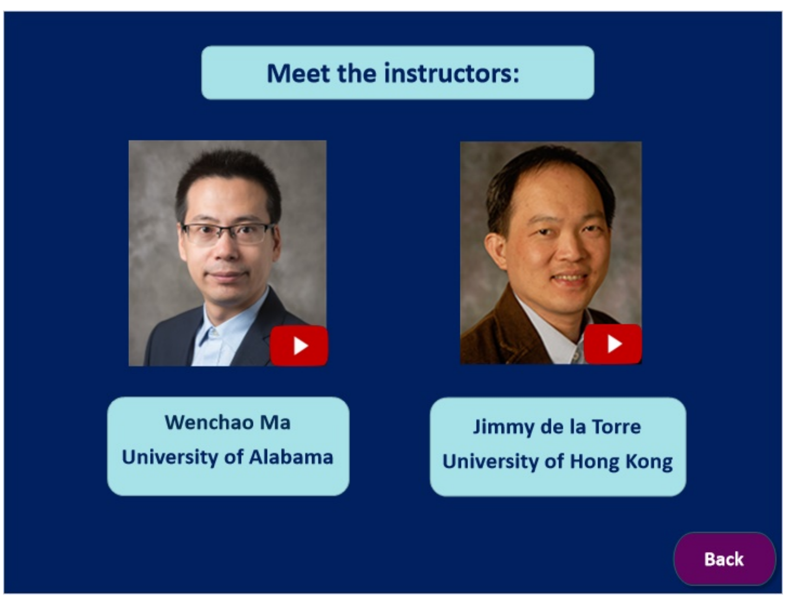
DM05 SLIDES (G-DINA Framework, Version 1.5)

