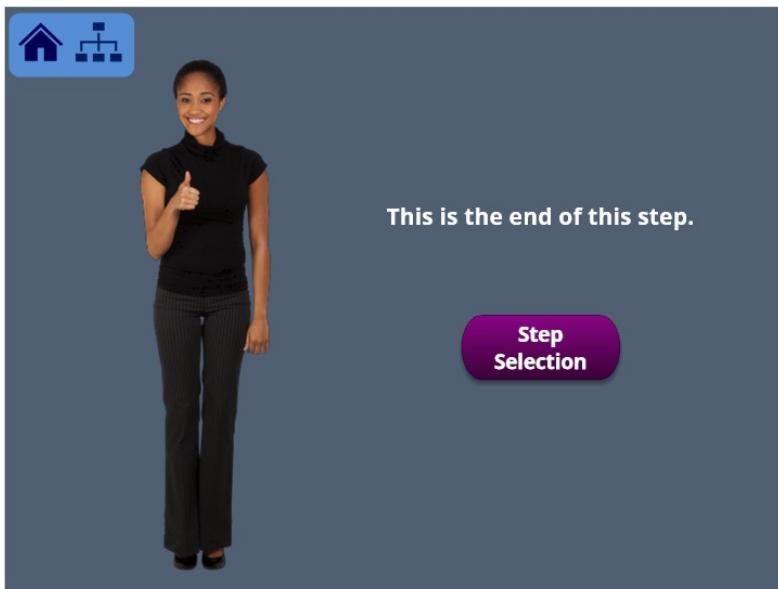
$\hat{\theta}_i$: Parameter estimate from replication i

θ_{True} : True parameter value

n : Total number of replications

[Back](#)

2.26 Bookend: Step 3



2.27 Simulating Data (I)

A bookend slide for "Step 4: Data Simulation". The title is at the top. Below it is a list of two steps: 1. Generate individual samples for each replication and 2. Generate a large number of responses (the population) and then randomly sample from the population so that each sample serves as a replication. At the bottom is a colorful silhouette of a diverse group of people.

2.28 Simulating Data (II)

Step 4: Person Parameters

Simulate each individual's latent ability

Common Choice: $\theta \sim N(0,1)$

2.29 Simulating Data (III)

Step 4: Response Probabilities

Combine latent ability with item parameters
to obtain a matrix of response probabilities

Dichotomous IRT

Polytomous IRT

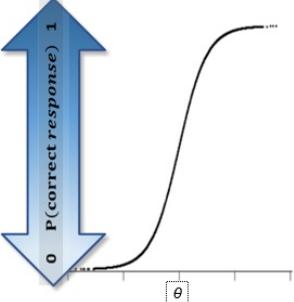
2.30 Simulating Data (IV)

Step 4: Dichotomous Responses

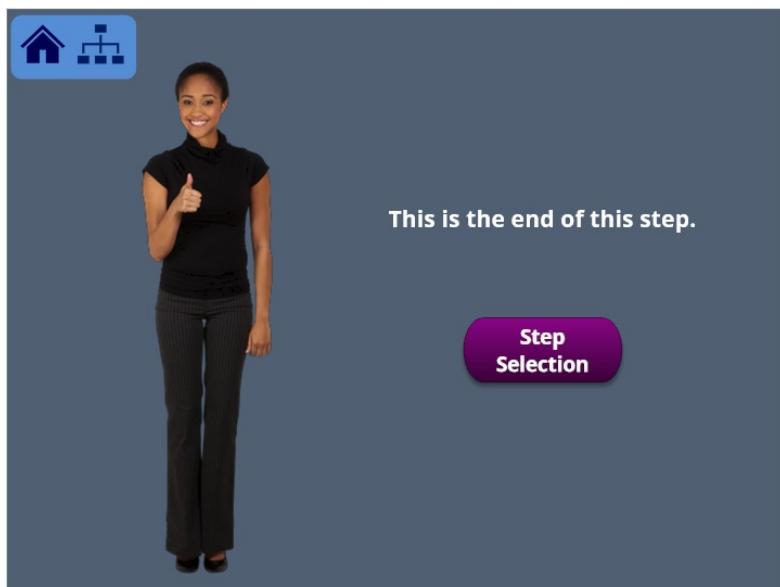
Transform probabilities into responses

- $P(\text{correct response}) = \text{high}$
Response = 1
Correct
- $P(\text{correct response}) = \text{low}$
Response = 0
Incorrect



The graph shows a sigmoid curve starting near zero and approaching 1. A vertical double-headed arrow is positioned next to the curve, with its top end labeled "P(correct response) 1" and its bottom end labeled "0". The horizontal axis is marked with a small square containing the Greek letter theta (θ).

2.31 Bookend: Step 4



2.32 Estimating Parameters (I)



Step 5: Parameter Estimation

- **Available tools:**
 - ✓ Commercially available programs (e.g., Winsteps or Bilog)
 - ✓ Open source packages (e.g., R package ltm or Stan)
 - ✓ Their own routine (e.g., Fortran, SAS, or Basic)
- **Adequacy** of the estimation algorithm must be documented
- **Validation evidence examples:**
 - ✓ Using the program to analyze a well-known dataset
 - ✓ Analyzing item responses with perfect fit to an item response function in which case the estimation routine should return the exact item parameters

2.33 Estimating Parameters (II)



Step 5: Example

Example from Drasgow (1989)

- A FORTRAN program was written to estimate the a (discrimination) and b (difficulty) parameters of the 2PL
- Validity evidence was not provided



2.34 Estimating Parameters (III)



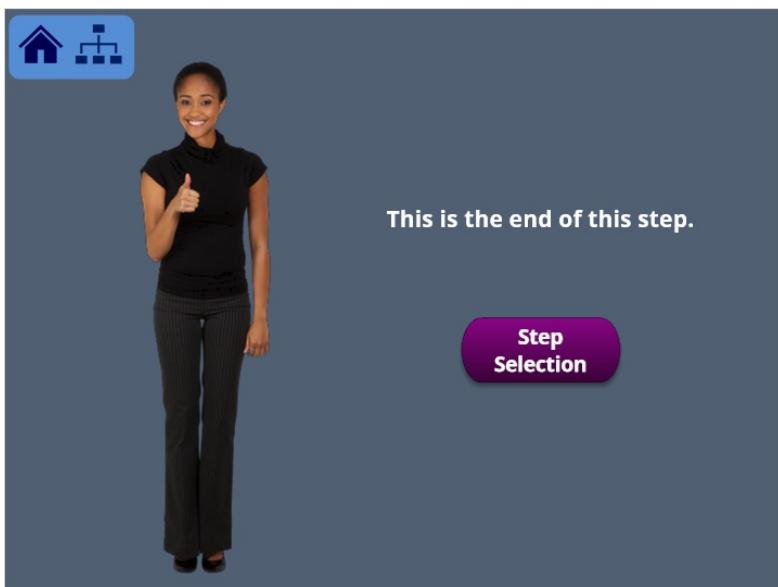
Step 5: Addendum

Example from Drasgow (1989)

Interest was in parameter recovery for the 2PL so data were generated from the 2PL

In some MCS studies the data-generating model is more complex than the analysis model so that misfit is introduced (e.g., Orlando & Thissen, 2000)

2.35 Bookend: Step 5



2.36 Outcome Measures

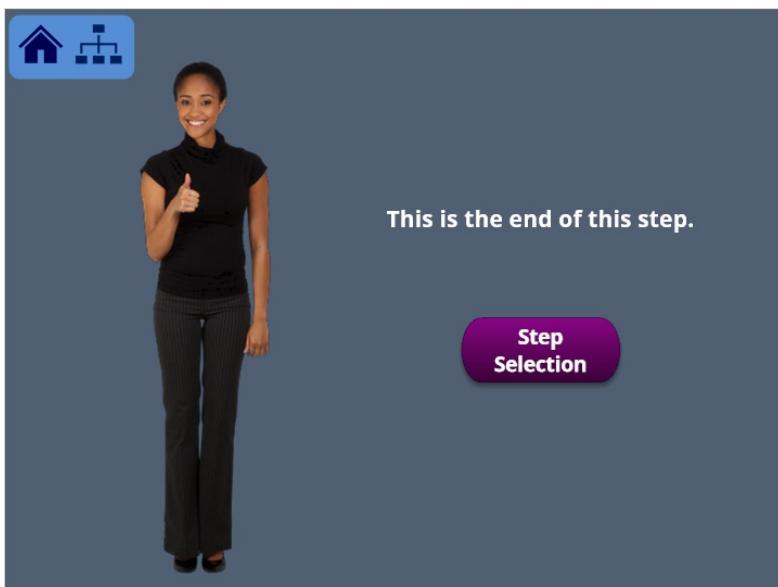


Step 6: Outcome Measures

Example from Drasgow (1989)

- Bias of discrimination and difficulty parameter estimates
- Distance between the true 2PL response curve and the estimated response curve
- Comparison of observed and mean estimated standard errors

2.37 Bookend: Step 6



2.38 Replications



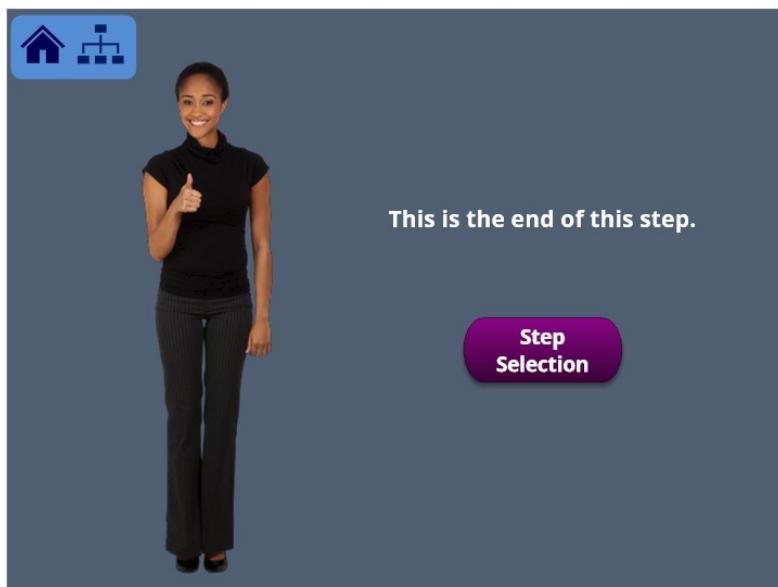
Step 7: Replications

"More replications are always better in terms of producing a more accurate and reliable estimate of the parameters of interest." Feinberg & Rubright, 2016

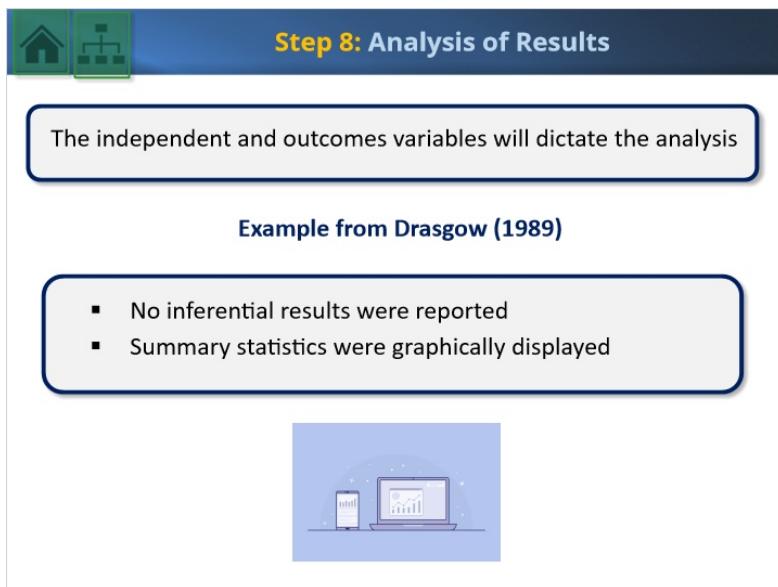
The purpose of the study has an important effect on the number of replications selected.

Study Purpose	Replications
Parameter Recovery	500-1000+
Bayesian MCS	10-100
Comparing IRT-based methodologies	10-100

2.39 Bookend: Step 7



2.40 Analyzing Results



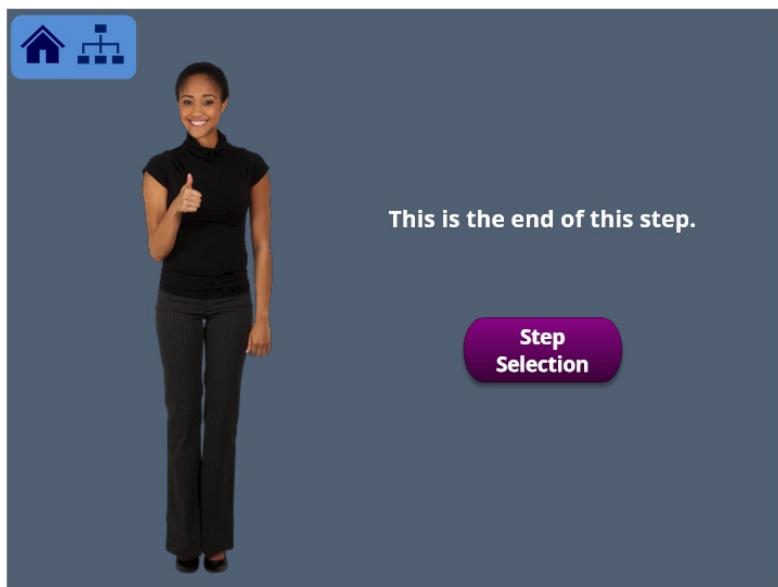
The independent and outcomes variables will dictate the analysis

Example from Drasgow (1989)

- No inferential results were reported
- Summary statistics were graphically displayed



2.41 Bookend: Step 8



This is the end of this step.

Step Selection

2.42 Simulation Standards



Standards for Monte Carlo Studies

1. Can the problem could be solved analytically?
2. Is the study a minor extension of existing results?
3. Is an appropriate experimental design and analysis of MC results used?
4. Are locally-written software or modifications of public software properly documented?
5. Do the results depend on the starting values for iterative parameter estimation methods?
6. Are the choices of distributional assumptions and independent variables and their values realistic?

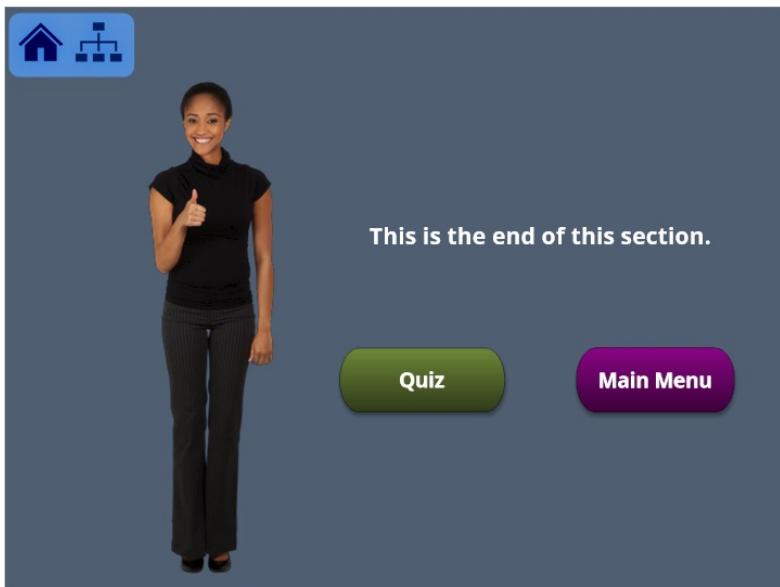
2.43 Summary



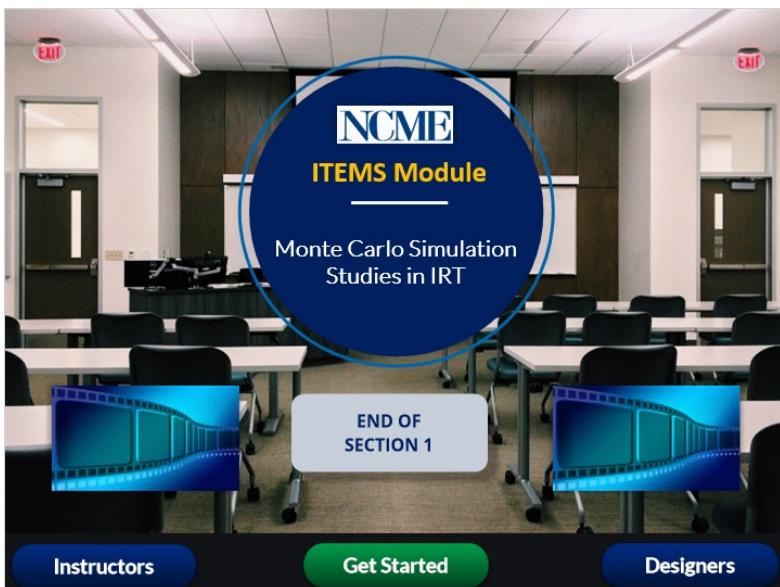
Summary

- Important steps of Monte Carlo simulation studies:
 - ✓ Always begins with a research question, and all aspects follow from the question
 - ✓ Simulations are not a substitution for empirical work as they are designed to answer methodological questions
- Demonstrated all steps from Drasgow's 1989 parameter recovery study
- Next steps: implement the software steps in SAS

2.44 Bookend: Section 1

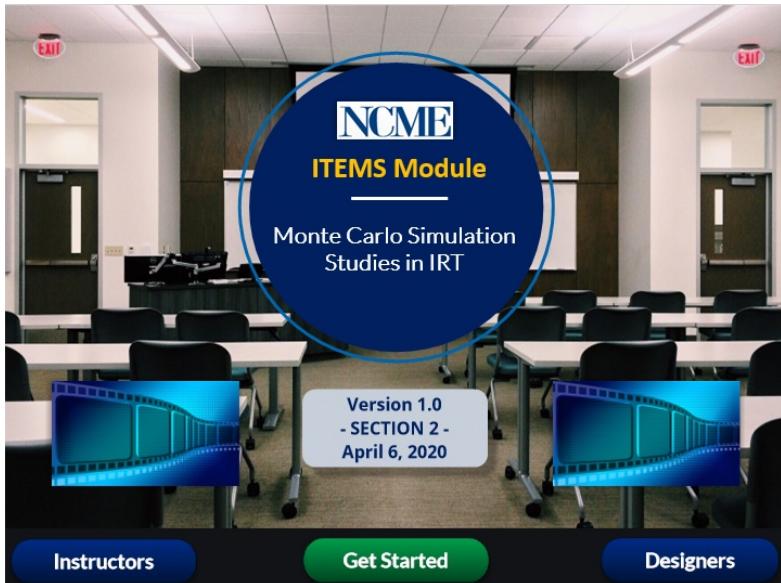


2.45 Module Cover (END)

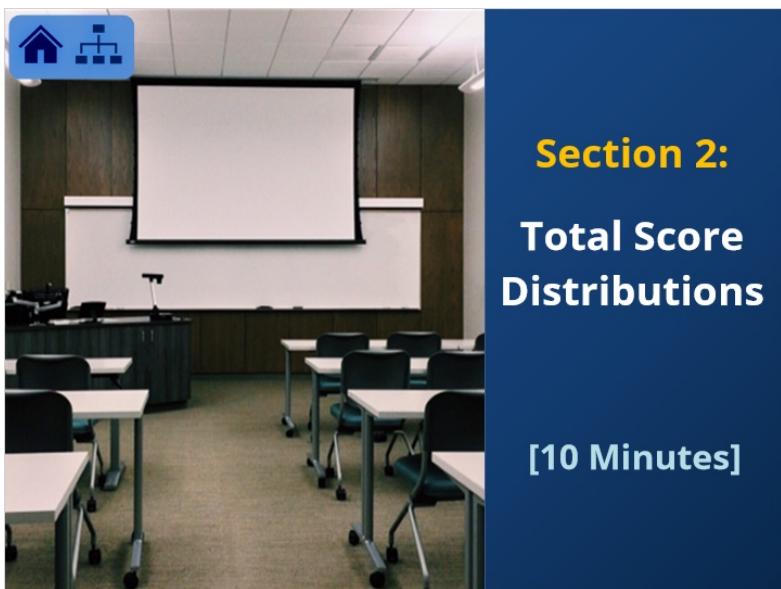


3. Section 2: Total Score Distributions

3.1 Module Cover (START)



3.2 Cover: Section 2



3.3 Objectives

Learning Objectives



1. Identify situations for total score simulations

2. Articulate rationale behind total score simulation methods

3. Describe the process for total score data simulation

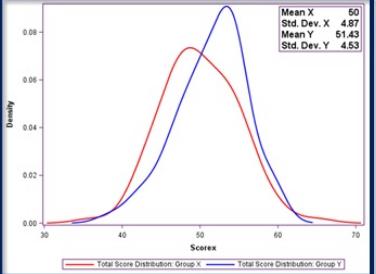
4. Modify example code for specific research scenarios

3.4 Total Score Distributions

Total Score Distributions

Total score:
sum of all the scored items

Total score distribution:
distribution of the sample's total scores



Group	Mean	Std. Dev.
X	50	4.87
Y	51.43	4.53

3.5 Simulation Goal

The screenshot shows a software interface titled "Simulation Goal". A text box contains the instruction: "Simulate total scores for 1000 individuals to a 50-item test with a specific mean, standard deviation, and other properties". Below this is a table with two columns: "Person" and "Score". The table has 1000 rows, with the first few rows shown as 1, 26; 2, 31; 3, 50; 4, 46; and so on down to 1000, 37. A callout box points to the first row with the text: "Each row represents an individual's total score". Two red arrows point down to the top of the "Person" and "Score" columns.

Person	Score
1	26
2	31
3	50
4	46
:	:
1000	37

3.6 Topic Selection

The screenshot shows a software interface with a navigation bar at the top featuring icons for home and tree. Below are three purple rectangular buttons labeled "Normal Distributions", "Skewed Distributions", and "Frequency Distributions". At the bottom right is a dark blue rounded rectangle labeled "Section End".

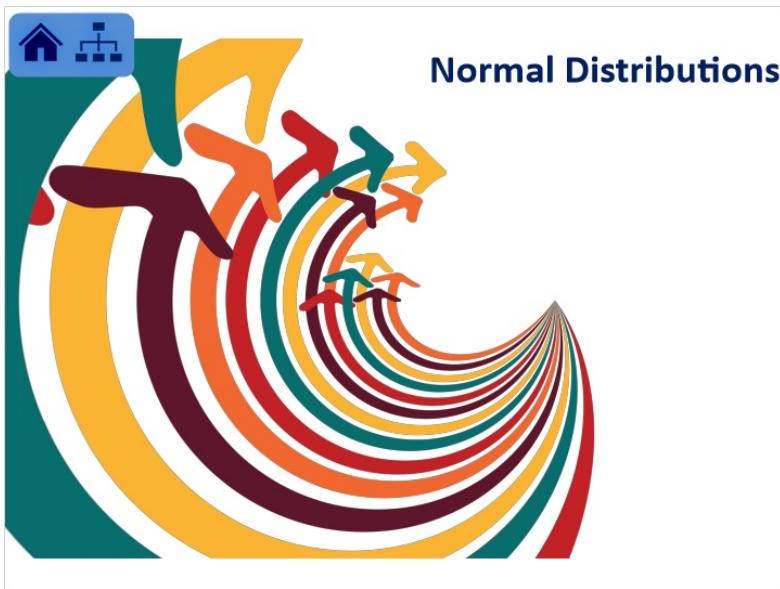
Normal Distributions

Skewed Distributions

Frequency Distributions

Section End

3.7 Bookmark: Normally Distributed Data



3.8 Standard Normal (I)

The screenshot shows a software interface with the title "Generate Random Normal Values".

Code snippet:

```
x = rand("Normal", 50, 5);
```

Description: Yields normally distributed data (mean = 50, standard deviation of = 5)

Result:

person	x
1	56.2034
2	44.9303
3	48.3771
4	46.6160
5	51.9494
6	53.7597
...	...

We want:

person	x
1	56
2	45
3	48
4	47
5	52
6	54
...	...

A callout box points from the "We want" table to the "Result" table with the text: "Test scores are typically recorded in whole numbers, not decimals."

3.9 Standard Normal (II)

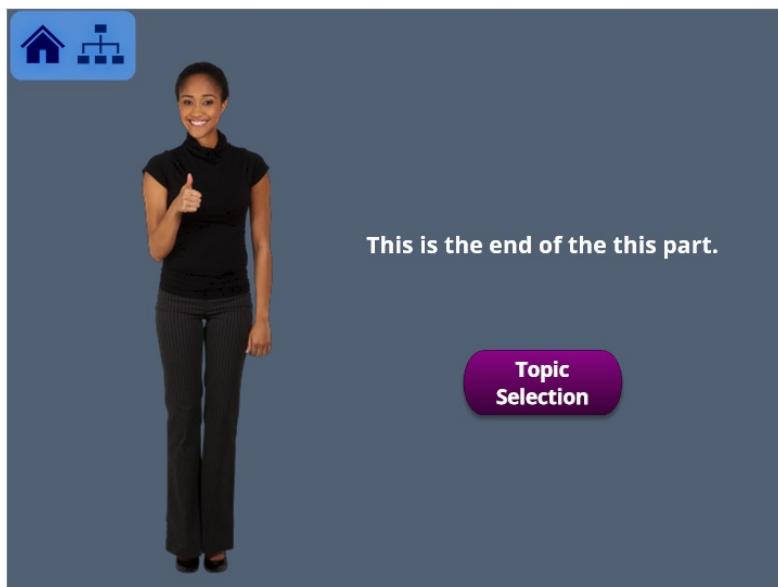
Truncate Generated Values

```
x = rand("Normal",50,5);  
Scorex=round(x,1);
```

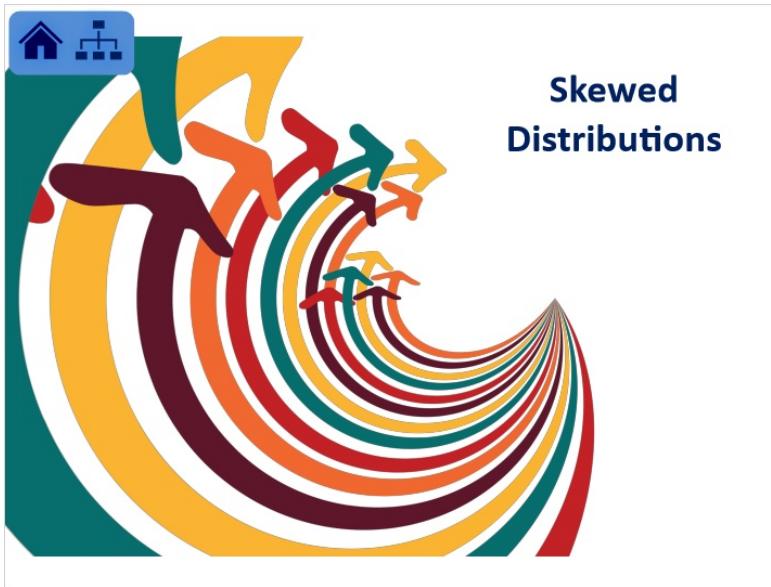
round(): Rounds the variable or constant to the nearest integer.

person	x	Scorex
1	56.2034	56
2	44.9303	45
3	48.3771	48
4	46.6160	47
5	51.9494	52
6	53.7597	54
...

3.10 Bookend: Normal Distributions



3.11 Bookmark: Skewed Data

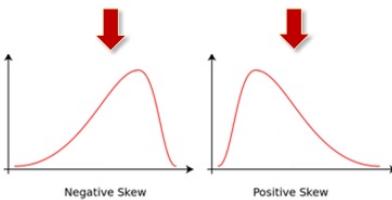


3.12 Skewed Distribution (I)

Skewness Formula

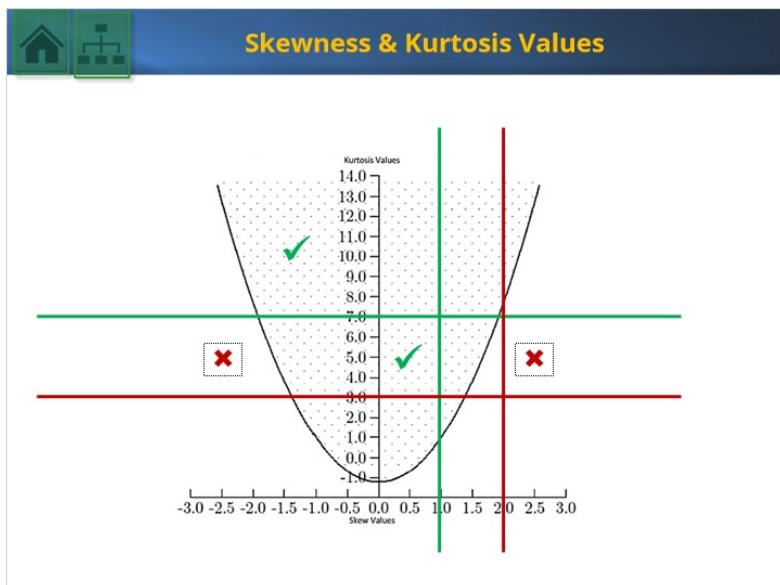
- Generate a random **standard normal value X** and compute:
$$skewX = -c + bX + cX^2 + dX^3$$
- b , c , and d come from **Fleishman's power method** implemented via coefficient table or *Proc NLP*

Example
skew = 1, kurtosis = 5
 $b = 0.108968$
 $c = 0.730401$
 $d = 0.079435$

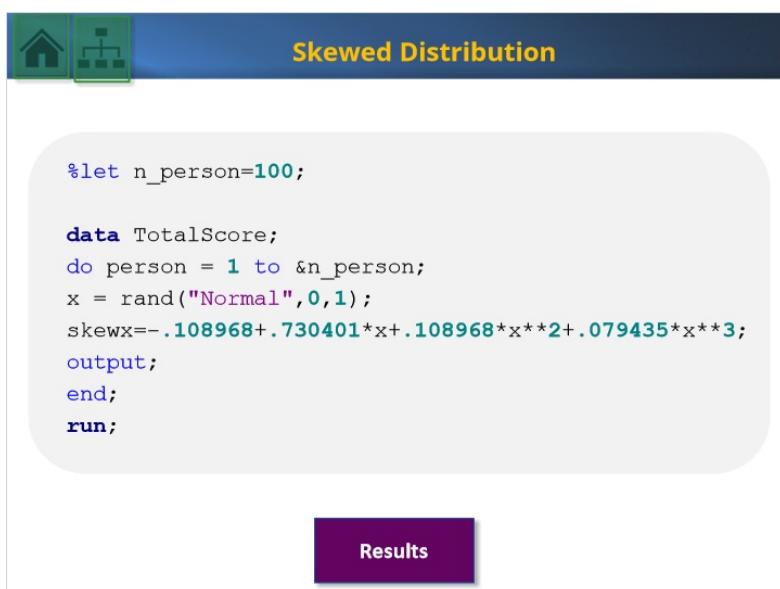


The figure shows two bell-shaped curves on a coordinate system. The left curve is skewed to the left (negative skew), with its peak shifted to the left of the center and a red arrow pointing down at the peak. The right curve is skewed to the right (positive skew), with its peak shifted to the right of the center and a red arrow pointing down at the peak.

3.13 Skewed Distribution (II)



3.14 Skewed Distribution (III)



Results (Slide Layer)

The screenshot shows a software interface with a blue header bar. On the left is a house icon, and on the right is the title "Adding Skew". Below the header is a code block:

```
proc means data=TotalScore skew kurt mean std;
var skewx;
run;
```

Below the code is a table:

Skewness	Kurtosis	Mean	Std. Dev.
1.021336	5.084481	0.005539	1.008835

The "Mean" cell (0.005539) is highlighted with a red border. A blue arrow points from this cell to a callout box containing the text: "These may not be the mean and standard deviation that we want so transformations are necessary". To the right of the table is a purple "Back" button.

3.15 Skewed Distribution (IV)

The screenshot shows a software interface with a green header bar. On the left is a house icon, and on the right is the title "Transformation & Rounding". Below the header is a code block:

```
%let n_person=100;
%let sd=5;
%let mean=50;

data TotalScore;
do person = 1 to &n_person;
x = rand("Normal",0,1);
skewx=-.108968+.730401*x+.108968*x**2+.079435*x**3;
score=round(skewx*&sd+&mean,1);
output;
end;
run;
```

A blue box highlights the formula $x = z * \sigma + \mu$. At the bottom is a purple "Results" button.

Results (Slide Layer)

Adding Skew

```
proc means data=TotalScore skew kurt mean std;  
var skewx score;  
run;
```

Variable	Skewness	Kurtosis	Mean	Std. Dev.
skewx	1.021336	5.084481	0.005539	1.008835
score	1.014059	5.009353	50.0309	5.049544

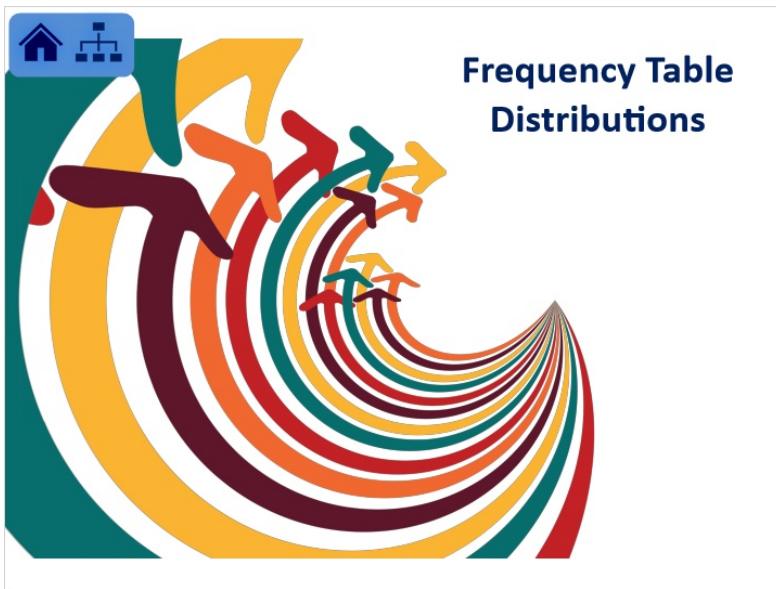
Back

3.16 Bookend: Skewed Distributions

This is the end of the this part.

Topic Selection

3.17 Bookmark: Data from Freq Distribution



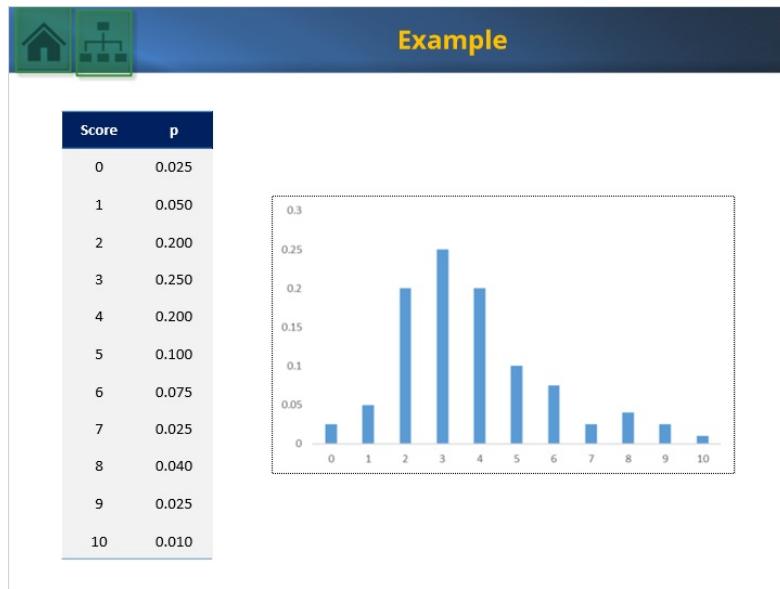
3.18 Frequency Table (I)

A bookmark titled "Generating Values from a Frequency Table". It includes two bullet points: "Simulate discrete values from a frequency distribution" and "The tabled distribution takes on the values 1, 2, ..., n with specified probabilities". Below the points is a code snippet:

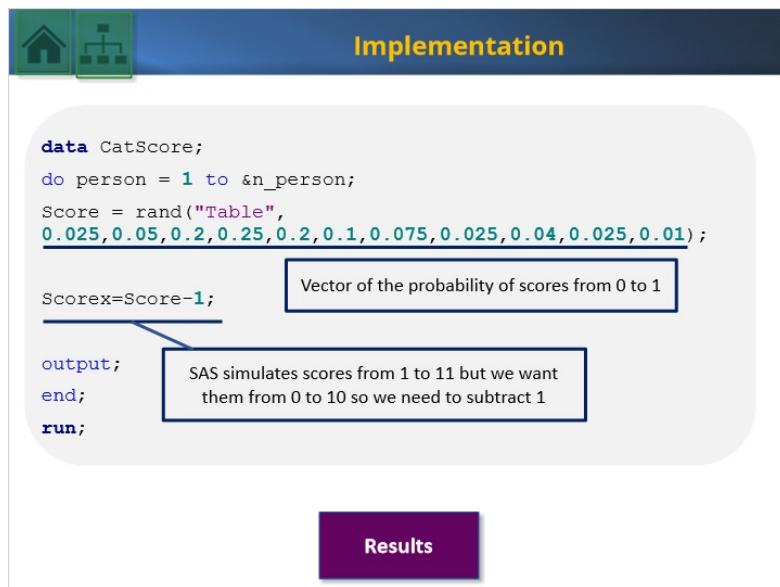
```
rand("Table", p1, p2...);
```

 At the bottom is an icon of a grid or table.

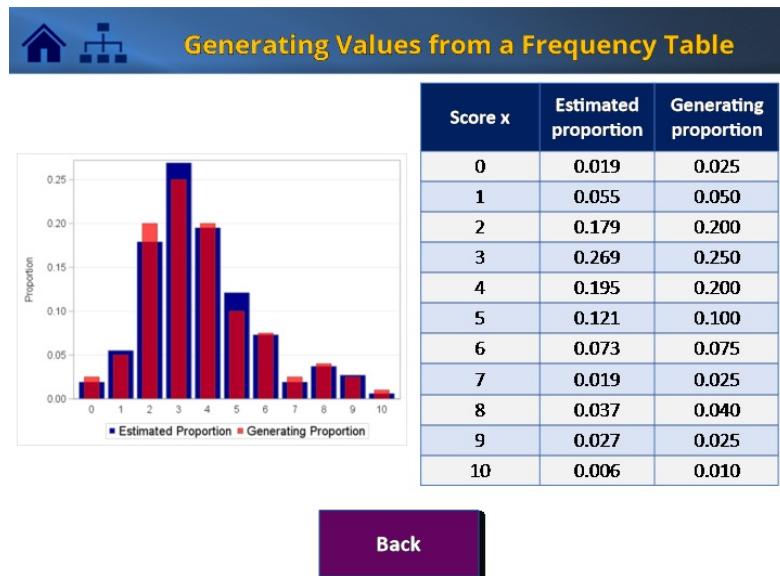
3.19 Frequency Table (II)



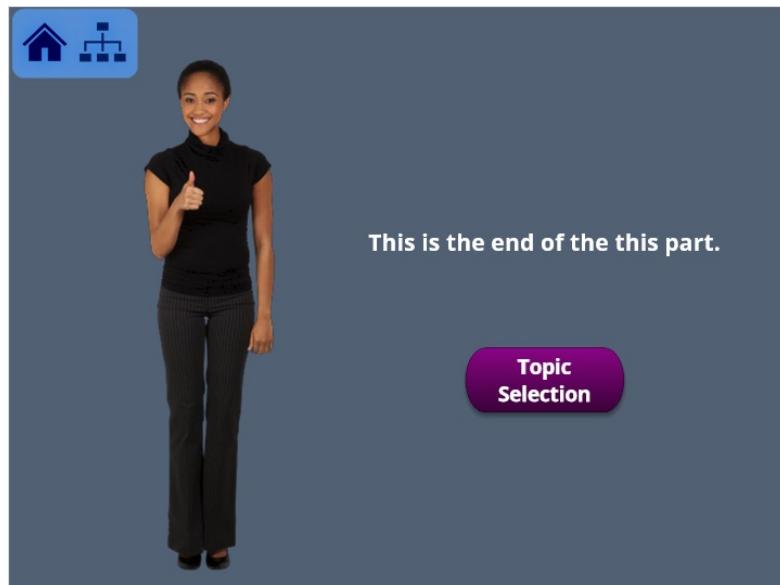
3.20 Frequency Table (III)