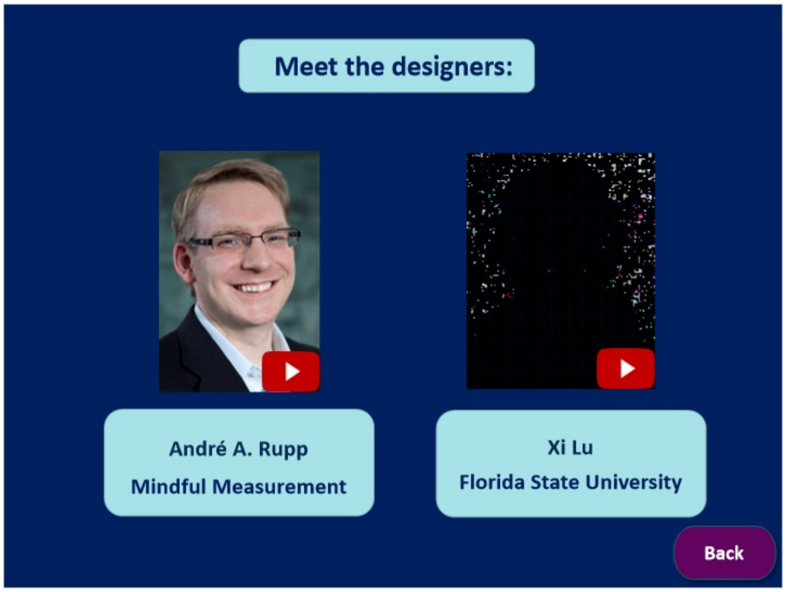
1. Module Overview

1.1 Module Cover (START)

1.2 Instructors

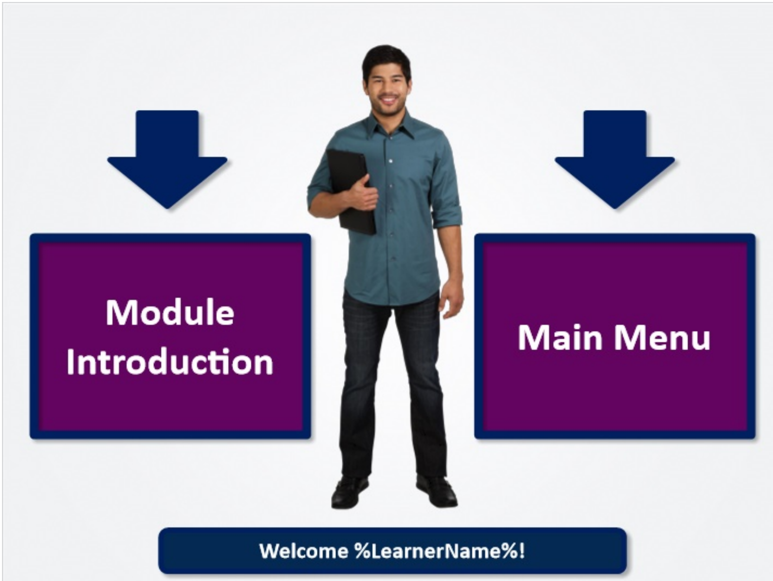


1.3 Designers



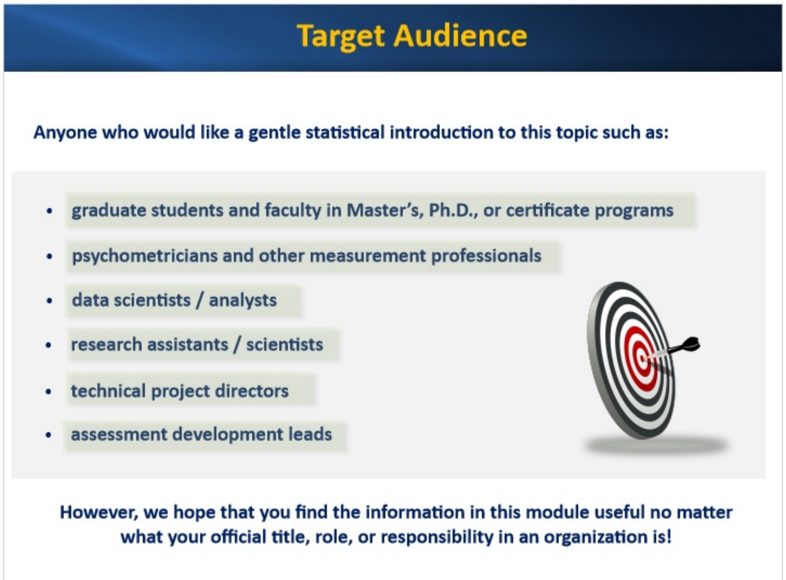
1.4 Welcome