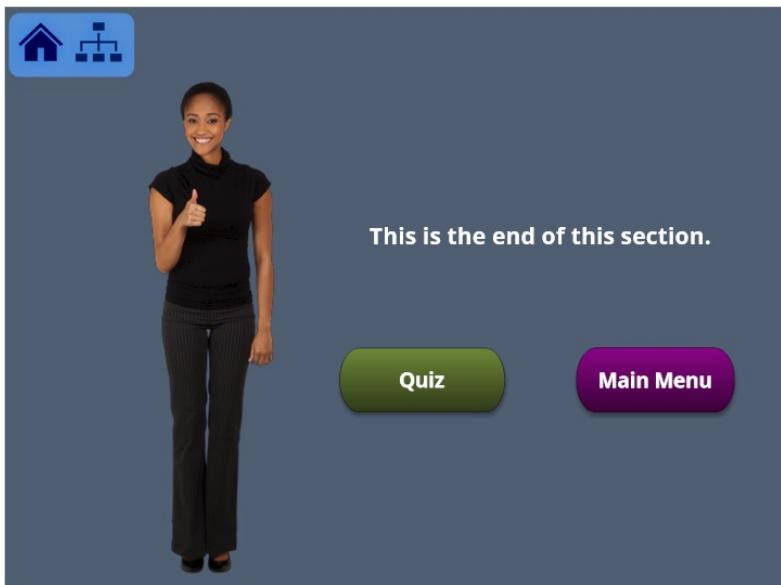
Results (Slide Layer)



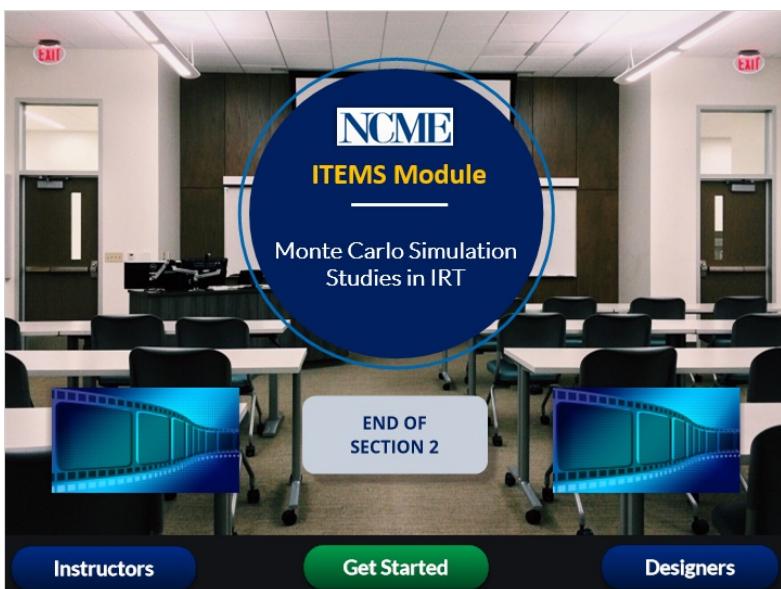
3.21 Bookend: Frequency Table Distributions



3.22 Bookend: Section 2

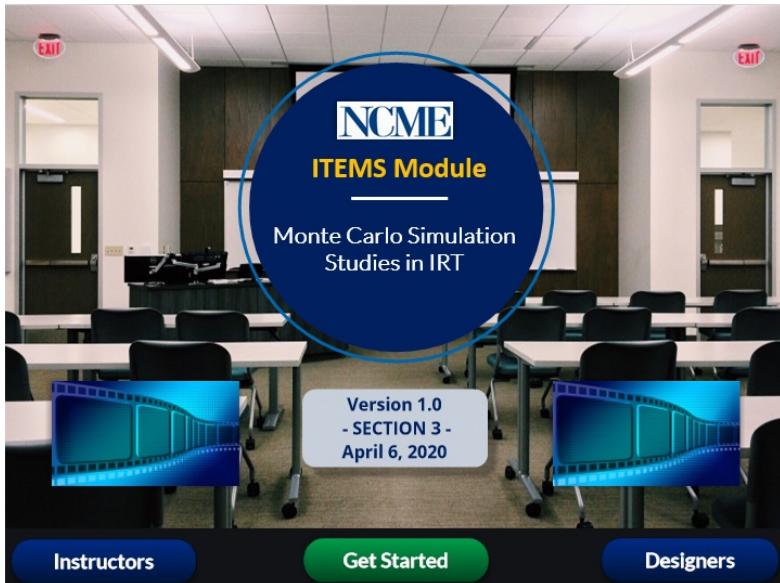


3.23 Module Cover (END)

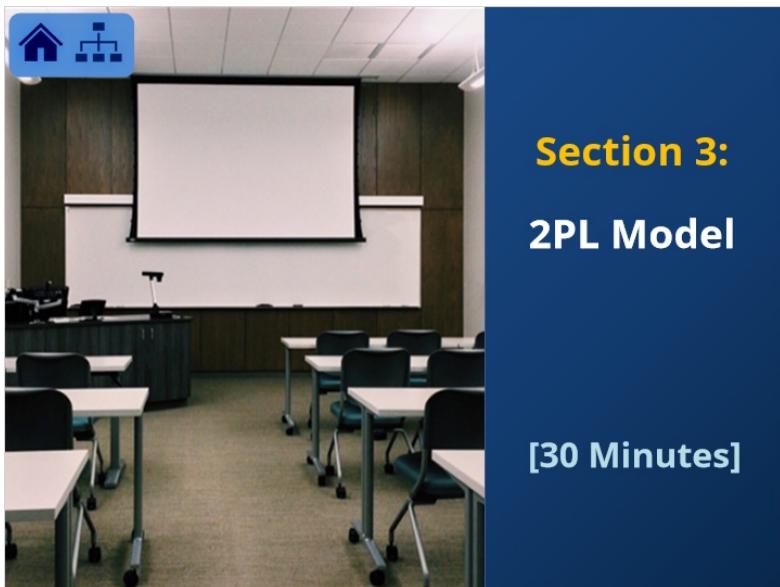


4. Section 3: Two-parameter Model

4.1 Module Cover (START)



4.2 Cover: Section 3



4.3 Objectives

Learning Objectives



1. Identify key parameters in the 2PL model for simulation

2. Describe methods of how to chose generating parameters

3. Articulate the process for item response simulation using the 2PL

4. Modify example code for specific research scenarios

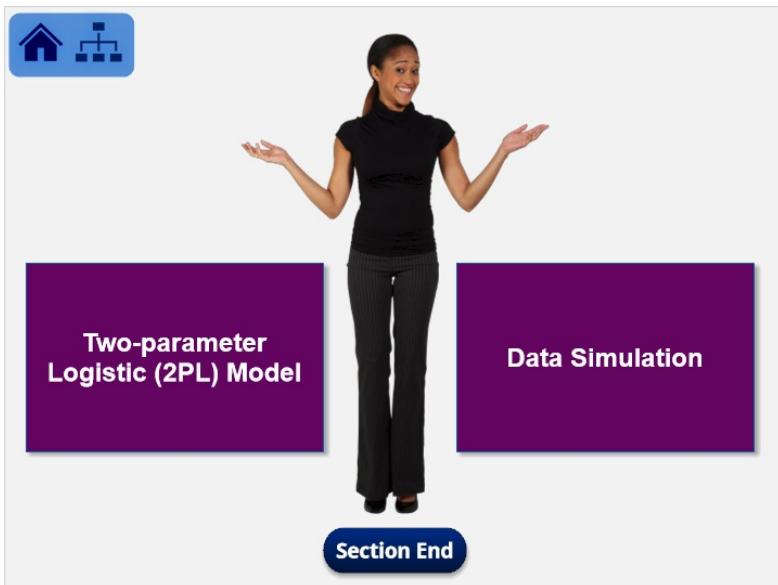
4.4 Simulation Goal

Simulation Goal

- Simulate item responses for 100 individuals to a 14-item test
- Each item is going to be dichotomously scored as
 - ✓ 1 = Correct
 - ✓ 0 = Incorrect
- Goal dataset:
 - Each row represents an individual's response set

	Item 1	Item 2	Item 3	...	Item 14
1	1	0	...	1	
0	1	1	...	0	
:	:	:	⋮	⋮	
0	1	1	...	0	

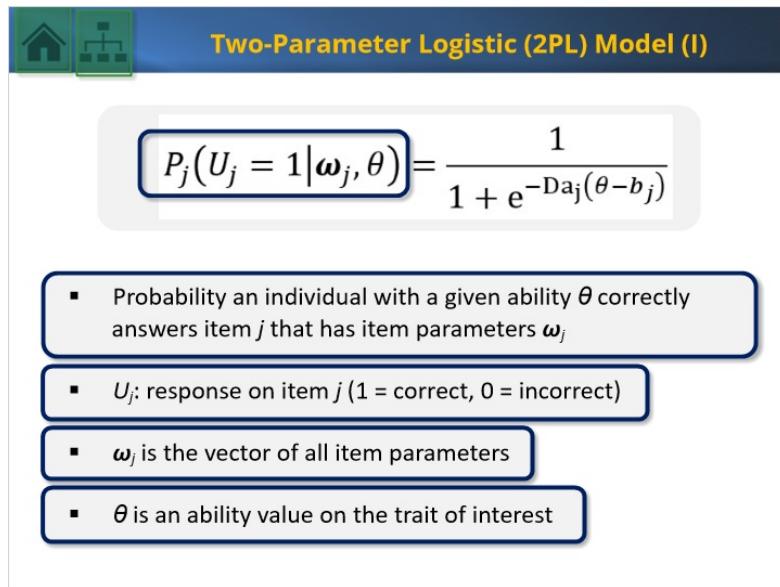
4.5 Topic Selection



4.6 Bookmark: Two-paramter Model



4.7 Two-parameter Model (I)

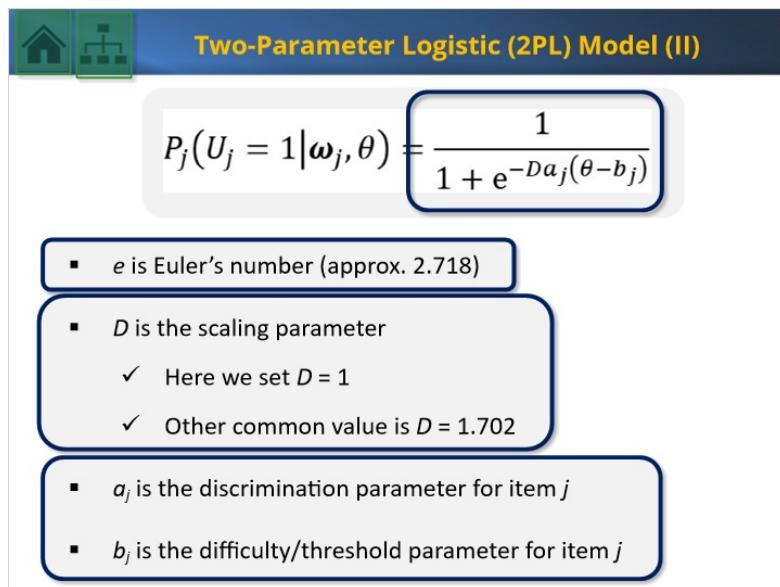


Two-Parameter Logistic (2PL) Model (I)

$$P_j(U_j = 1 | \omega_j, \theta) = \frac{1}{1 + e^{-D a_j (\theta - b_j)}}$$

- Probability an individual with a given ability θ correctly answers item j that has item parameters ω_j
- U_j : response on item j (1 = correct, 0 = incorrect)
- ω_j is the vector of all item parameters
- θ is an ability value on the trait of interest

4.8 Two-parameter Model (II)

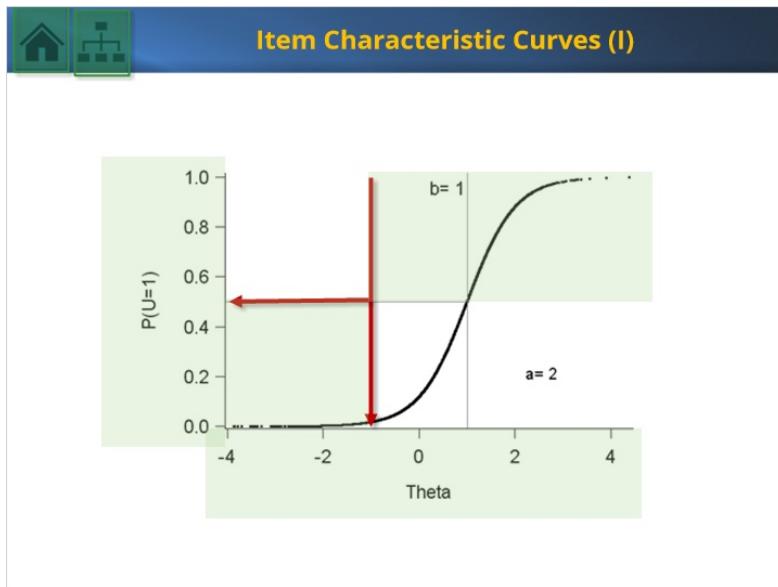


Two-Parameter Logistic (2PL) Model (II)

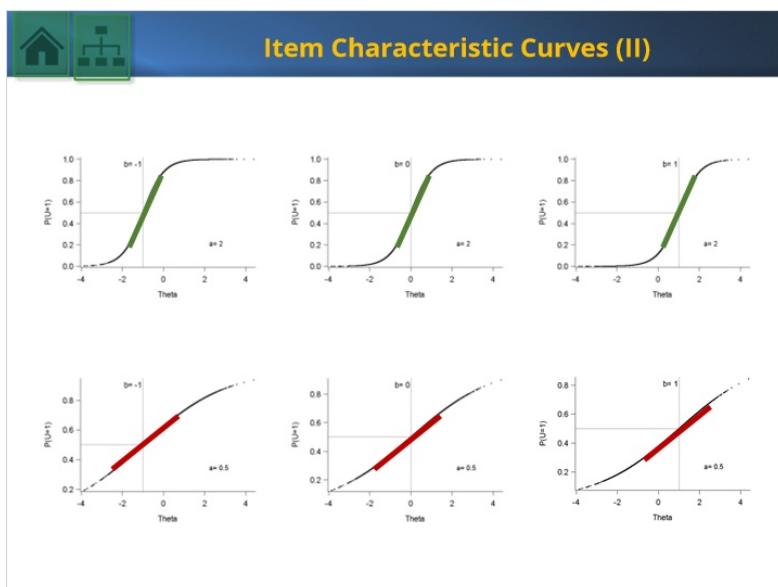
$$P_j(U_j = 1 | \omega_j, \theta) = \frac{1}{1 + e^{-D a_j (\theta - b_j)}}$$

- e is Euler's number (approx. 2.718)
- D is the scaling parameter
 - ✓ Here we set $D = 1$
 - ✓ Other common value is $D = 1.702$
- a_j is the discrimination parameter for item j
- b_j is the difficulty/threshold parameter for item j

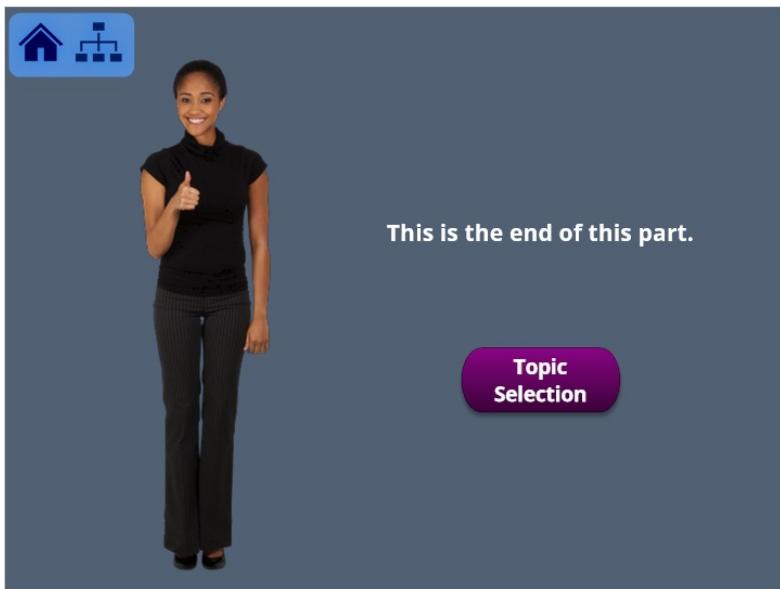
4.9 Item Characteristic Curves (I)



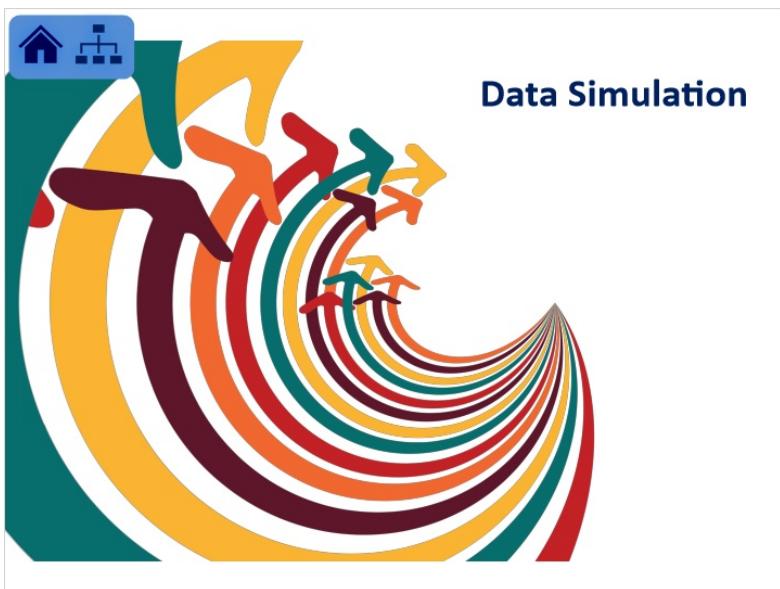
4.10 Item Characteristic Curves (II)



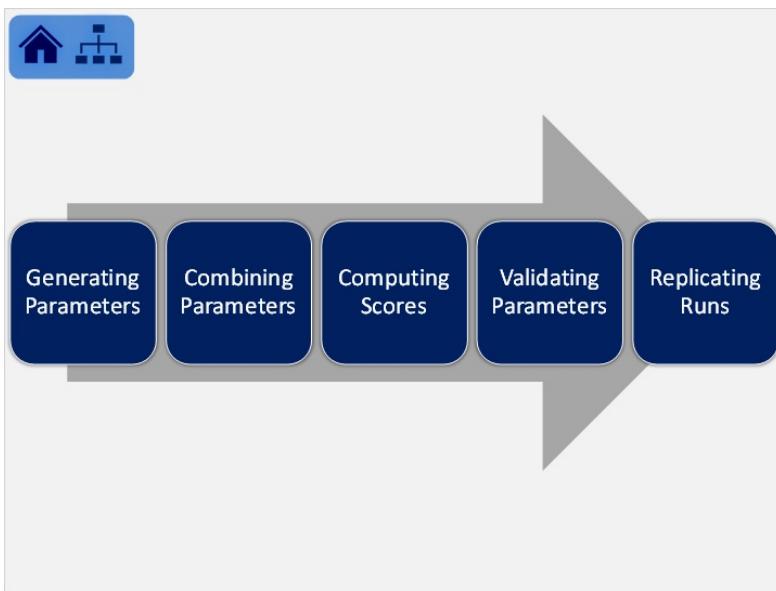
4.11 Bookend: Two-parameter Model



4.12 Bookmark: Generating Parameters with 2PLM



4.13 Step Selection



4.14 Person Parameters

The screenshot shows a software interface with a dark blue header bar. On the left, there are icons for a house and a tree. To the right of the icons, the title "Person Parameters" is displayed in yellow. Below the header, there is a code editor window containing SAS code:

```
%let n_person = 100;
data person_parm;
  do i = 1 to &n_person;
    theta=rand('normal',0,1);
    output;
  end;
  keep theta;
run;
```

4.15 Item Parameters

The screenshot shows a software interface with a blue header bar containing icons for a house and a tree, and the text "Item Parameters". Below the header, there are three rounded rectangular boxes, each containing a bulleted list of steps:

- Create a dataset that contains item parameters.
- This may be based off of empirical estimates, previous literature, etc.
- You may randomly select item parameters as well.
- Literature can be helpful in determining what distribution to randomly select from.

To the right of these boxes is a code editor window containing SAS code:

```
data item_parms;
  input a b;
  cards;
1.7 -2
1.7 -1
1.7 -.5
1.7 0
1.7 .5
1.7 1
1.7 2
1.9 -2
1.9 -1
1.9 -.5
1.9 0
1.9 .5
1.9 1
1.9 2
;
run;
```

4.16 Bookend: Step 1

The slide features a dark blue background. In the top left corner, there are icons for a house and a tree. In the center, a woman in a black turtleneck and black pants is smiling and giving a thumbs-up. To her right, the text "This is the end of this part." is displayed. In the bottom right corner, there is a purple rounded rectangle containing the text "Topic Selection".