1.5 Path Choice



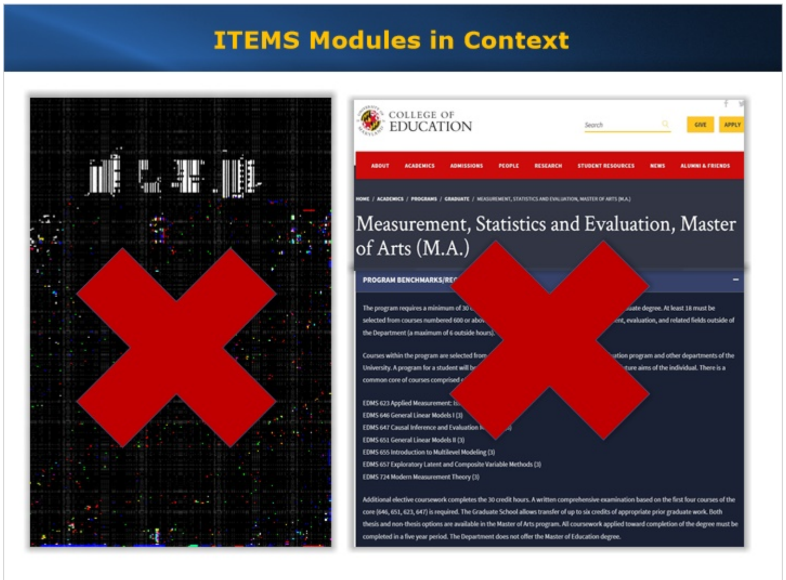
1.6 Overview

1.7 Target Audience

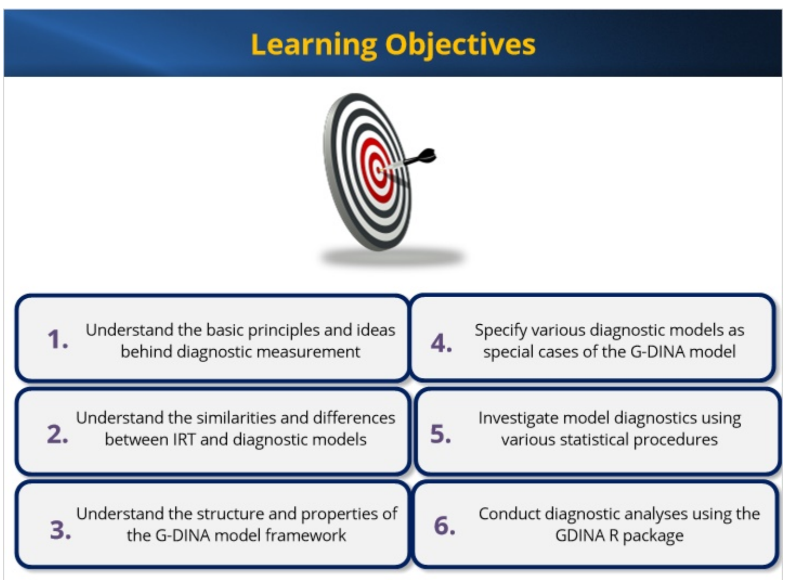


1.8 Expecations (I)

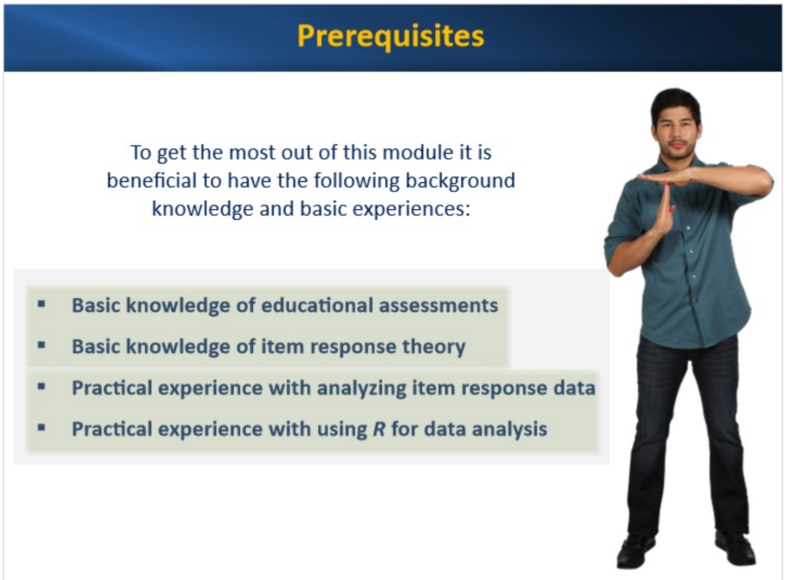
1.9 Expectations (II)



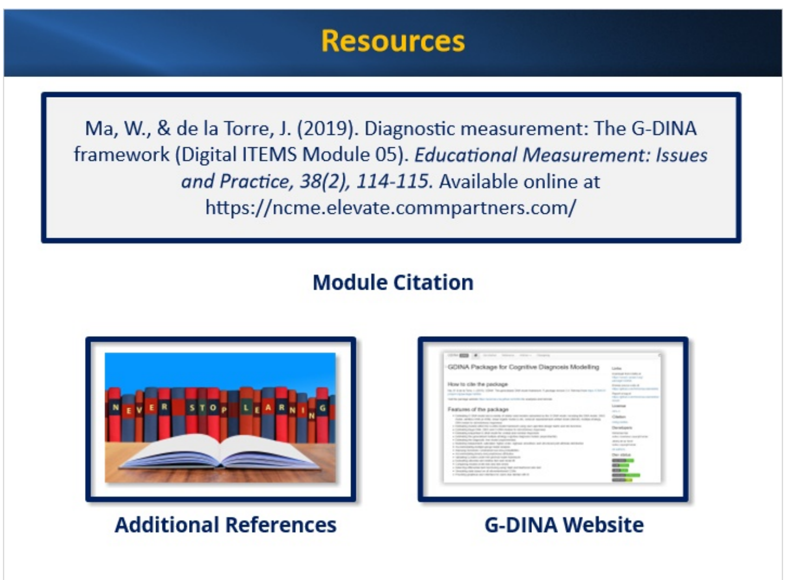
1.10 Learning Objectives



1.11 Prerequisites



1.12 Resources



References (Slide Layer)

References (I)

The G-DINA Model Framework

de la Torre, J. (2011). The generalized DINA framework. *Psychometrika*, *76*, 179-199. https://doi.org/10.1007/s11336-011-9207-7

Q-matrix Validation Procedures

de la Torre, J., & Chiu, C. Y. (2016). A General Method of Empirical Q-matrix Validation. *Psychometrika*, *81*, 253-273. https://doi.org/10.1007/s11336-015-9467-8

Ma, W., & de la Torre, J. (2019). An empirical Q-matrix validation method for the sequential generalized DINA model. *British Journal of Mathematical and Statistical Psychology*. https://doi.org/10.1111/bmsp.12156

Nájera, P., Sorrel, M. A., & Abad, F. J. (2019). Reconsidering Cutoff Points in the General Method of Empirical Q-Matrix Validation. *Educational and Psychological Measurement*. https://doi.org/10.1177/0013164418822700

More References

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More References (Slide Layer)

References (II)

Item-level Model Comparison

de la Torre, J., & Lee, Y. S. (2013). Evaluating the Wald test for item-level comparison of saturated and reduced models in cognitive diagnosis. *Journal of Educational Measurement, 50*, 355-373. https://doi.org/10.1111/jedm.12022

Ma, W., Iaconangelo, C., & de la Torre, J. (2016). Model Similarity, Model Selection, and Attribute Classification. *Applied Psychological Measurement*, *40*, 200-217. https://doi.org/10.1177/0146621615621717

Other References

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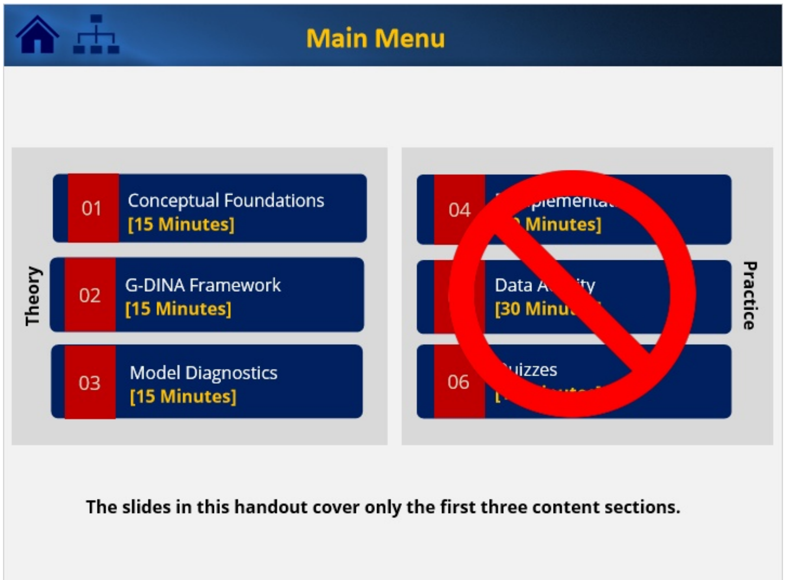
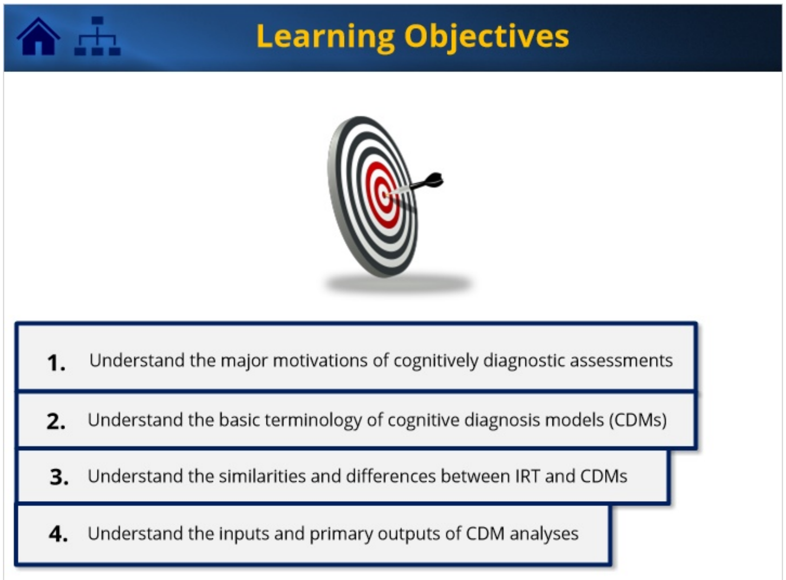
1.13 Main Menu