4.17 Parameter Matrix (I)



Parameter Matrix (I)

- We need in the same data set / parameter matrix:
 - ✓ an individual's ability (θ)
 - ✓ item parameters (a_j and b_j)
- We want our data set / parameter matrix to look like this:

4.18 Parameter Matrix (II)

Parameter Matrix (II)

	a	b
1	1.7	-2
2	1.7	-1
3	1.7	-0.5
4	1.7	0
5	1.7	0.5
6	1.7	1
7	1.7	2
8	1.9	-2
9	1.9	-1
10	1.9	-0.5
11	1.9	0
12	1.9	0.5
13	1.9	1
14	1.9	2

```
proc transpose data=item_parms out=a_wide prefix=var a;
run;
```

NAME OF POSITIONAL VARIABLE	a1	a2	a3	a4	a5	a6	a7
1	1.7	1.7	1.7	1.7	1.7	1.7	1.7
2							

```
proc transpose data=item_parms out=b_wide prefix=var b;
run;
```

NAME OF POSITIONAL VARIABLE	b1	b2	b3	b4	b5	b6	b7	b8
1	-2	-1	-0.5	0	0.5	1	2	-2
2								

```
data items_wide;
merge a_wide b_wide;
drop _name_;
run;
```

a1	a2	a3	a4	a5	a6	a7	a8	a9	a10	a11	a12	a13	a14	b1	b2	b3
1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.9	1.9	1.9	1.9	1.9	1.9	1.9	-2	-1	-0.5

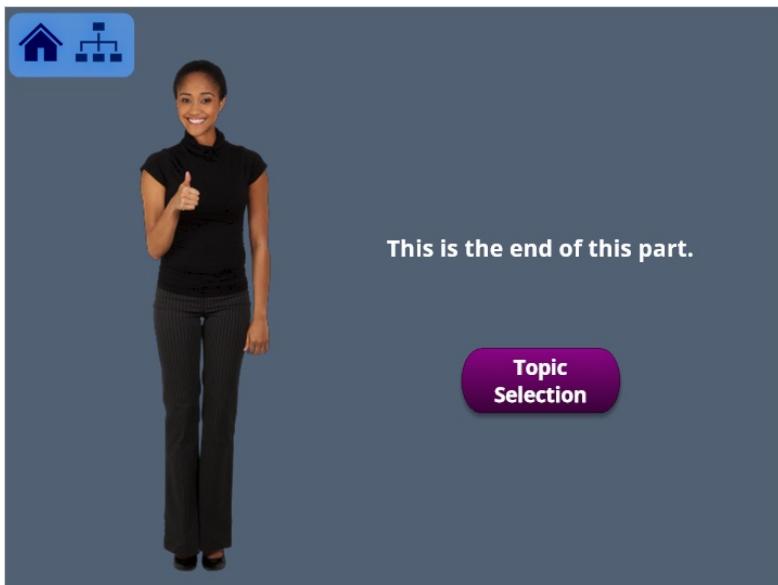
Goal:

4.19 Parameter Matrix (III)

```
data all_parms;
  set person_parm;
  If _N_=1 then set items_wide ;
run;
```

	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13	x14	x15	x16	x17	x18
1	0.307762395	17	17	17	17	17	17	17	19	19	19	19	19	19	19	2	-1	-5
2	-0.40642094	17	17	17	17	17	17	17	19	19	19	19	19	19	19	2	-1	-5
3	1.38817271	17	17	17	17	17	17	17	19	19	19	19	19	19	19	2	-1	-5
4	0.926362426	17	17	17	17	17	17	17	19	19	19	19	19	19	19	2	-1	-5
5	-1.3684250	17	17	17	17	17	17	17	19	19	19	19	19	19	19	2	-1	-5
6	0.4208451	17	17	17	17	17	17	17	19	19	19	19	19	19	19	2	-1	-5
7	1.2019637	17	17	17	17	17	17	17	19	19	19	19	19	19	19	2	-1	-5
8	0.49952147	17	17	17	17	17	17	17	19	19	19	19	19	19	19	2	-1	-5
9	-0.49801163	17	17	17	17	17	17	17	19	19	19	19	19	19	19	2	-1	-5
10	0.20847442	17	17	17	17	17	17	17	19	19	19	19	19	19	19	2	-1	-5
11	0.81248501	17	17	17	17	17	17	17	19	19	19	19	19	19	19	2	-1	-5
12	-1.10138548	17	17	17	17	17	17	17	19	19	19	19	19	19	19	2	-1	-5
13	-2.35019434	17	17	17	17	17	17	17	19	19	19	19	19	19	19	2	-1	-5
14	-0.50321252	17	17	17	17	17	17	17	19	19	19	19	19	19	19	2	-1	-5
15	-0.89990594	17	17	17	17	17	17	17	19	19	19	19	19	19	19	2	-1	-5
16	0.291595042	17	17	17	17	17	17	17	19	19	19	19	19	19	19	2	-1	-5
17	0.48479986	17	17	17	17	17	17	17	19	19	19	19	19	19	19	2	-1	-5
18	0.995703426	17	17	17	17	17	17	17	19	19	19	19	19	19	19	2	-1	-5

4.20 Bookend: Step 2



4.21 Score Computation (I)

Response Probabilities

We want a data file that looks like:

Item 1	Item 2	Item 3	...	Item 14
1	1	0	...	1
0	1	1	...	0
:	:	:	⋮	:
0	1	1	...	0

Let's consider one individual:

Theta	a_1	b_1	...	a_{14}	b_{14}
-1.43	1.7	-2	...	1.9	2

$$P_j(U_j = 1 | \omega_j, \theta) = \frac{1}{1 + e^{-a_j(\theta - b_j)}}$$

For item 1: $P(U_1 = 1) = \frac{1}{1 + e^{-1.7(-1.43 - (-2))}} = .725$

⋮

For item 14: $P(U_{14} = 1) = \frac{1}{1 + e^{-1.9(-1.43 - (2))}} = .0015$

4.22 Score Computation (II)

Response Computation (I)

- Incorporate **randomness** into the process
- Choose a **random number** between 0 and 1 with **equal probability** from a **uniform distribution**
 - If the randomly selected value is **less than or equal to .725**, assign a score of **1**
 - If the randomly selected value is **greater than .725**, assign a score of **0**

For item 1: $P(U_1 = 1) = \frac{1}{1 + e^{-1.7(-1.43 - (-2))}} = .725$

4.23 Score Computation (III)



Response Computation (II)

```
data responses;
  set all_parms;
  array a{&n_item} a1-a&n_item;
  array b{&n_item} b1-b&n_item;
  array p{&n_item} p1-p&n_item;
  array item{&n_item} item1 - item&n_item;
```

4.24 Score Computation (IV)



Response Computation (III)

```
data responses;
  set all_parms;
  array a{&n_item} a1-a&n_item;
  array b{&n_item} b1-b&n_item;
  array p{&n_item} p1-p&n_item;
  array item{&n_item} item1 - item&n_item;
  do j = 1 to &n_item;
    b[j]=1/(1+exp(-(a[j]*(theta-b[j)))));
    y=rand('uniform');
    if y<=p[j] then item[j]=1;
    else if y>p[j] then item[j]=0;
  end;
run;
```

Results

Results (Slide Layer)



Results

Obs	p1	p2	p3	p4	p5	p6	p7	item1	item2	item3	item4	item5	item6	item7
20	0.96695	0.84241	0.69557	0.49407	0.29448	0.15139	0.03156	1	1	1	0	1	1	0
21	0.90752	0.64193	0.43382	0.24671	0.12279	0.05645	0.01081	0	1	0	0	0	0	0
22	0.74623	0.34946	0.18673	0.08937	0.04026	0.01761	0.00326	1	1	0	1	1	0	0
23	0.99891	0.09404	0.98616	0.96821	0.92866	0.84764	0.50406	1	1	1	1	1	1	1
24	0.98711	0.93327	0.85669	0.71870	0.52200	0.31822	0.07857	1	1	1	1	0	0	1
25	0.90371	0.63161	0.42290	0.23851	0.11807	0.05412	0.01034	1	1	0	1	0	0	0

Back

4.25 Score Computation (V)

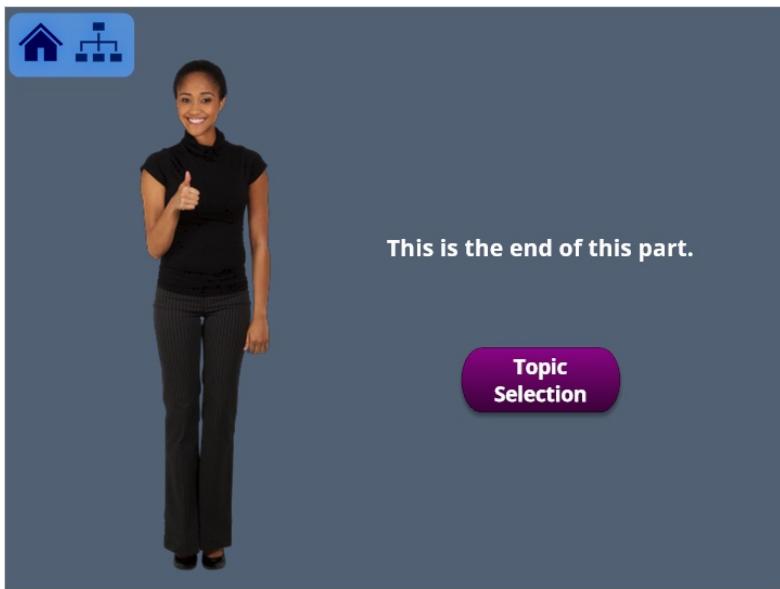


Data Set Cleanup

```
data responses_only;
set responses;
keep item:;
run;
```

	item1	item2	item3	item4	item5	item6	item7	item8	item9	item10	item11	item12	item13	item14	item15	item16
1	1	1	1	1	1	0	0	0	1	1	1	1	0	1	1	1
2	1	1	0	1	1	0	1	0	0	0	1	0	1	0	1	0
3	1	1	1	1	1	0	0	0	1	1	1	1	0	1	1	1
4	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1	1
5	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1	1
6	1	0	0	0	0	0	0	0	1	1	1	1	0	1	1	0
7	1	1	1	1	1	1	1	0	1	1	0	1	1	0	1	1
8	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1
9	1	1	1	1	1	1	1	0	0	1	1	0	1	0	1	1
10	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	1
11	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
12	1	1	0	1	0	1	0	1	0	1	1	1	0	1	1	0
13	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1
14	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1
15	1	1	1	1	1	1	1	0	0	1	1	0	1	1	1	1
16	1	1	1	1	1	1	1	0	0	1	1	0	1	1	1	1

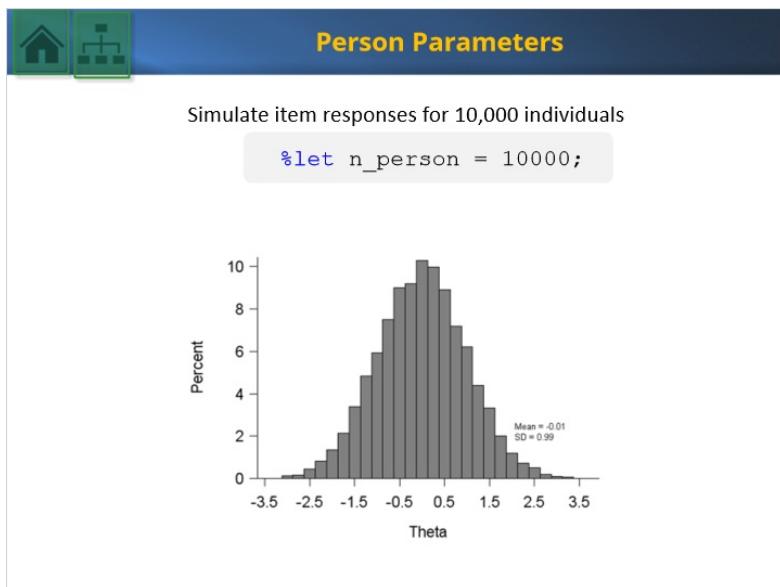
4.26 Bookend: Step 3



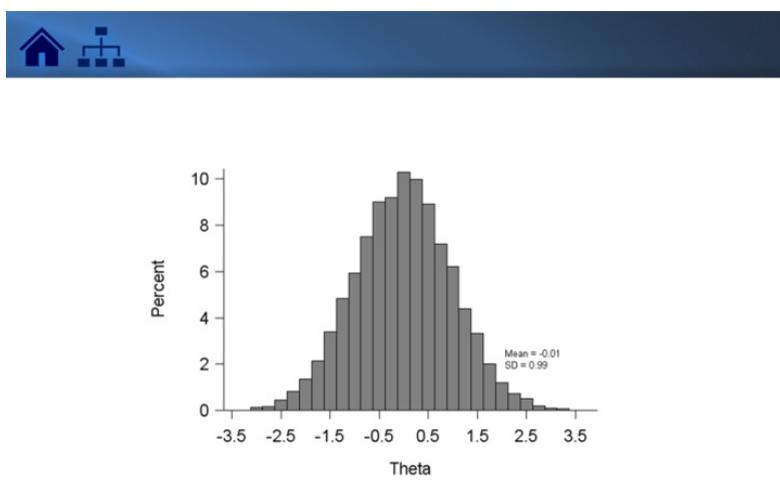
4.27 Parameter Validation (I)

A slide titled "Parameter Validation" with a green header bar containing icons for home and search. The main content area contains text about simulation techniques and two methods listed in a callout box, followed by a large green checkmark icon.

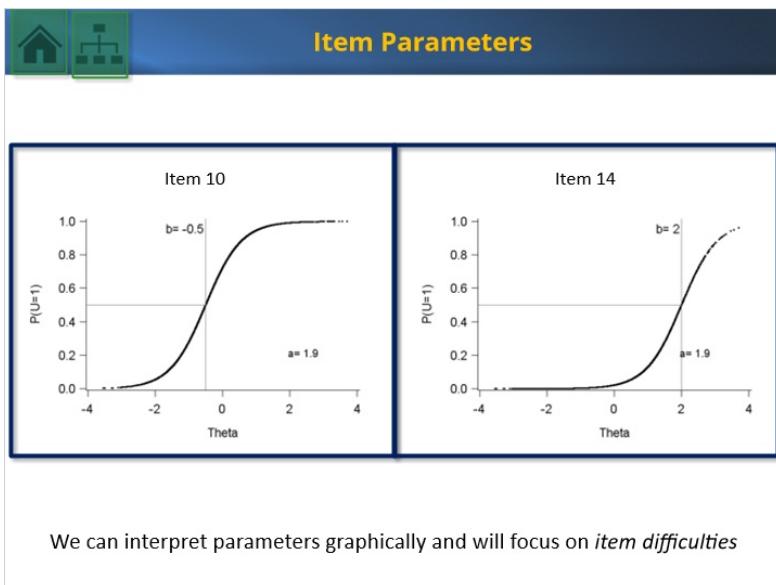
4.28 Parameter Validation (II)



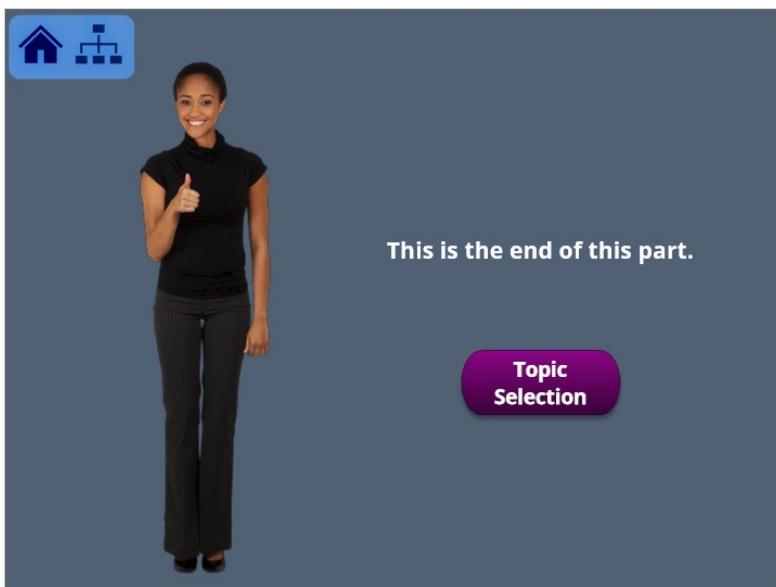
Results (Slide Layer)



4.29 Parameter Validation (III)



4.30 Bookend: Step 4



4.31 Replications (I)

The slide has a blue header bar with the title "Replications" in yellow. On the left, there are two icons: a house and a tree. Below the title is a quote in a grey box:

"within each condition, there are often a certain number of replications. That is, for many research questions, it may be necessary to repeat the data generation process over multiple occasions—that is, replications—so that the empirical estimate of the sampling distribution (due to the simulation) of various statistics of interest may be observed"

Feinberg and Rubright, 2016

Below the quote is a large grey circular icon containing a white lightning bolt symbol.

4.32 Replications (II)

The slide has a blue header bar with the title "Replications" in yellow. On the left, there are two icons: a house and a tree. Below the title are two code snippets in a grey box:

```
%let n_reps=50;
%macro repeatIRTgen;
  %do rep = 1 %to &n_reps;
    [data generation commands]
  %end;
%mend repeatIRTgen;
%repeatIRTgen;
```



```
libname sim 'C:\Users\desktop\simulation';
data sim.Data&rep;
  set responses;
  keep item: theta;
run;
```

Below the code snippets is a large grey circular icon containing a white lightning bolt symbol.

4.33 Replications (III)

What Do You Output?

What you save will depend on your research questions

Examples

- If you are looking at item parameter recovery
→ save true item parameters for each replication
- If you are investigating person parameter recovery
→ save true person ability parameters for each replication
- If you are investigating a new model that does not have the same parameters and are looking at a comparison between estimated expected score and true score
→ save the true scores of persons

4.34 Replications (IV)

General Setup

```
libname sim 'C:/Users/.../Desktop/simulation';
*****macro variables *****/
%let n_person = 100;
%let n_reps=50;
***** *****
data item_parms;
  input a b;
  cards;
  1.7 -2
  1.7 -1
  1.7 -.5
  1.7 0
  1.7 .5
  1.7 1
  1.7 2
  1.9 -2
  1.9 -1
  1.9 -.5
  1.9 0
  1.9 .5
  1.9 1
  1.9 2
;
run;
proc sql noprint;
select count(a) into :n_item trimmed from item_parms;
run;
```

4.35 Replications (V)



Generating Person Parameters

```
%macro repeatIRTgen;
%do rep = 1 %to &n_reps;
proc datasets library=work noprint;
  save item_parms;
run;
quit;

data person_parm;
  do i = 1 to &n_person;
    theta=rand('normal',0,1);
    output;
  end;
  keep theta;
run;
```

Full code is available in the “Resources” of the module

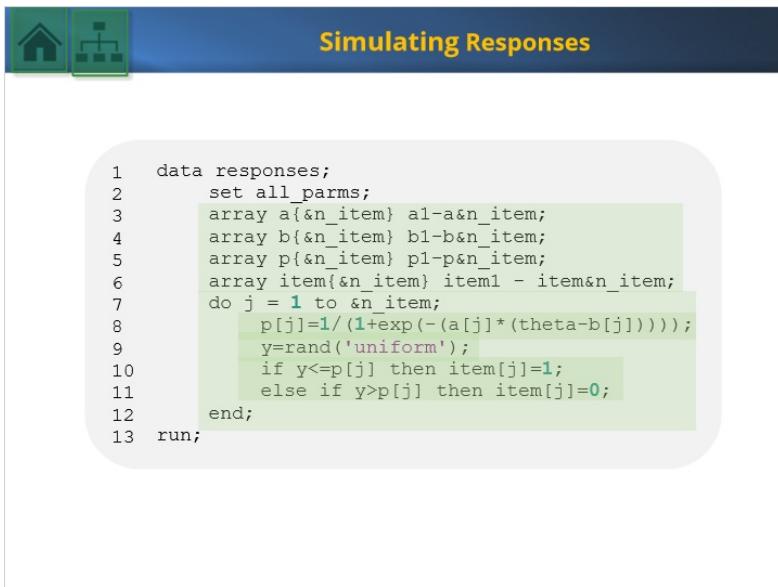
4.36 Replications (VI)



Merging Item and Person Parameters

```
1  proc transpose data=item_parms out=a_wide prefix=a;
2    var a;
3  run;
4
5  proc transpose data=item_parms out=b_wide prefix=b;
6    var b;
7  run;
8
9  data items_wide;
10   merge a_wide b_wide;
11   drop _name_;
12 run;
13
14 data all_parms;
15   set person_parm;
16   If _N_=1 then set items_wide ;
17 run;
```