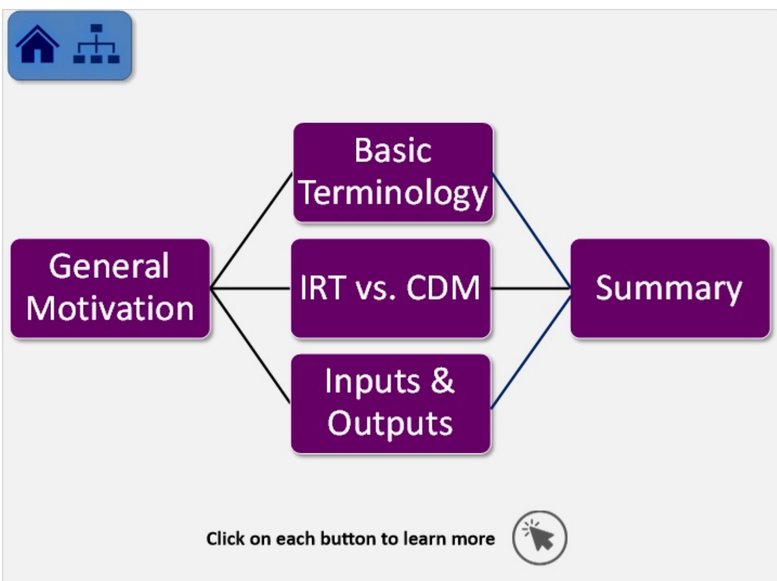
2. Section 1: Conceptual Foundations

2.1 Cover: Section 1

2.2 Objectives: Section 1

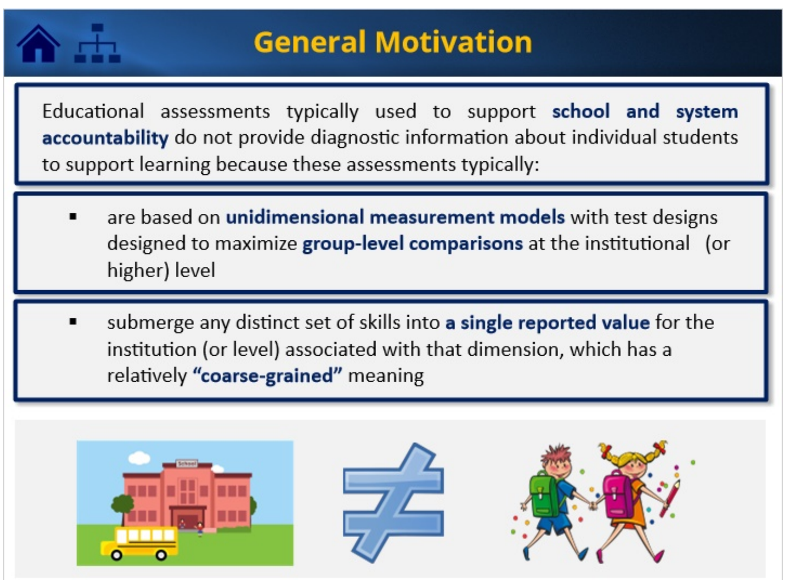


2.3 Topic Selection

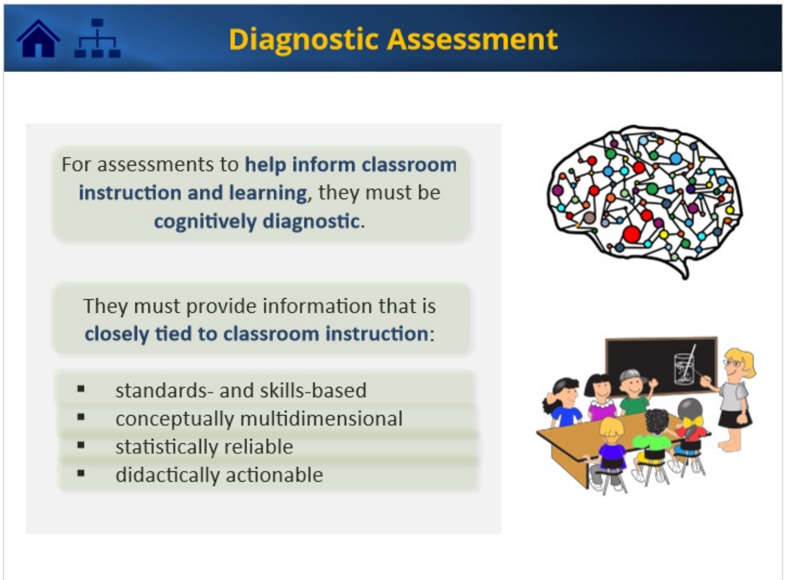


2.4 Bookmark: General Motivation

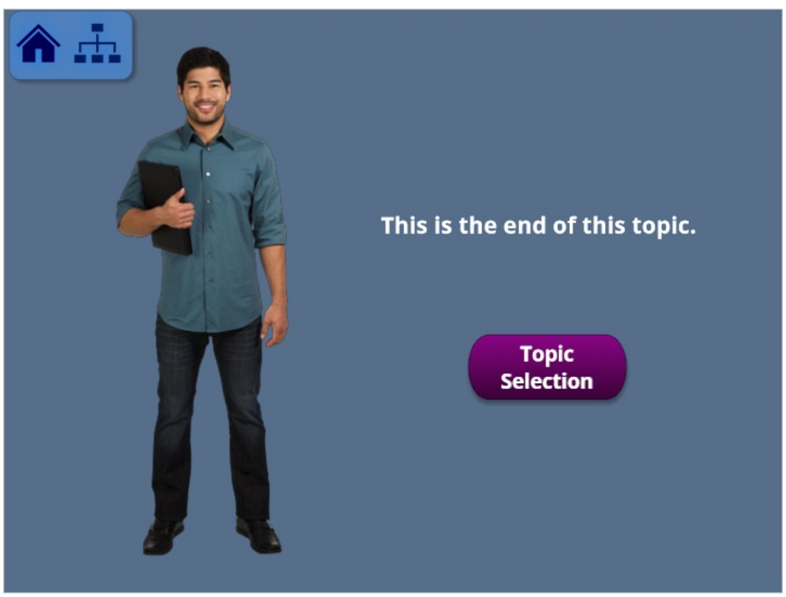
2.5 Motivation (I)