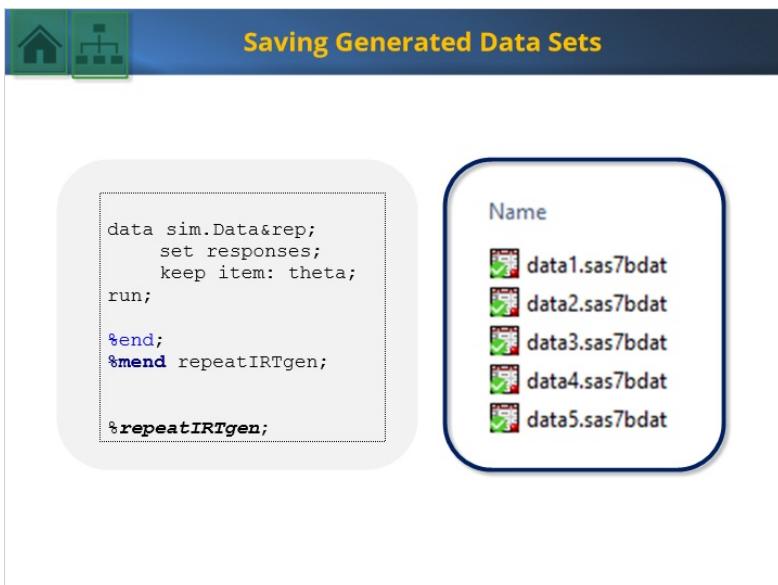
4.37 Replications (VII)



The screenshot shows a SAS interface with a title bar "Simulating Responses". Below it is a code editor containing the following SAS code:

```
1  data responses;
2    set all_parms;
3    array a{&n_item} a1-a&n_item;
4    array b{&n_item} b1-b&n_item;
5    array p{&n_item} p1-p&n_item;
6    array item{&n_item} item1 - item&n_item;
7    do j = 1 to &n_item;
8      p[j]=1/(1+exp(-(a[j]*(theta-b[j]))));
9      y=rand('uniform');
10     if y<=p[j] then item[j]=1;
11     else if y>p[j] then item[j]=0;
12   end;
13 run;
```

4.38 Replications (VIII)



The screenshot shows a SAS interface with a title bar "Saving Generated Data Sets". Below it is a code editor containing the following SAS code:

```
data sim.Data&rep;
  set responses;
  keep item: theta;
run;

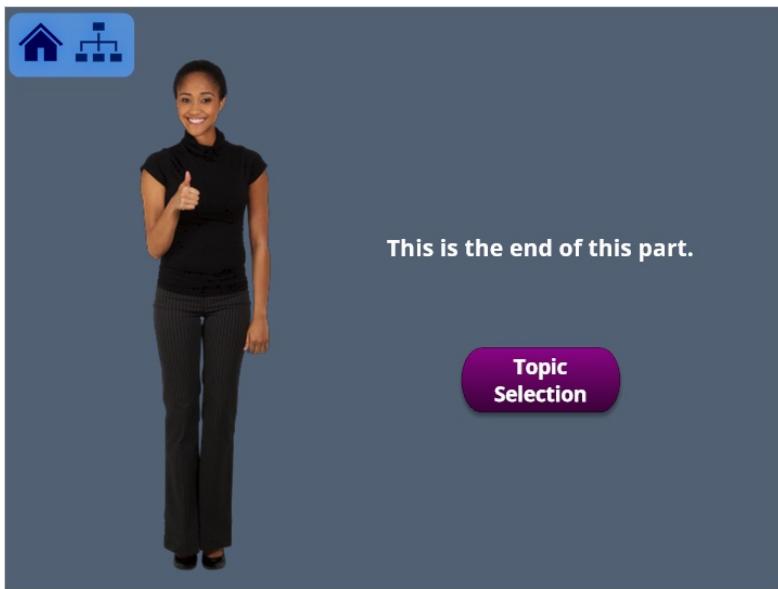
%end;
%mend repeatIRTgen;

%repeatIRTgen;
```

To the right of the code editor is a preview pane titled "Name" which lists five generated data sets:

Name
data1.sas7bdat
data2.sas7bdat
data3.sas7bdat
data4.sas7bdat
data5.sas7bdat

4.39 Bookend: Step 5

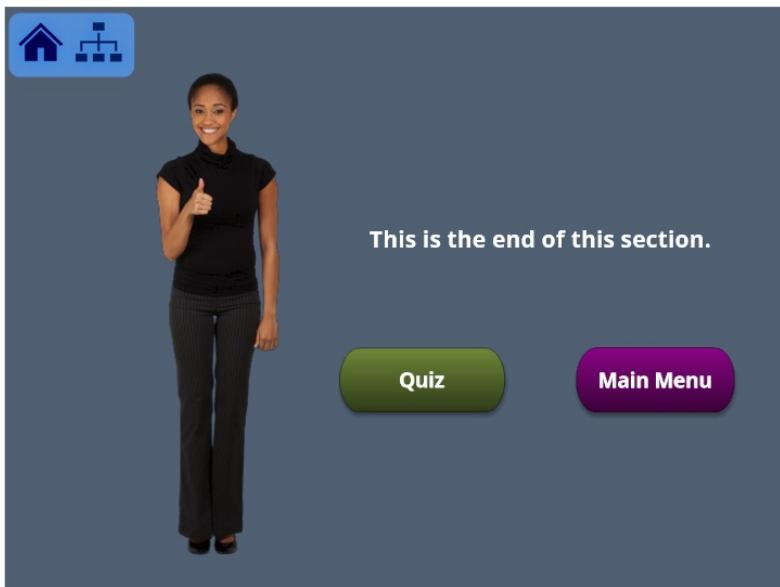


4.40 Other Useful Tips

A slide titled "Other Useful Tips" with a blue header bar. The slide contains a bulleted list of tips with corresponding arrows pointing to actions:

- You want your results to be reproduced
→ use a random seed
- Macros are very difficult to debug
→ develop macros only at the last step
- It is tedious to change after hard-coding values
→ start with macro variables
- Data validation usually doesn't go into a publication but is incredibly important
→ perform data validation
- Don't reinvent the wheel unnecessarily
→ call outside programs when appropriate

4.41 Bookend: Section 3

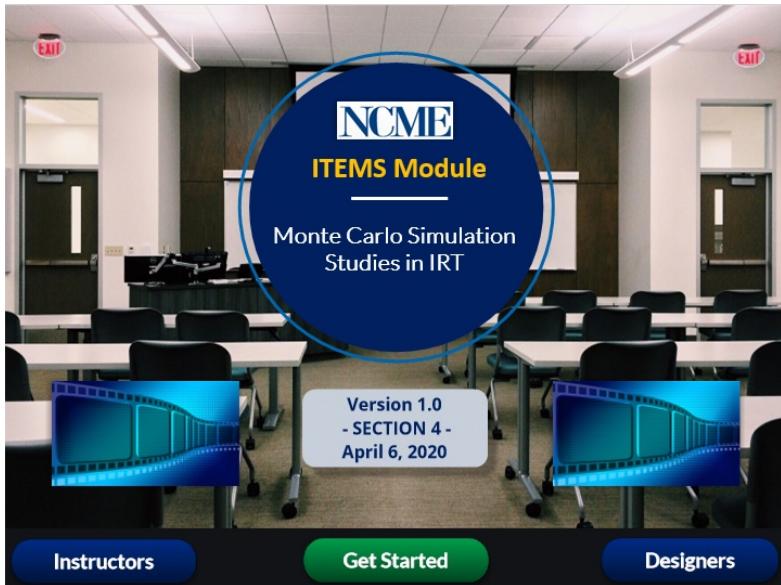


4.42 Module Cover (END)

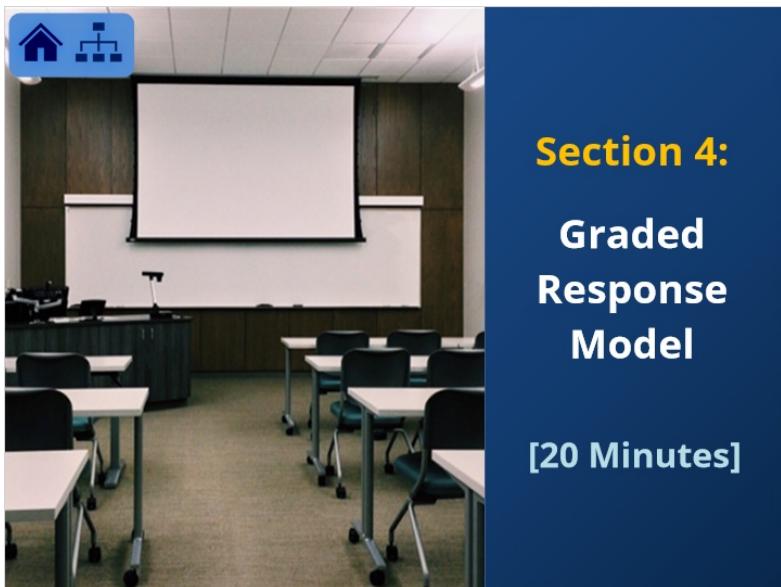


5. Section 4: Graded Response Model

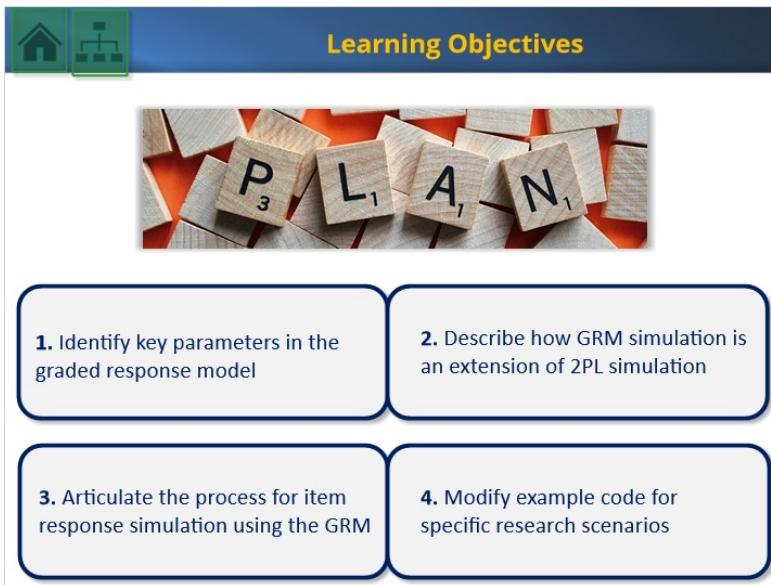
5.1 Module Cover (START)



5.2 Cover: Section 4



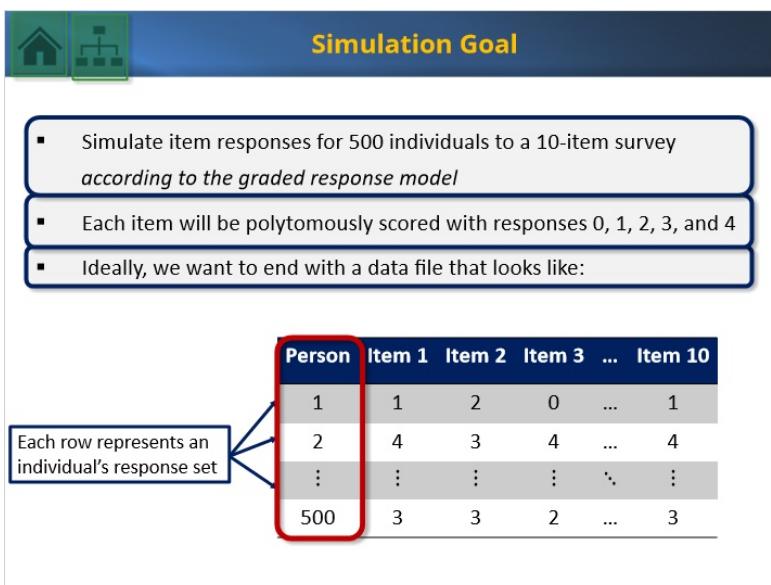
5.3 Objectives



The slide has a blue header bar with the title "Learning Objectives". Below the header is a photograph of wooden blocks spelling out "PLAN" (P, L, A, N) on a surface with a red star in the background. The main content area contains four numbered objectives in white boxes with a dark blue border:

1. Identify key parameters in the graded response model
2. Describe how GRM simulation is an extension of 2PL simulation
3. Articulate the process for item response simulation using the GRM
4. Modify example code for specific research scenarios

5.4 Simulation Goal



The slide has a blue header bar with the title "Simulation Goal". Below the header is a list of simulation requirements in a white box with a dark blue border:

- Simulate item responses for 500 individuals to a 10-item survey according to the graded response model
- Each item will be polytomously scored with responses 0, 1, 2, 3, and 4
- Ideally, we want to end with a data file that looks like:

Each row represents an individual's response set

Person	Item 1	Item 2	Item 3	...	Item 10
1	1	2	0	...	1
2	4	3	4	...	4
:	:	:	:	:	:
500	3	3	2	...	3

5.5 Topic Selection



Section End

5.6 Bookmark: GRM



5.7 GRM (I)



Overview

Probabilities (p_{ijk}) for individual score categories are defined as differences between cumulative probabilities (p_{ijk}^*)

Example

$$p_{ijk=3}^* - p_{ijk=4}^* = p_{ijk=3}$$



Each p_{ijk}^* is modeled as a 2PL

5.8 GRM (II)



Core Formula

$$P_j^*(U_j \geq k | \theta) = \frac{1}{1 + e^{-D a_j (\theta - b_{jk})}}$$

- Cumulative probability an individual with a given ability or trait θ selects or receives a category score of k or higher for item j
- U_j : response on item j
- θ is an ability value on the trait of interest
- a_j is the slope/discrimination parameter for item j
- b_{jk} is the threshold parameter for item j and category k
- D is the scaling parameter
 - ✓ Here we set $D = 1$
 - ✓ Other common value is $D = 1.702$

Reference

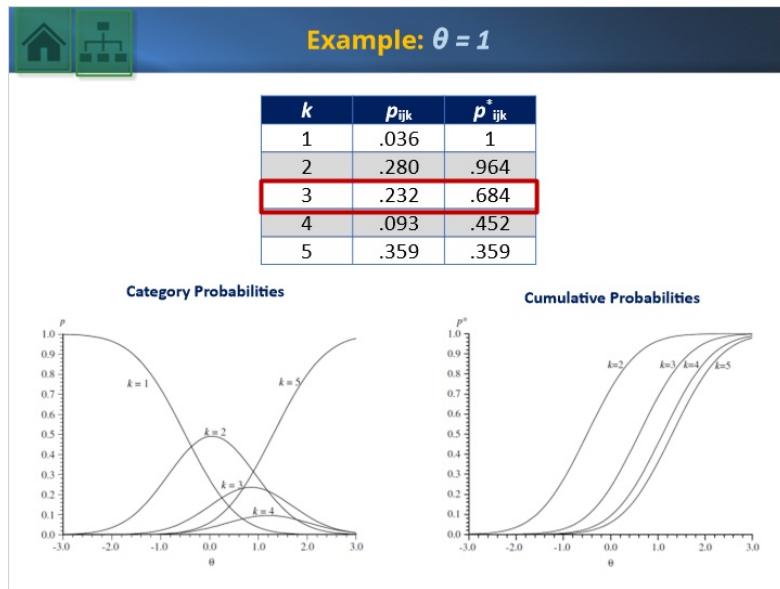
References (Slide Layer)

The screenshot shows a slide with a dark blue header containing icons for home and navigation, and the word "Reference" in yellow. Below the header is a white content area. At the top left of the content area is a small thumbnail of a book cover for "Handbook of Modern Item Response Theory". To its right is the title "Graded Response Model". Below the title are sections for "Authors" (Fumiko Samejima) and "Authors and affiliations". Under "Authors" are links for "Chapter" (302), "Citations" (1.7k), and "Downloads". A "Abstract" section follows, containing a brief description of the Graded Response Model. Below the abstract is a "Keywords" section listing "Latent Trait", "Item Parameter", "Homogeneous Case", "Partial Credit", and "Grade Response Model". A note at the bottom states: "These keywords were added by machine and not by the authors. This process is experimental and the keywords may be updated as the learning algorithm improves." A purple "Back" button is located at the bottom right of the slide.

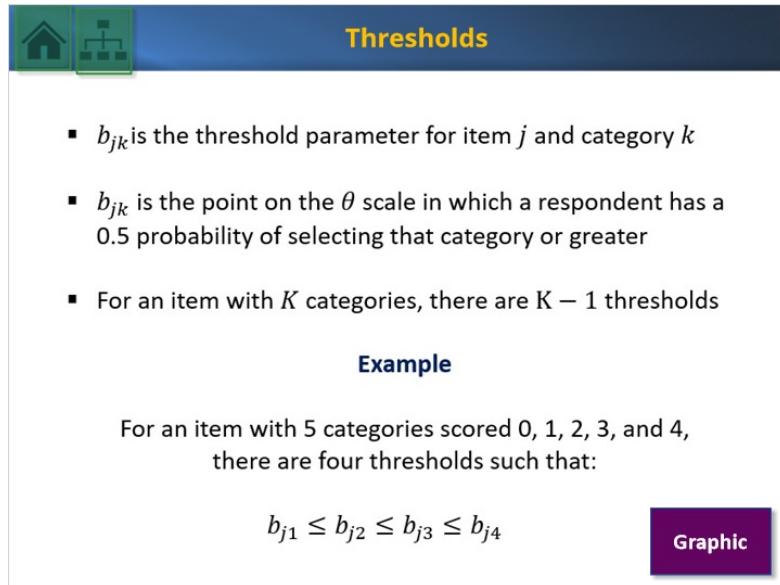
5.9 GRM (III)

The screenshot shows a slide with a dark blue header containing icons for home and navigation, and the word "Response Probabilities" in yellow. Below the header is a white content area. In the center is a large mathematical equation:
$$P_j^*(U_j \geq k | \theta) = \frac{1}{1 + e^{-D a_j (\theta - b_{jk})}}$$
. Below the equation is the text "Item j with Categories $k = 0, 1, \dots, K$ ". Below this are three equations: $P_j(U_j = 0) = 1 - P_j^*(U_j \geq 1 | \theta)$ for $k = 0$, $P_j(U_j = k) = P_j^*(U_j \geq k | \theta) - P_j^*(U_j \geq k + 1 | \theta)$ for $1 < k < K - 1$, and $P_j(U_j = K) = P_j^*(U_j \geq K | \theta)$ for $k = K$.