2.6 Motivation (II)

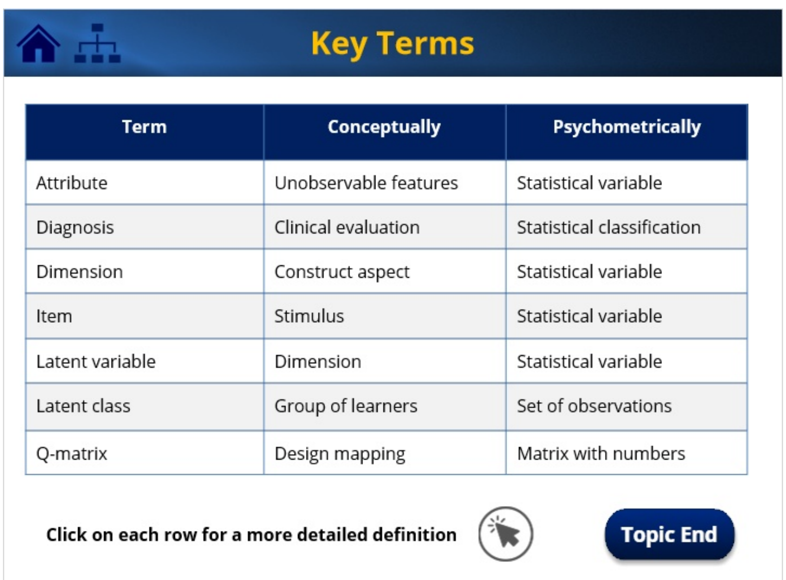


2.7 Bookend: General Motivation



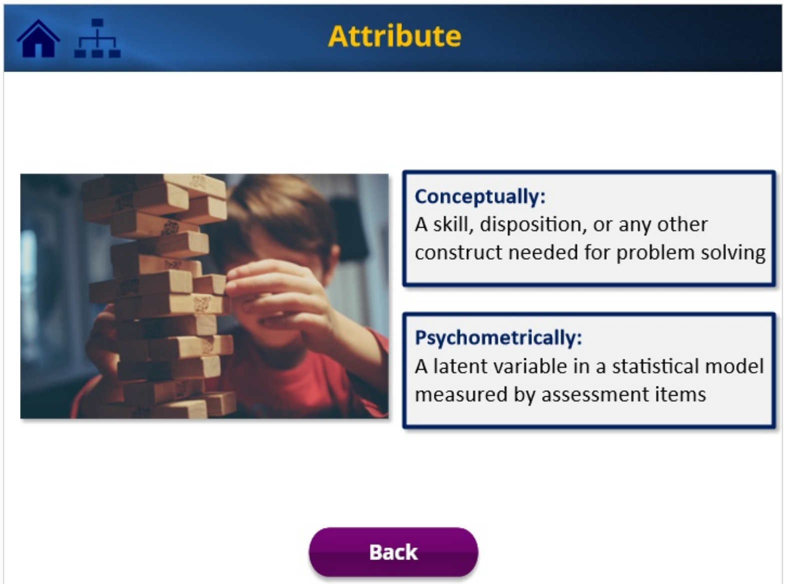
2.8 Bookmark: Terminology

2.9 Term Selection

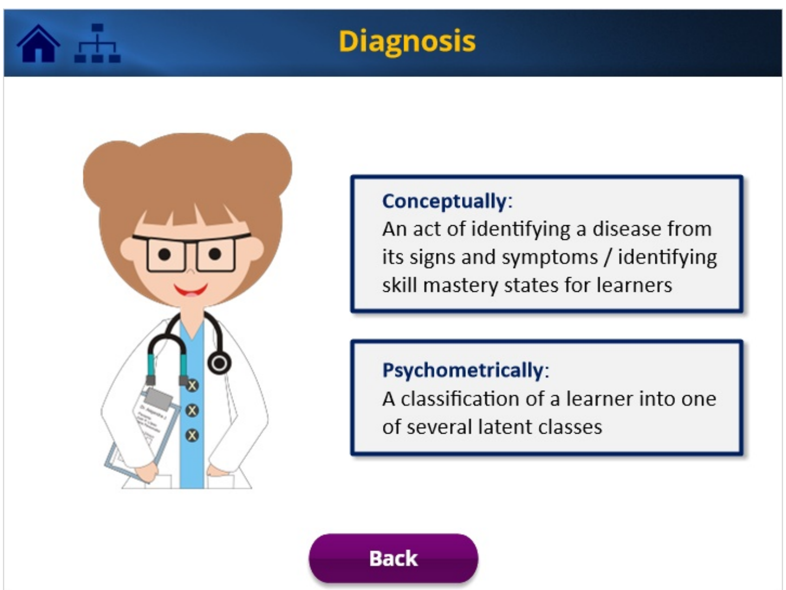


Term	Conceptually	Psychometrically
Attribute	Unobservable features	Statistical variable
Diagnosis	Clinical evaluation	Statistical classification
Dimension	Construct aspect	Statistical variable
Item	Stimulus	Statistical variable
Latent variable	Dimension	Statistical variable
Latent class	Group of learners	Set of observations
Q-matrix	Design mapping	Matrix with numbers