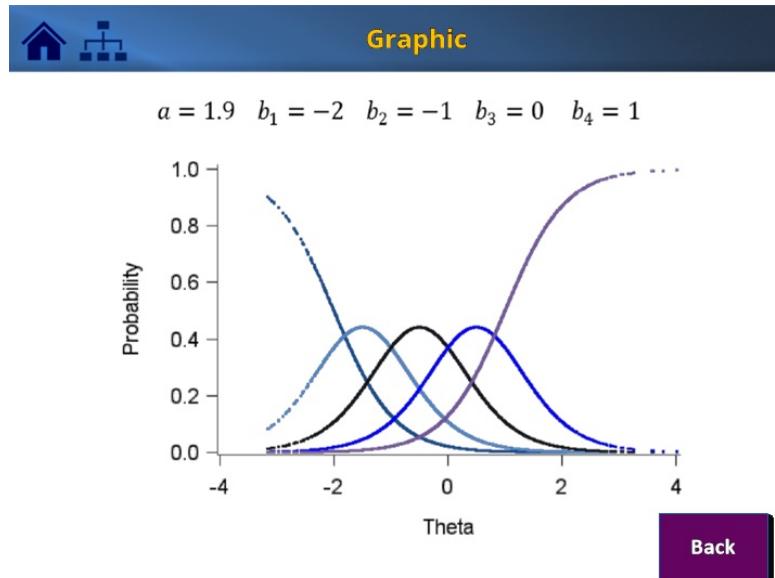
5.10 GRM (IV)



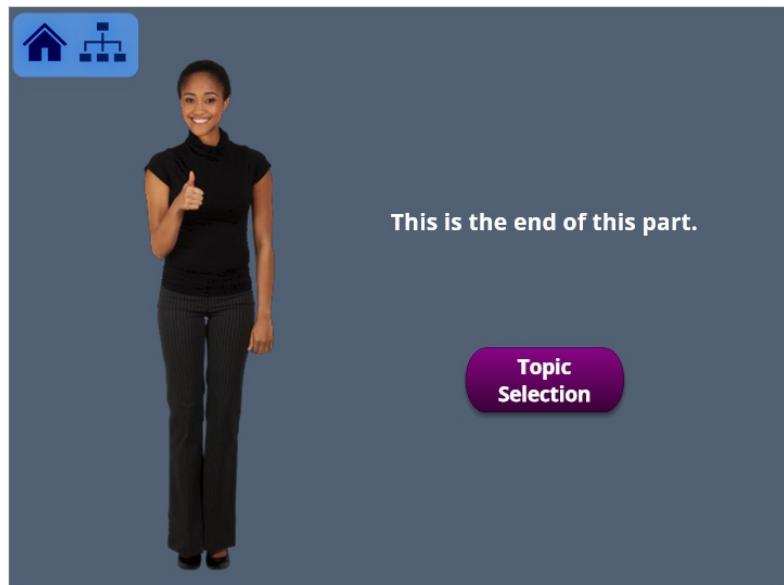
5.11 GRM (V)



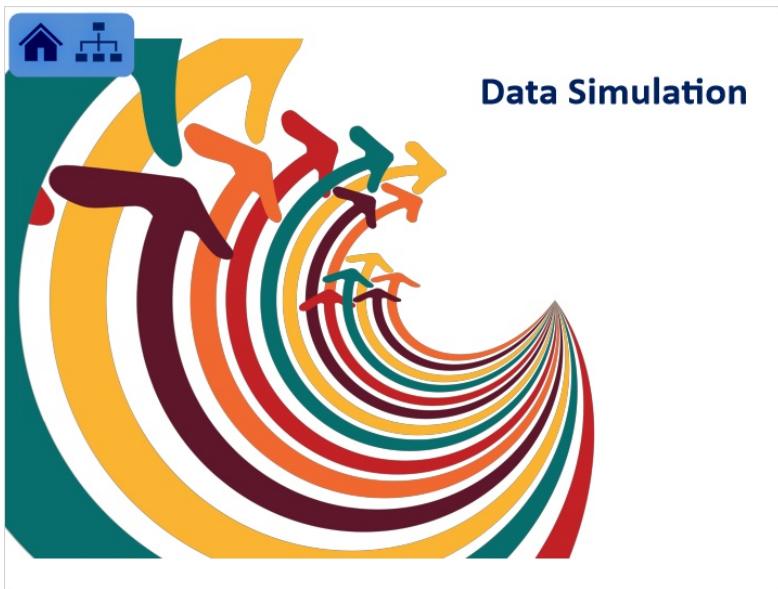
Graphic (Slide Layer)



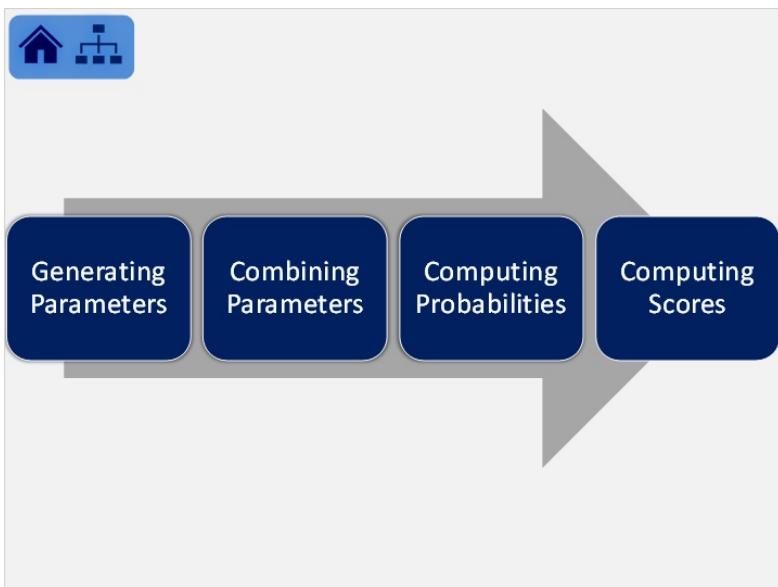
5.12 Bookend: GRM



5.13 Bookmark: Data Simulation



5.14 Step Selection



5.15 Person Parameters

The screenshot shows a SAS interface with a dark blue header bar. On the left, there are two small green icons: a house-like symbol and a tree-like symbol. To the right of these icons, the text "Person Parameters" is displayed in yellow. Below the header is a light gray workspace containing a portion of a SAS program. The program code is as follows:

```
%let n_person =500;
data person_parm;
do i = 1 to &n_person;
    theta=rand('normal',0,1);
    output;
end;
keep theta;
run;
```

A callout box with a blue border and white text is positioned to the right of the line "theta=rand('normal',0,1);". An arrow from the text box points to the "theta" variable in the code. The text in the callout box reads: "Create a value named **theta**. Randomly select theta value from a normal distribution with mean = 0 and var = 1".

5.16 Item Parameters

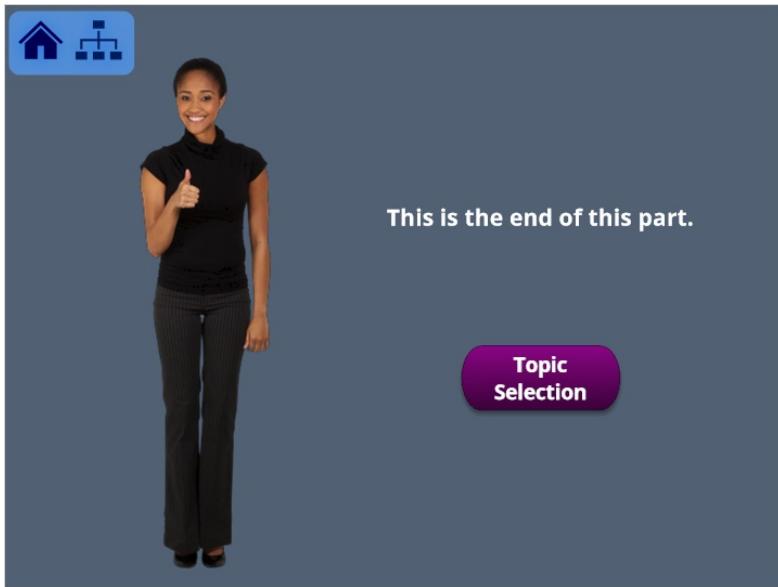
The screenshot shows a SAS interface with a dark blue header bar. On the left, there are two small green icons: a house-like symbol and a tree-like symbol. To the right of these icons, the text "Item Parameters" is displayed in yellow. Below the header is a light gray workspace containing a portion of a SAS program. The program code is as follows:

```
data item_parms;
input a b1 b2 b3 b4;
cards;
1.2   -2     -1     0     1
1.2   -1.5   -.5    .5    1.5
1.2   -1     0     1     2
1.2   -.5    .5    1.5   2.5
1.2   0     .5    1     1.5
1.9   -2     -1     0     1
1.9   -1.5   -.5    .5    1.5
1.9   -1     0     1     2
1.9   -.5    .5    1.5   2.5
1.9   0     .5    1     1.5
;
run;
```

To the left of the code, there are four blue-outlined rounded rectangles containing bulleted text:

- Create a data set that contains **item parameters**
- This may be based off of **empirical estimates, previous literature, etc.**
- You may **randomly select** item parameters as well
- **Literature** can be helpful in determining what **distribution** to randomly select from

5.17 Bookend: Step 1



5.18 Parameter Matrix (I)

Parameter Matrix (I)

- We need in the same data set / parameter matrix:
 - ✓ an individual's ability parameter θ
 - ✓ item parameters a_j and b_{jk}
- We want our data set / parameter matrix to look like this:

Theta	a_1	$b_{1,1}$	$b_{1,2}$	$b_{1,3}$	$b_{1,4}$...	a_{10}	$b_{10,1}$	$b_{10,2}$	$b_{10,3}$	$b_{10,4}$
1.43	1.2	-2	-1	0	1	...	1.9	0	.5	1	1.5
2.81	1.2	-2	-1	0	1	...	1.9	0	.5	1	1.5
:	:	:	:	:	:	:	:	:	:	:	:
0.09	1.2	-2	-1	0	1	...	1.9	0	.5	1	1.5

5.19 Parameter Matrix (II)

Parameter Matrix (II)

```

proc transpose data=item_parms out=a_wide prefix=a;
var a;
run;

proc transpose data=item_parms out=b1_wide prefix=b1_;
var b1;
run;

proc transpose data=item_parms out=b2_wide prefix=b2_;
var b2;
run;

proc transpose data=item_parms out=b3_wide prefix=b3_;
var b3;
run;

proc transpose data=item_parms out=b4_wide prefix=b4_;
var b4;
run;

data items_wide;
merge a_wide b1_wide b2_wide b3_wide b4_wide;
drop _name_;
run;

```

Turn parameters into wide form

5.20 Parameter Matrix (III)

Parameter Matrix (III)

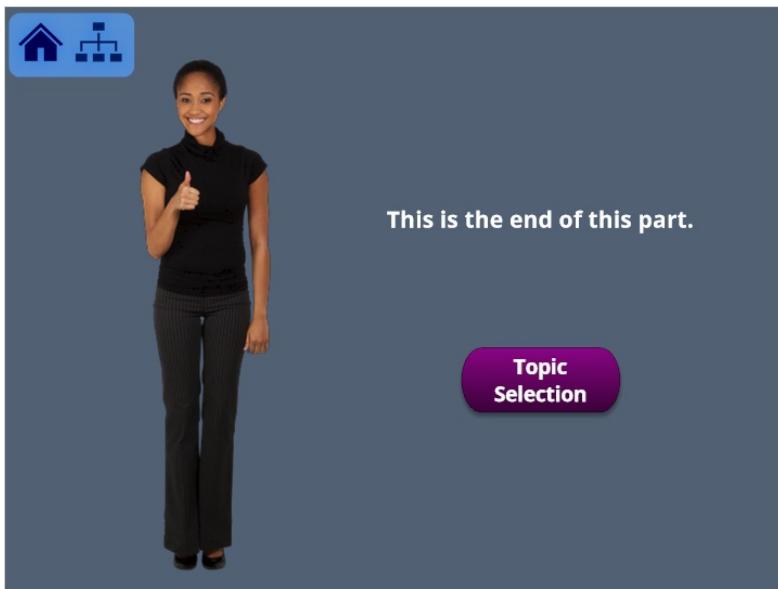
```

data all_parms;
  set person_parm;
  if _N_=1 then set items_wide ;
run;

```

ID	I ₁	I ₂	I ₃	I ₄	I ₅	I ₆	I ₇	I ₈	I ₉	I ₁₀	I ₁₁	I ₁₂	I ₁₃	I ₁₄	I ₁₅	I ₁₆	I ₁₇	I ₁₈	I ₁₉	I ₂₀	I ₂₁	I ₂₂	I ₂₃	I ₂₄	I ₂₅	I ₂₆	I ₂₇	I ₂₈	I ₂₉	I ₃₀				
1	0.00011724	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	0.00011724	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	0.00011724	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	0.00011724	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	0.00011724	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
6	0.00011724	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
7	0.00011724	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
8	0.00011724	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
9	0.00011724	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
10	0.00011724	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
11	0.00011724	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
12	0.00011724	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13	0.00011724	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
14	0.00011724	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
15	0.00011724	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
16	0.00011724	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
17	0.00011724	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
18	0.00011724	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
19	0.00011724	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
20	0.00011724	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
21	0.00011724	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
22	0.00011724	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
23	0.00011724	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
24	0.00011724	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
25	0.00011724	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
26	0.00011724	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
27	0.00011724	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
28	0.00011724	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
29	0.00011724	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
30	0.00011724	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

5.21 Bookend: Step 2



5.22 Cumulative Probabilities

A slide titled "Cumulative Probabilities" with a green header bar. It shows a table of parameters and several equations for calculating cumulative probabilities using the logistic function.

Theta	a_1	b_{11}	b_{12}	b_{13}	b_{14}	...
1.43	1.2	-2	-1	0	1	...
2.81	1.2	-2	-1	0	1	...
:	:				:	..
0.09	1.2	-2	-1	0	1	...

$$P_j^*(U_j \geq k | \theta) = \frac{1}{1 + e^{-a_j(\theta - b_{jk})}}$$
$$P(U_1 \geq 1) = \frac{1}{1 + e^{-1.2(1.43 - (-2))}} = .984$$
$$P(U_1 \geq 2) = \frac{1}{1 + e^{-1.2(1.43 - (-1))}} = .949$$
$$P(U_1 \geq 3) = \frac{1}{1 + e^{-1.2(1.43 - (0))}} = .848$$
$$P(U_1 \geq 4) = \frac{1}{1 + e^{-1.2(1.43 - (1))}} = .626$$

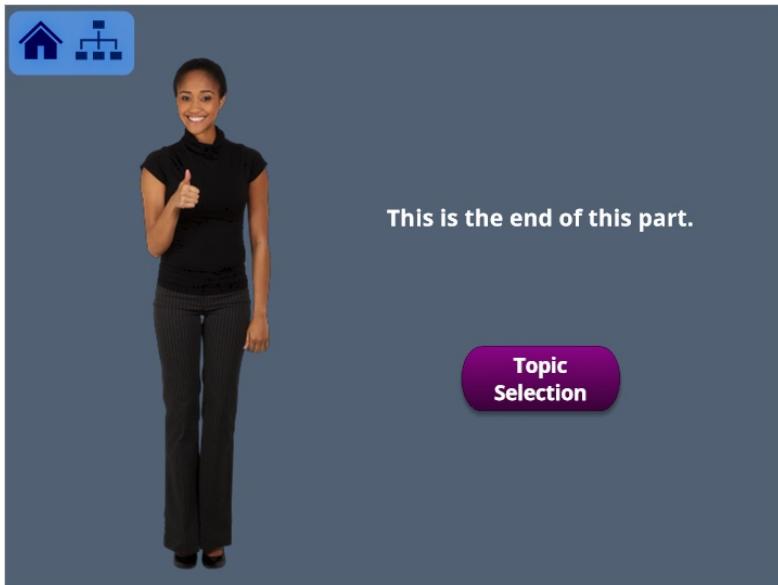

5.23 Category Probabilities



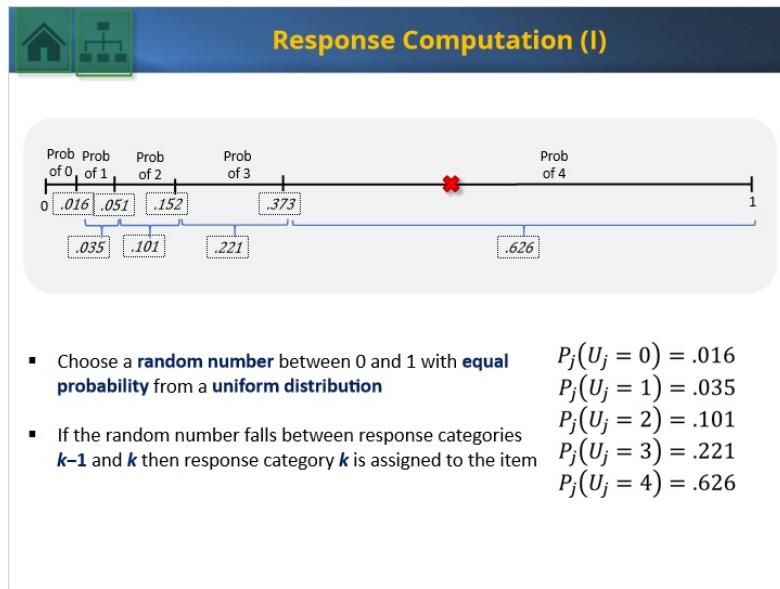
Category Probabilities

$$P_j(U_j = 0) = P_j^*(U_j \geq 1|\theta)$$
$$= 1 - .984 = .016$$
$$P_j(U_j = 1) = P_j^*(U_j \geq 1|\theta) - P_j^*(U_j \geq 2|\theta)$$
$$= .984 - .949 = .035$$
$$P_j(U_j = 2) = P_j^*(U_j \geq 2|\theta) - P_j^*(U_j \geq 3|\theta)$$
$$= .949 - .848 = .101$$
$$P_j(U_j = 3) = P_j^*(U_j \geq 3|\theta) - P_j^*(U_j \geq 4|\theta)$$
$$= .848 - .626 = .221$$
$$P_j(U_j = 4) = P_j^*(U_j \geq 4|\theta)$$
$$= .626$$

5.24 Bookend: Step 3



5.25 Response Computation (I)



- Choose a **random number** between 0 and 1 with **equal probability** from a **uniform distribution** $P_j(U_j = 0) = .016$ $P_j(U_j = 1) = .035$
- If the random number falls between response categories **k-1** and **k** then response category **k** is assigned to the item $P_j(U_j = 2) = .101$ $P_j(U_j = 3) = .221$ $P_j(U_j = 4) = .626$

5.26 Response Computation (II)

```
data responses;
  set all_parms;
  array a{&n_item} a1-a&n_item;
  array b1_{&n_item} b1_1-b1_&n_item;
  array b2_{&n_item} b2_1-b2_&n_item;
  array b3_{&n_item} b3_1-b3_&n_item;
  array b4_{&n_item} b4_1-b4_&n_item;

  array p0{&n_item} p0_1-p0_&n_item;
  array p1{&n_item} p1_1-p1_&n_item;
  array p2{&n_item} p2_1-p2_&n_item;
  array p3{&n_item} p3_1-p3_&n_item;
  array p4{&n_item} p4_1-p4_&n_item;

  array item{&n_item} item1 - item&n_item;
```

5.27 Response Computation (III)

Response Computation (III)

```

do j = 1 to &n_item;
  p1_star=1/(1+exp(-(a[j]*(theta-b1_[j]))));
  p2_star=1/(1+exp(-(a[j]*(theta-b2_[j]))));
  p3_star=1/(1+exp(-(a[j]*(theta-b3_[j]))));
  p4_star=1/(1+exp(-(a[j]*(theta-b4_[j]))));

  p0[j]=1-p1_star;
  p1[j]=p1_star-p2_star;
  p2[j]=p2_star-p3_star;
  p3[j]=p3_star-p4_star;
  p4[j]=p4_star;

  score_temp=rand("Table", p0[j], p1[j], p2[j], p3[j], p4[j]);
  item[j]=score_temp-1;
end;

```

Calculate cumulative probabilities
 Calculate score probabilities
 Assign scores {1-5}
 Convert scores (0-4)

5.28 Data Set Cleanup

Data Set Cleanup

```

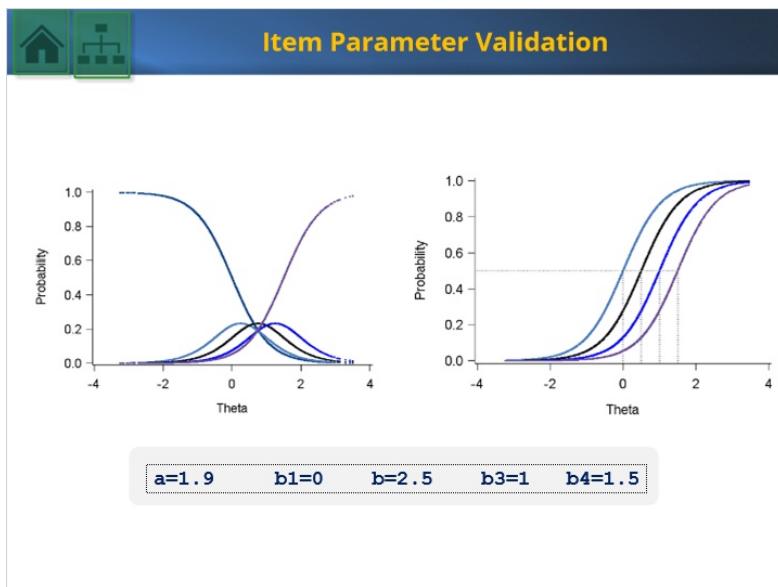
data responses_only;
  set responses;
  keep item:;
run;

```

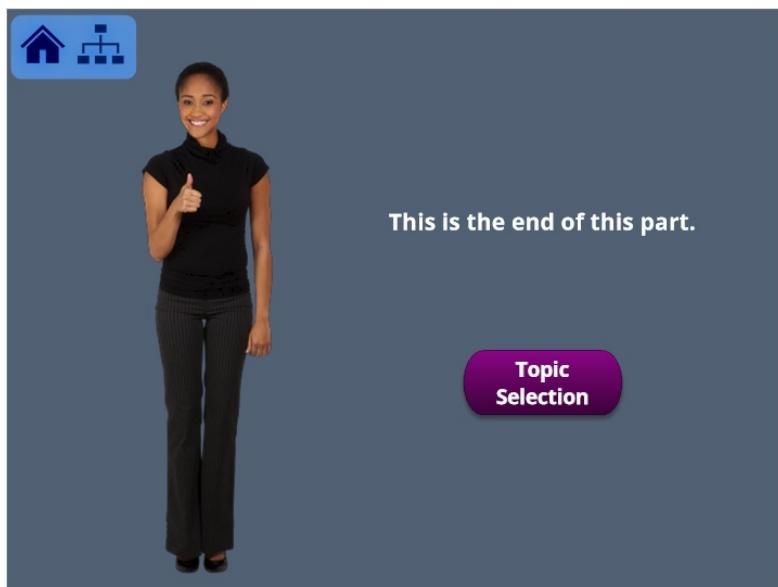
Keep only variables with prefix "item" →
 data responses_only;
set responses;
keep item:;
run;

	Item1	Item2	Item3	Item4	Item5	Item6	Item7	Item8	Item9	Item10
1	4	3	3	4	4	4	4	3	2	3
2	4	3	2	2	4	4	4	1	3	3
3	4	3	2	2	4	4	3	4	3	4
4	3	2	2	0	0	3	1	1	0	0
5	4	3	3	1	0	3	3	2	3	0
6	3	3	2	1	0	4	3	2	1	4
7	3	1	2	1	1	2	3	3	2	1
8	3	2	0	0	4	2	3	0	0	0
9	3	1	1	3	0	1	0	1	2	0
10	1	2	0	1	4	3	1	2	2	0
11	2	4	4	4	4	4	4	1	4	1
12	3	2	2	2	0	3	3	3	1	3
13	1	3	0	0	2	0	0	1	0	0
14	2	4	0	1	1	1	2	3	1	1
15	3	0	0	0	0	3	0	0	0	0
16	1	0	0	0	0	3	0	0	0	0
17	3	1	3	0	0	2	2	0	2	0
18	2	4	1	0	3	1	3	4	1	2
19	1	2	0	0	1	0	0	0	0	0
20	0	3	0	0	0	0	0	0	0	0
21	2	2	1	3	3	2	3	2	3	3
22	2	1	1	0	0	3	2	0	1	4
23	2	3	2	0	1	1	2	0	1	4
24	4	4	3	1	0	3	4	2	2	3

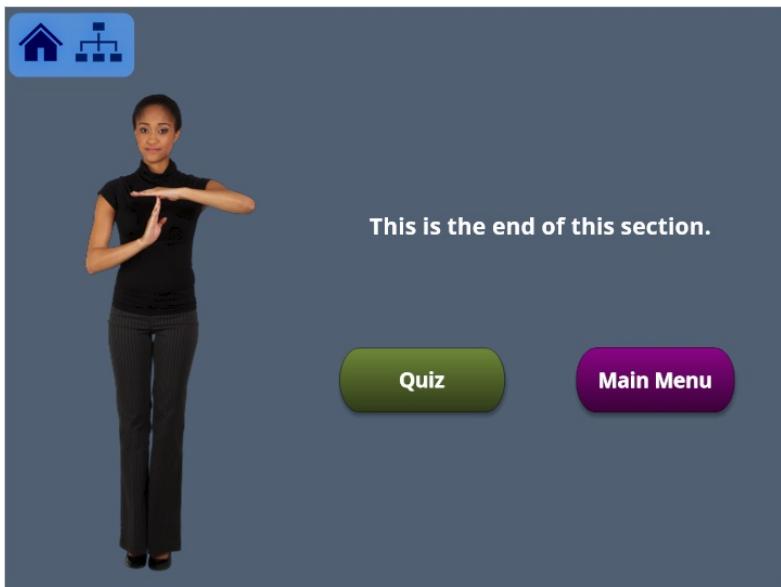
5.29 Item Parameter Validation



5.30 Bookend: Step 4



5.31 Bookend: Section 4

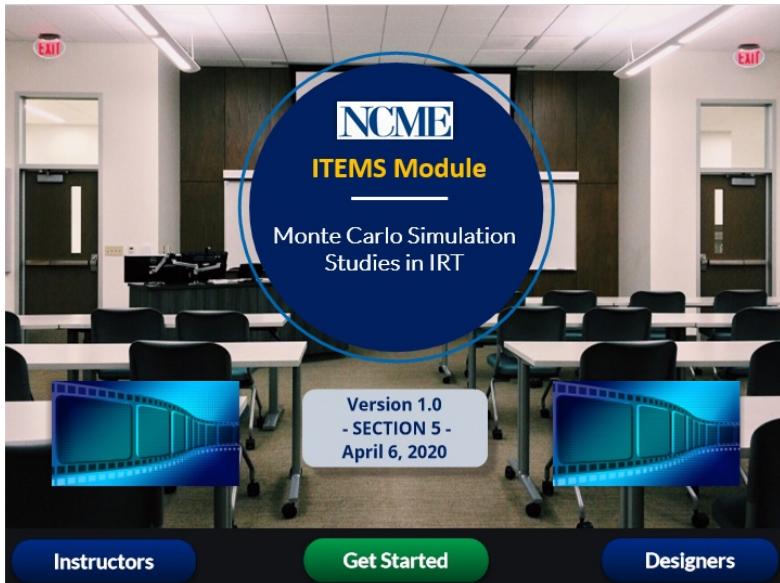


5.32 Module Cover (END)

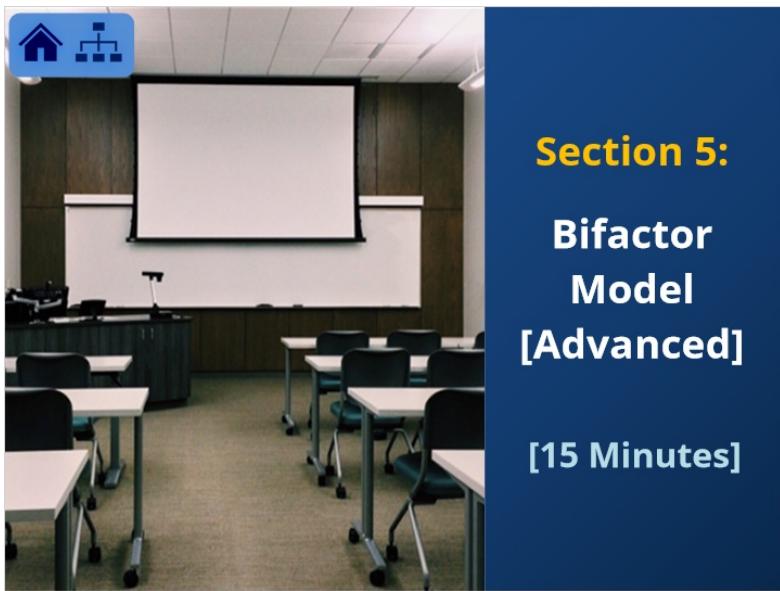


6. Section 5: Bifactor Model [Advanced]

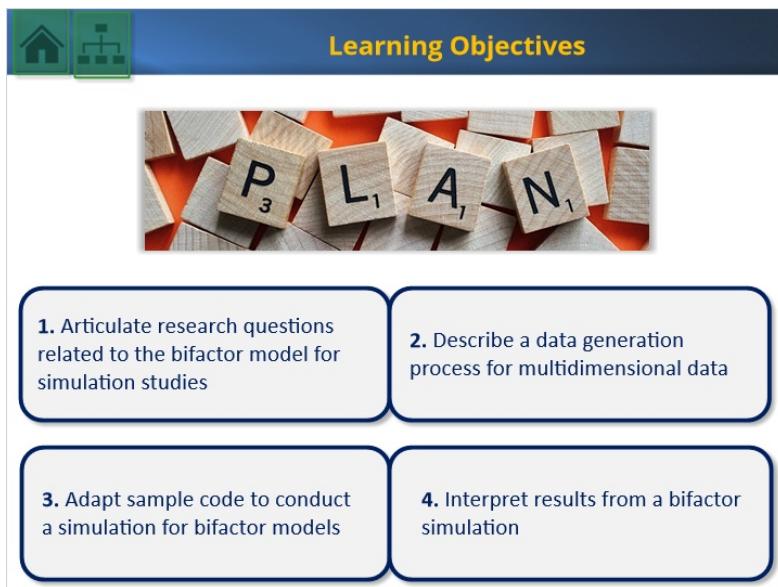
6.1 Module Cover (START)



6.2 Cover: Section 4



6.3 Objectives



The slide features a blue header bar with the text "Learning Objectives". Below the header is a photograph of wooden blocks spelling out "PLAN" against a red background. The main content area is divided into four rounded rectangular boxes, each containing one of four numbered objectives:

1. Articulate research questions related to the bifactor model for simulation studies
2. Describe a data generation process for multidimensional data
3. Adapt sample code to conduct a simulation for bifactor models
4. Interpret results from a bifactor simulation

6.4 Topic Selection



The slide features a blue header bar with icons for a house and a tree. Below the header is a central image of a woman in a black turtleneck and pants, standing with her arms outstretched. To her left is a purple box labeled "Bifactor Model" and to her right is another purple box labeled "Data Simulation". At the bottom center is a dark blue button labeled "Section End".

6.5 Bookmark: Bifactor Model

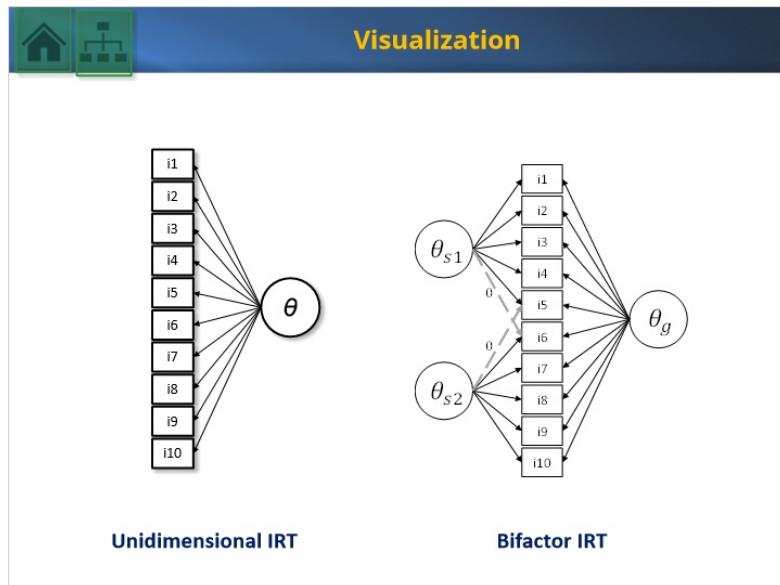


6.6 Bifactor Model (I)

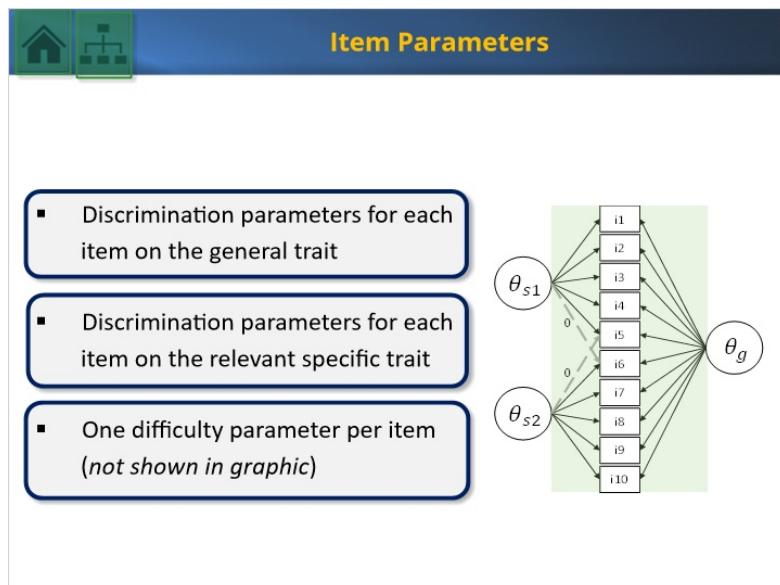
The slide has a dark blue header bar with a green icon of a house and a tree on the left, and the word "Overview" in yellow text on the right. The main content area contains a bulleted list of five items, each enclosed in a rounded rectangular box with a thin blue border:

- Bifactor models are multidimensional
 - ✓ General factor: Primary trait of interest
 - ✓ Specific (secondary) factors: Capture the residual item relationships beyond those accounted by the general factor
- Each response is determined by the general and no more than one specific factor
- The specific factors are orthogonal to the general factor (uncorrelated)

6.7 Bifactor Model (II)



6.8 Bifactor Model (III)



6.9 Bifactor Model (IV)

Core Formula

$$P_j(U_j = 1 | \omega_j, \theta) = \frac{1}{1 + e^{-(a_{js1}\theta_{s1} + a_{js2}\theta_{s2} + a_{jg}\theta_g + d_j)}}$$

P_j : Response probability for item j

U_j : Response on item j
1: correct
0: incorrect

$U_j = 1$: We are specifically modeling the probability of a correct response

ω_j : Vector of all item parameters for item j

θ : Vector of person parameters (general and specific)

6.10 Bifactor Model (V)

Item Parameters

$$P_j(U_j = 1 | \omega_j, \theta) = \frac{1}{1 + e^{-(a_{js1}\theta_{s1} + a_{js2}\theta_{s2} + a_{jg}\theta_g + d_j)}}$$

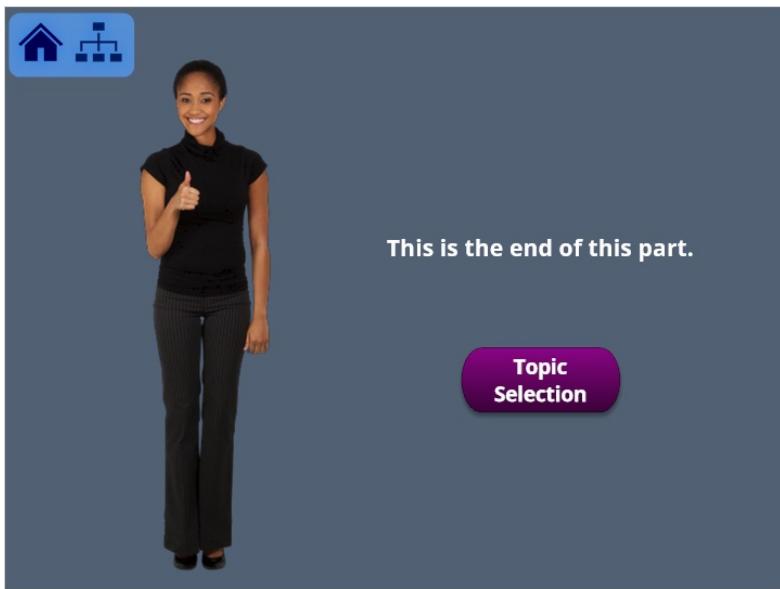
a_{js1} : discrimination parameter for item j on specific trait 1

a_{js2} : discrimination parameter for item j on specific trait 2

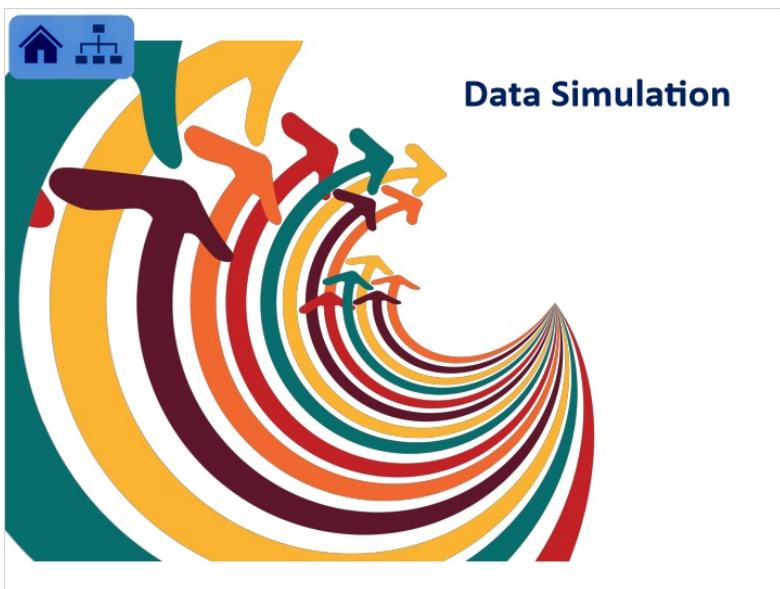
a_{jg} : discrimination parameter for item j on the general trait

d_j : difficulty / threshold parameter

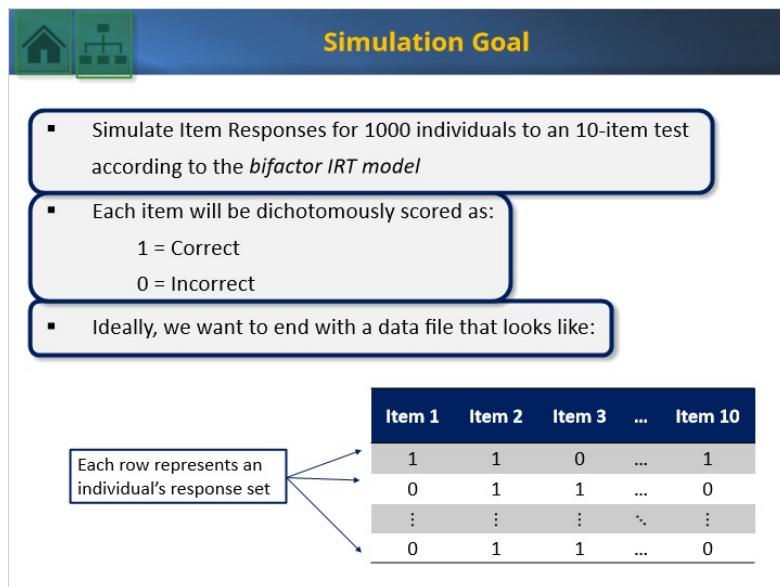
6.11 Bookend: Bifactor Model



6.12 Bookmark: Data Simulation



6.13 Simulation Goal



The slide has a blue header bar with icons for home and simulation. The title "Simulation Goal" is in yellow. Below the title is a bulleted list:

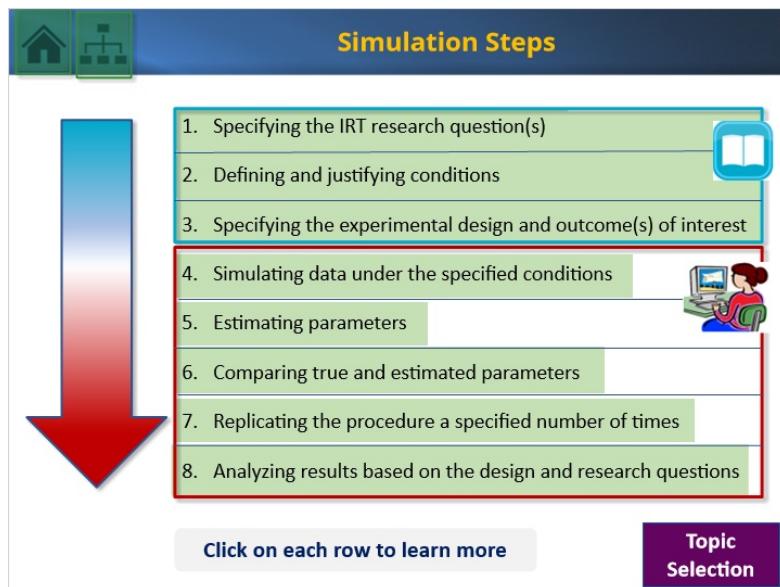
- Simulate Item Responses for 1000 individuals to an 10-item test according to the *bifactor IRT model*
- Each item will be dichotomously scored as:
 - 1 = Correct
 - 0 = Incorrect
- Ideally, we want to end with a data file that looks like:

Below the list is a table illustrating the data structure:

	Item 1	Item 2	Item 3	...	Item 10
1	1	1	0	...	1
0	0	1	1	...	0
:	:	:	:	..	:
0	0	1	1	...	0

A callout box points to the first row of the table with the text: "Each row represents an individual's response set".

6.14 Step Selection



The slide has a blue header bar with icons for home and simulation. The title "Simulation Steps" is in yellow. To the left of the steps is a large red downward arrow.

1. Specifying the IRT research question(s) 
2. Defining and justifying conditions
3. Specifying the experimental design and outcome(s) of interest
4. Simulating data under the specified conditions 
5. Estimating parameters
6. Comparing true and estimated parameters
7. Replicating the procedure a specified number of times
8. Analyzing results based on the design and research questions

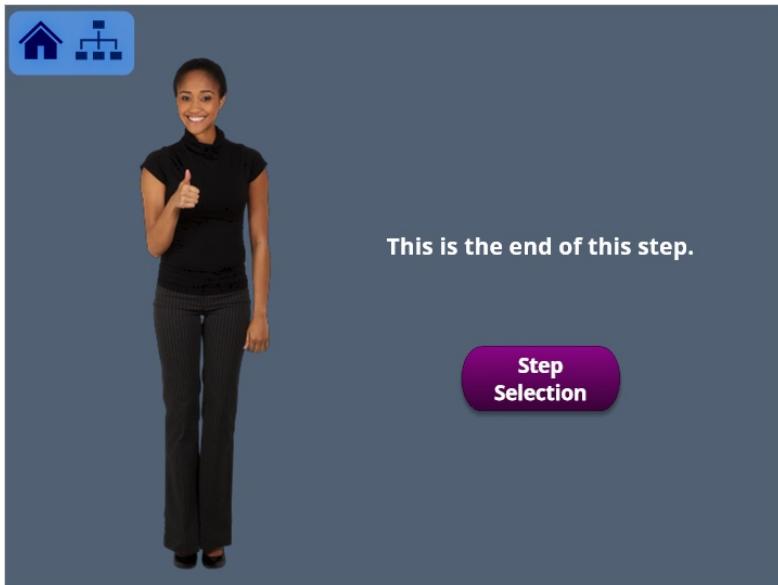
At the bottom left is a button: "Click on each row to learn more". At the bottom right is a button: "Topic Selection".