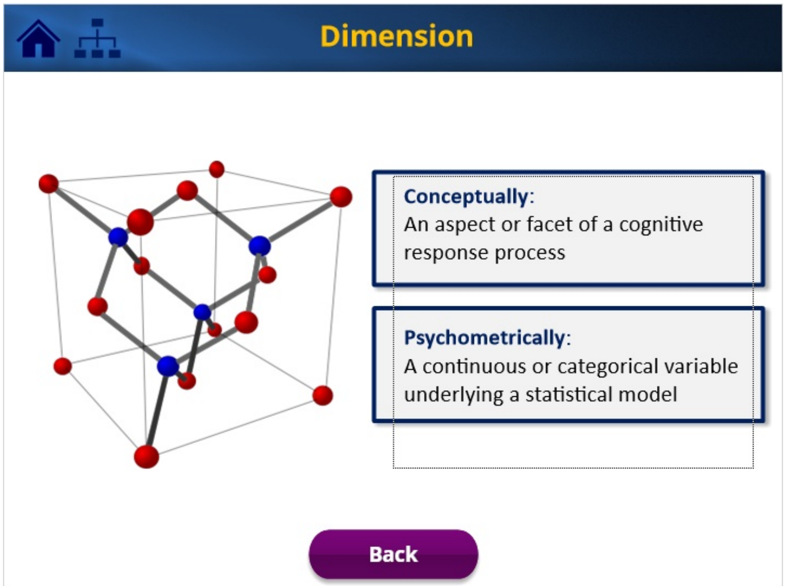
2.10 Attribute



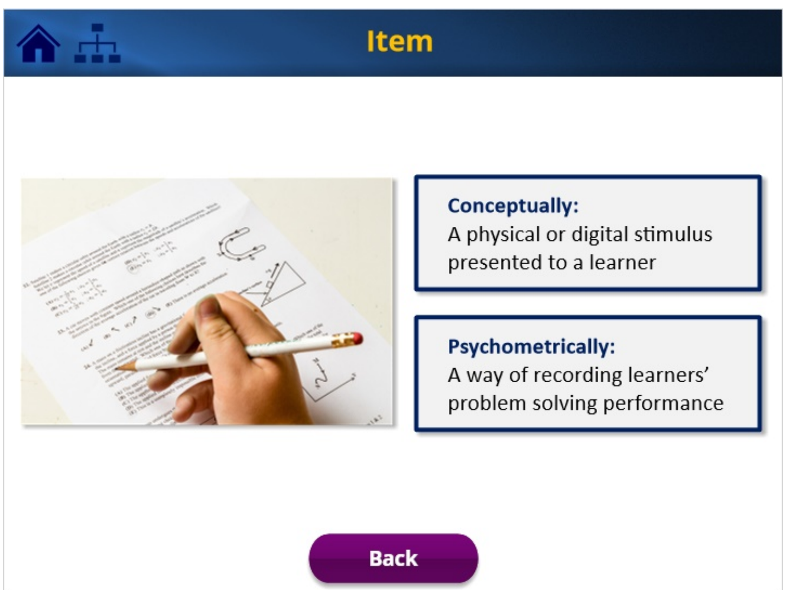
2.11 Diagnosis



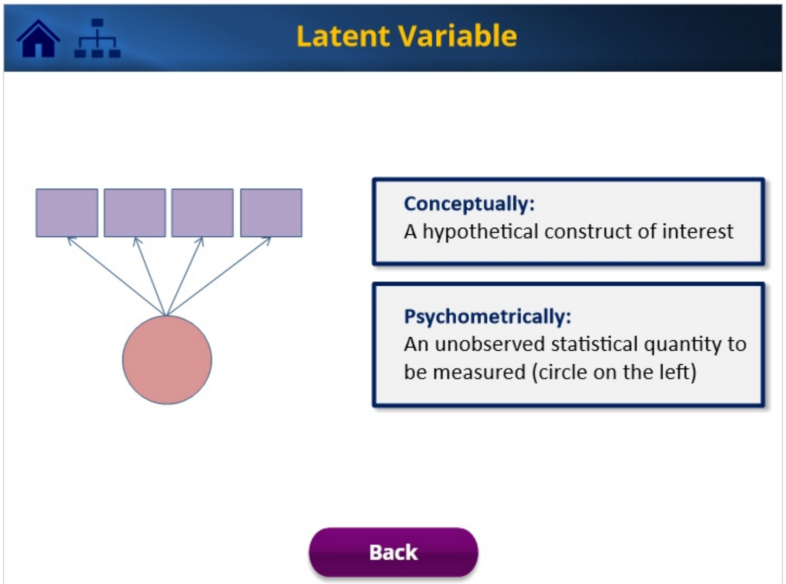
2.12 Dimension



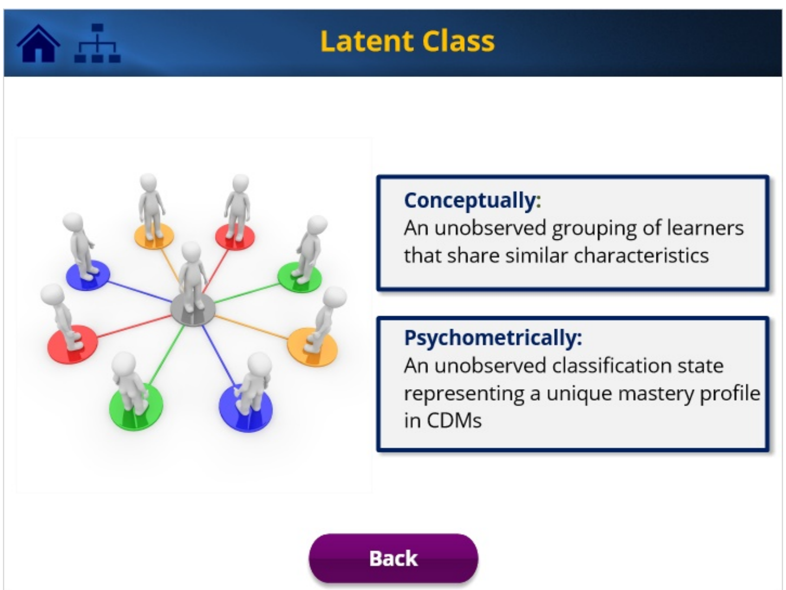
2.13 Item



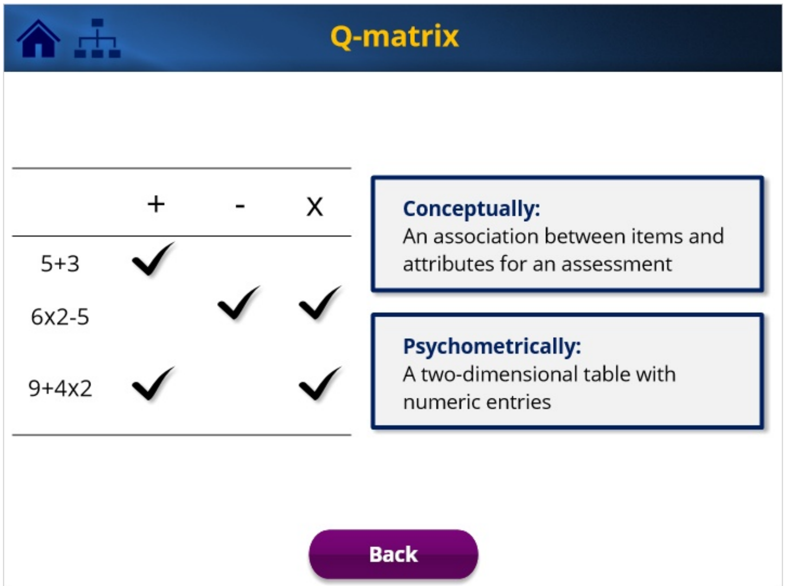
2.14 Latent Variable



2.15 Latent Class

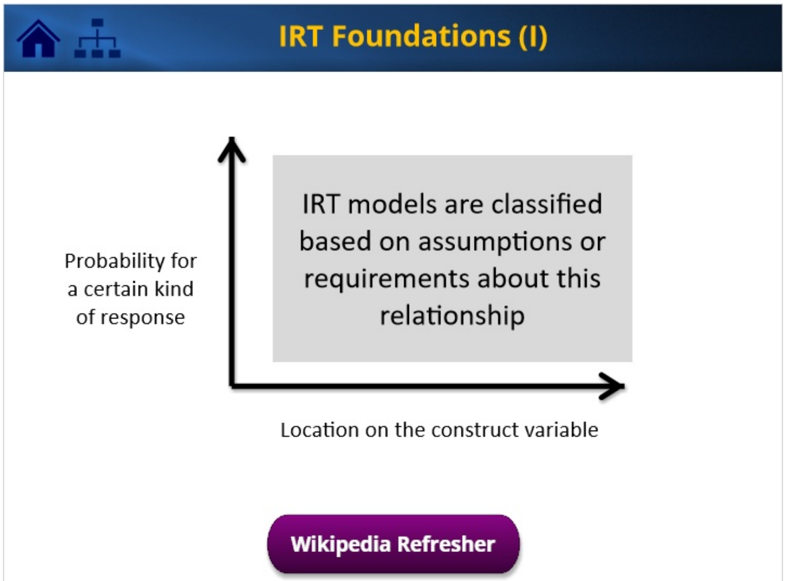


2.16 Q-matrix

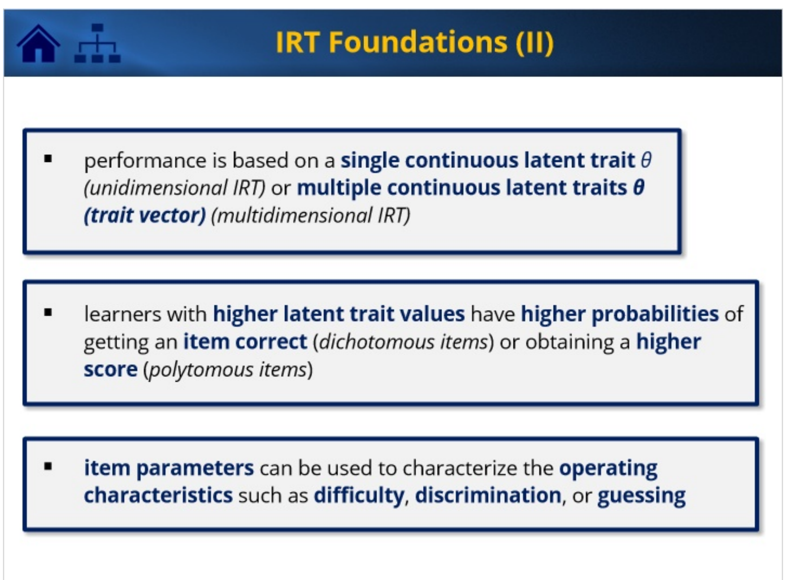


2.17 Bookmark: IRT vs CDM

2.18 Item Response Function (IRF)



2.19 IRT Principles



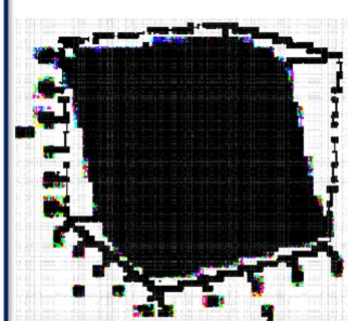
2.20 CDM Principles

CDM Foundations

- performance is based on **multiple latent variables**, which are also called **attributes** $\boldsymbol{\alpha} = (\alpha_1,\ldots,\alpha_K)$