6.15 Research Questions

The screenshot shows a software interface with a blue header bar containing icons for a house and a tree, followed by the text "Step 1: Research Question(s)". Below the header are two rounded rectangular boxes, each containing an example and a research question.

- Example:** Cai, L., Yang, J., & Hansen, M. (2011). Generalized full-information item bifactor analysis. *Psychological Methods*, 16(3), 221-248.
Research Question: What are the bias and precision of the item parameters from a mixed format (dichotomous and polytomously scored) bifactor assessment?
Parameter Matrix (I)
- Example:** Fukuhara, H., & Kamata, A. (2011). A bifactor multidimensional item response theory model for differential item functioning analysis on testlet-based items. *Applied Psychological Measurement*, 35(8), 604-622.
Research Question: What is the accuracy of estimating DIF magnitude by the proposed bifactor MIRT DIF model?

6.16 Bookend: Step 1



6.17 Justifying Conditions

Step 2: Justifying Conditions

- Cai, Yang, & Hansen (2011)
 - Discrimination on the dichotomous and polytomously scored items
- Our simple example will manipulate sample size
 - $n = 500$
 - $n = 2000$



6.18 Bookend: Step 2



This is the end of this step.

Step Selection

6.19 Experimental Design



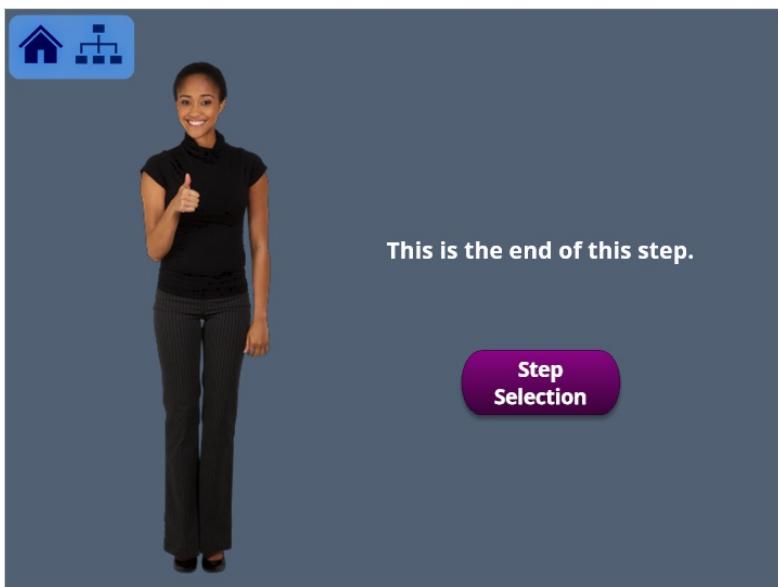
Step 3: Experimental Design

- Cai, Yang, & Hansen (2011):
Bias and estimated **standard errors** of the **item parameters**
- Our simple example will focus on **bias** of the **item parameters**

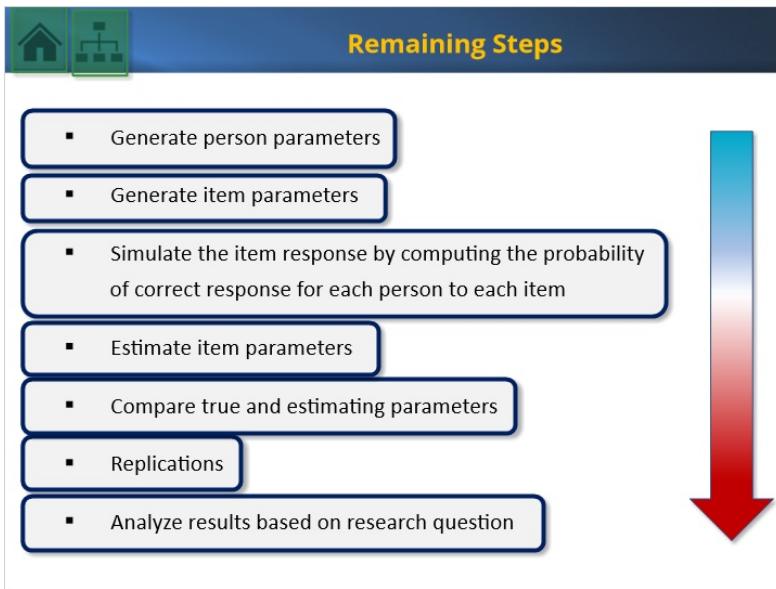
$$bias = \frac{\sum_{i=1}^n (\hat{\omega}_i - \omega_{true})}{n}$$

- $\hat{\omega}_i$ is the estimated item parameter for replication i
- ω_{true} is the generating item parameter
- n is the number of replications

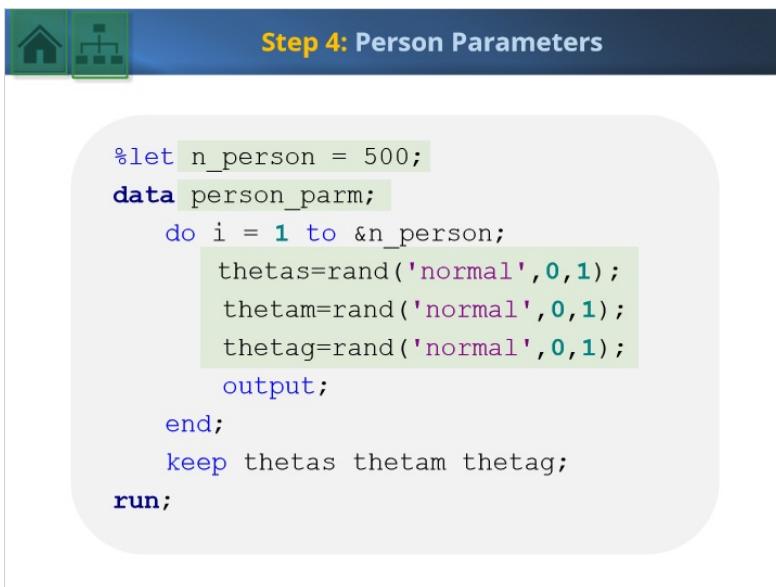
6.20 Bookend: Step 3



6.21 Remaining Steps



6.22 Person Parameters



6.23 Item Parameters

Step 4: Item Parameters

- Borrow item parameters from DeMars (2013)
- Loadings not in the model are fixed to 0

Item	a_{ig}	a_{js}	a_{jm}	d_i
1	1.06	0.44	0	-0.01
2	1.59	0.45	0	-0.13
3	0.98	0.36	0	0.44
4	0.88	0.48	0	-0.34
5	0.27	0.33	0	1.26
6	1.04	0	0.51	-0.77
7	0.61	0	0.23	-0.07
8	1.34	0	0.45	-0.14
9	0.96	0	0.46	2.22
10	1.20	0	0.42	0.18

6.24 Parameter Matrix (I)

Step 4: Parameter Matrix (I)

```

data item_parms;
input ag as am d;
cards;
1.06 .44 0 -.01
1.59 .45 0 -.13
0.98 .36 0 0.44
0.88 .48 0 -.34
0.27 .33 0 1.26
1.04 0 .51 -.77
0.61 0 .23 -.07
1.34 0 .45 -.14
0.96 0 .46 2.22
1.20 0 .42 0.18
;
run;

```

6.25 Parameter Matrix (II)



Step 4: Parameter Matrix (II)

- We want our data set / parameter matrix to look like this:

Thetag	Thetas	Thetam	a_g1	a_s1	a_m1	d_1	...
-1.43	1.02	0.22	1.06	0.44	0	-2	...
2.81	-0.05	0.57	1.06	0.44	0	-2	...
:	:	:	:	:	:	:	:
0.09	1.3	-0.25	1.06	0.44	0	-2	...

- Each row represents an individual
- Individuals are all answering the same items so the item parameters are the same for each line

6.26 Parameter Matrix (III)



Step 4: Parameter Matrix (III)

```
proc transpose data=item_parms out=as_wide prefix=as;
var as;
run;

proc transpose data=item_parms out=am_wide prefix=am;
var am;
run;

proc transpose data=item_parms out=ag_wide prefix=ag;
var ag;
run;

proc transpose data=item_parms out=d_wide prefix=d;
var d;
run;

data items_wide;
merge as_wide am_wide ag_wide d_wide;
drop _name_;
run;
```

6.27 Parameter Matrix (IV)

Step 4: Parameter Matrix (IV)

- Let's consider one individual and the first item:

thetas	thetam	thetag	as1	am1	ag1	d1
1.54	-1.30	-0.81	0.44	0	1.06	-0.01

- We want a data file that looks like this:

↓

Item 1	Item 2	Item 3	...	Item 10
1	1	0	...	1
0	1	1	...	0
:	:	:	..	:
0	1	1	...	0

6.28 Parameter Matrix (V)

Step 4: Parameter Matrix (V)

```
data responses;
set all_parms;
array as{&n_item} as1-as&n_item;
array am{&n_item} am1-am&n_item;
array ag{&n_item} ag1-ag&n_item;
array d{&n_item} d1-d&n_item;
array p{&n_item} p1-p&n_item;
array item{&n_item} item1 - item&n_item;
do j = 1 to &n_item;
  p[j]=1/(1+exp(-(as[j]*thetas+am[j]*thetam+ag[j]*thetag+d[j]))
  y=rand('uniform');
  if y<=p[j] then item[j]=1;
  else if y>p[j] then item[j]=0;
end;
run;
```

$P_i(U_i = 1 | \omega_j, \theta) = \frac{1}{1 + e^{-(a_{s1}\theta_s + a_{m1}\theta_m + a_{g1}\theta_g + d_1)}}$

6.29 Bookend: Step 4



6.30 Parameter Estimation

A slide titled "Step 5: Parameter Estimation". It contains a code snippet for PROC IRT to estimate a bifactor model.

```
proc irt data=responses;
  var item1-item10;
  factor  Factorg====> item1-item10,
          Factors====> item1-item5= 1. load2 load3
                           load4 load5,
          FactorM====> item6-item10= 1. load7 load8
                           load9 load10;
  ods output ParameterEstimates=d1 slope=a1;
run;
quit;
```

6.31 Bookend: Step 5



6.32 Parameter Comparison (I)

A slide titled "Step 6: Parameter Comparison (I)" showing SAS code in a code editor window. The code is as follows:

```
data bias;
merge item_parms d1 a1;
biasd=d-Estimate;
biasag=ag-Factorg;
biasas=as-Factors;
biasam=am-Factorm;
item=_n_;
run;
```

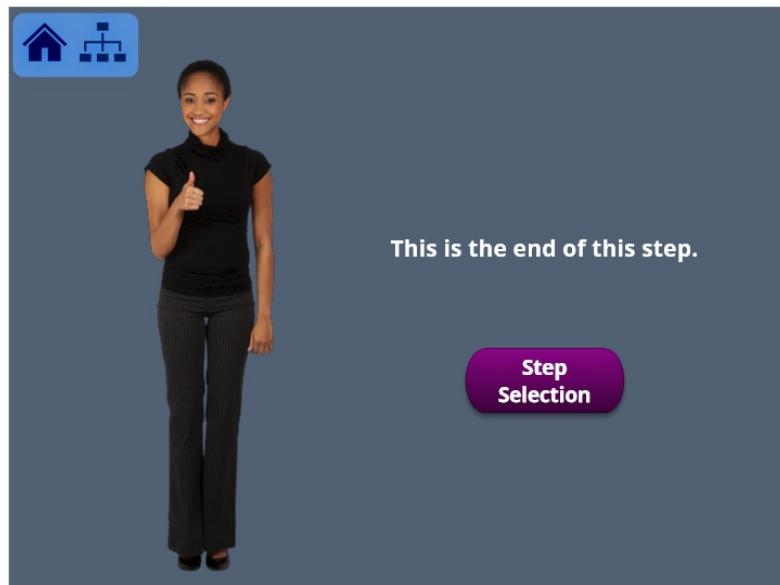
6.33 Parameter Comparison (II)



Step 6: Parameter Comparison (II)

items	ag	biasag	as	biasas	am	biasam	d	biasd
1	1.06	-0.1135	0.44	-0.56	-	-	-0.01	0.00695
2	1.59	-0.0178	0.45	0.16476	-	-	-0.13	-0.0107
3	0.98	0.03178	0.36	0.11529	-	-	0.44	0.01344
4	0.88	-0.0007	0.48	0.12659	-	-	-0.34	0.0003
5	0.27	-0.0299	0.33	0.18134	-	-	1.26	0.00216
6	1.04	-0.1116	-	-	0.51	-0.49	-0.77	0.11083
7	0.61	-0.0184	-	-	0.23	0.12497	-0.07	-0.0003
8	1.34	-0.0416	-	-	0.45	0.22995	-0.14	0.00957
9	0.96	-0.0267	-	-	0.46	0.23003	2.22	0.026
10	1.2	0.01481	-	-	0.42	0.14133	0.18	-0.0067

6.34 Bookend: Step 6



6.35 Replications (I)

Step 7: Replications

```
%macro IRTsim;  
%do rep = 1 %to &n_reps;  
    [Data generation steps]  
    ...  
    [Model estimation steps]  
    [Compute bias steps]  
%end;  
%mend IRTsim;  
%IRTsim;
```

6.36 Replications (II)

Step 7: Keeping Track of Conditions (I)

```
%macro IRTsim;  
%do np = 500 %to 2000 %by 1500;  
    %do rep = 1 %to &n_reps;  
        [Data generation steps]  
        ...  
        [Model estimation steps]  
        [Compute bias steps]  
    %end;  
    %end;  
%mend IRTsim;  
%IRTsim;
```

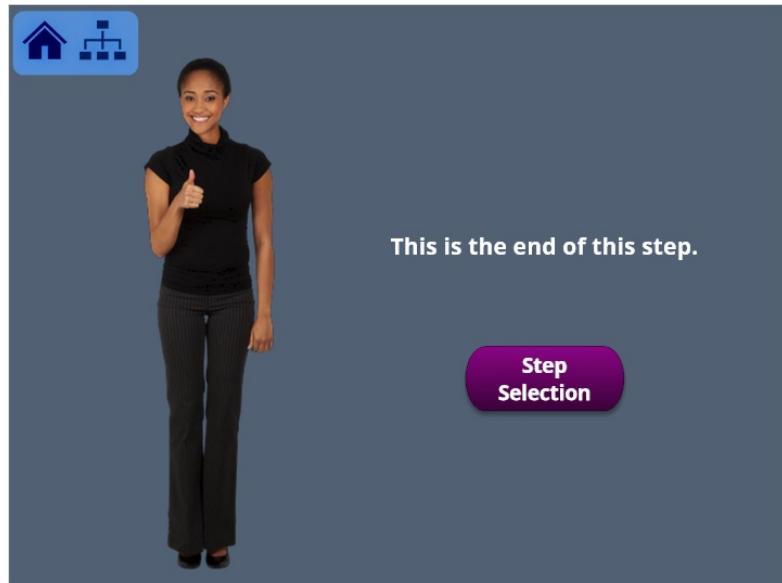
6.37 Replications (III)



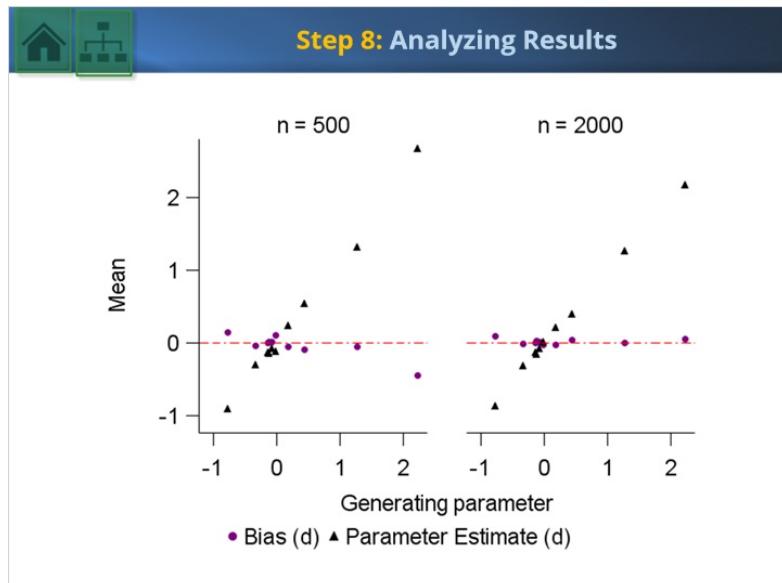
Step 7: Keeping Track of Conditions (II)

```
data responses;  set all_parms;
    array as{&n_item} as1-as&n_item;
    array am{&n_item} am1-am&n_item;
    array ag{&n_item} ag1-ag&n_item;
    array d{&n_item} d1-d&n_item;
    array p{&n_item} p1-p&n_item;
    array item{&n_item} item1 - item&n_item;
    do j = 1 to &n_item;
        p[j]=1/(1+exp(-(as[j]*thetas+am[j]*thetam+ag[j]*thet
+d[j])));
        y=rand('uniform');
        if y<=p[j] then item[j]=1;
        else if y>p[j] then item[j]=0;
    end;
    drop j;
    rep=&rep;
    n=&np;
run;
```

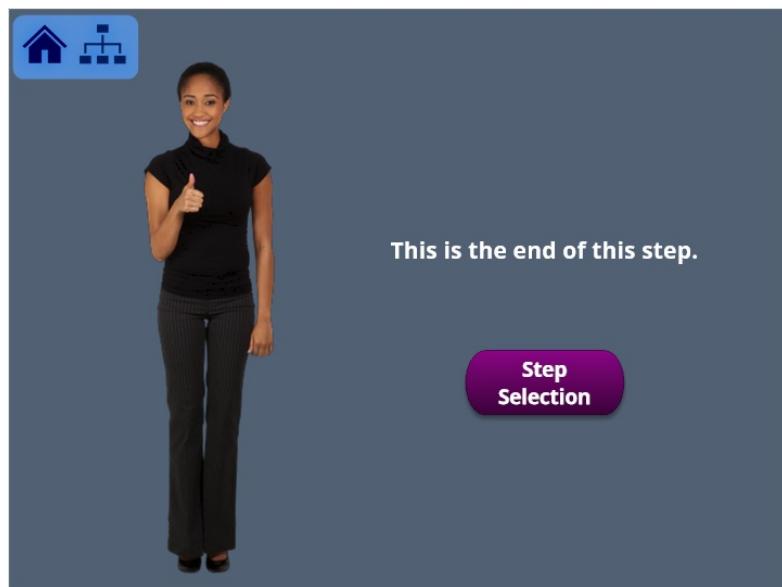
6.38 Bookend: Step 7



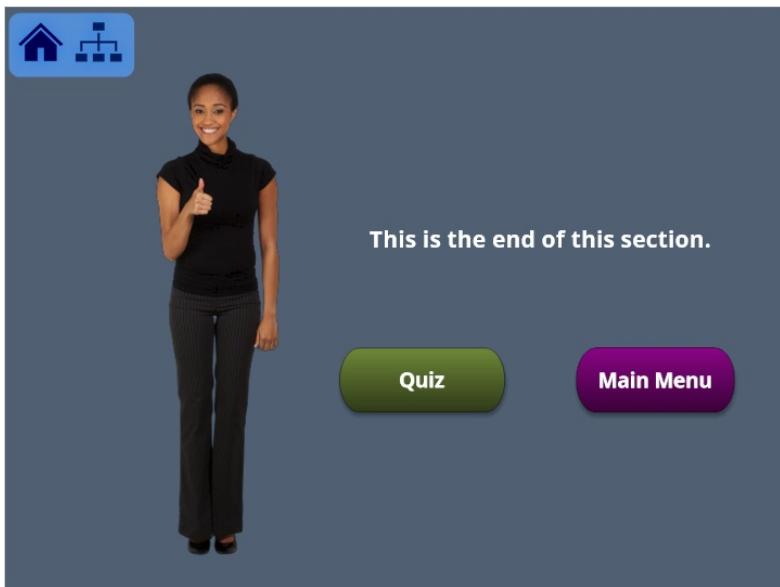
6.39 Analyzing Results



6.40 Bookend: Step 8



6.41 Bookend: Section 5



6.42 Module Cover (END)