- attributes are typically **binary (0-1)** to indicate **mastery status**, which results in $\mathbf{2^K}$ **latent classes**, each with a **unique attribute profile**
- goal is to **estimate learners' attribute profiles** by classifying them into different **latent classes** based on the **discrete attribute variables**
- each item is designed to **measure one or more** of the latent attributes with the exact design captured in a **Q-matrix** with 0s and 1s
- **response probabilities are class-specific** and depends on which attributes are **mastered in the particular latent class**

2.21 Illustration

Illustration



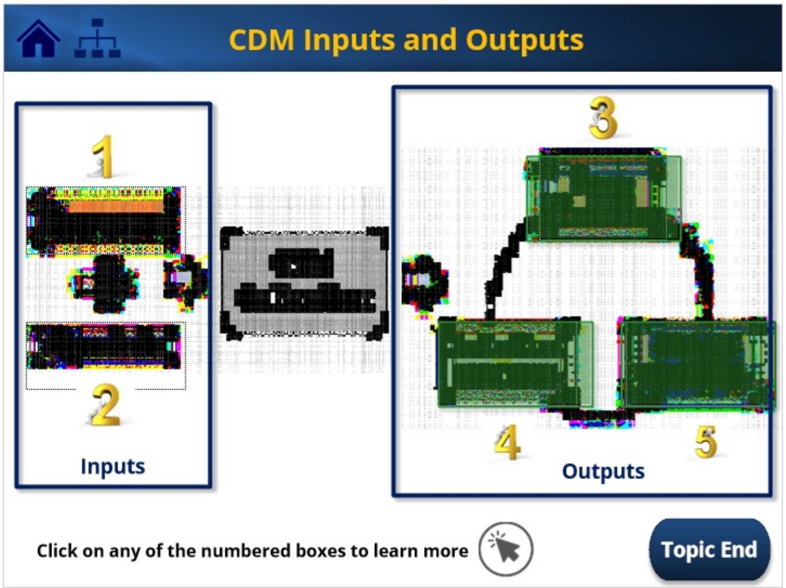
IRT Model

CDM

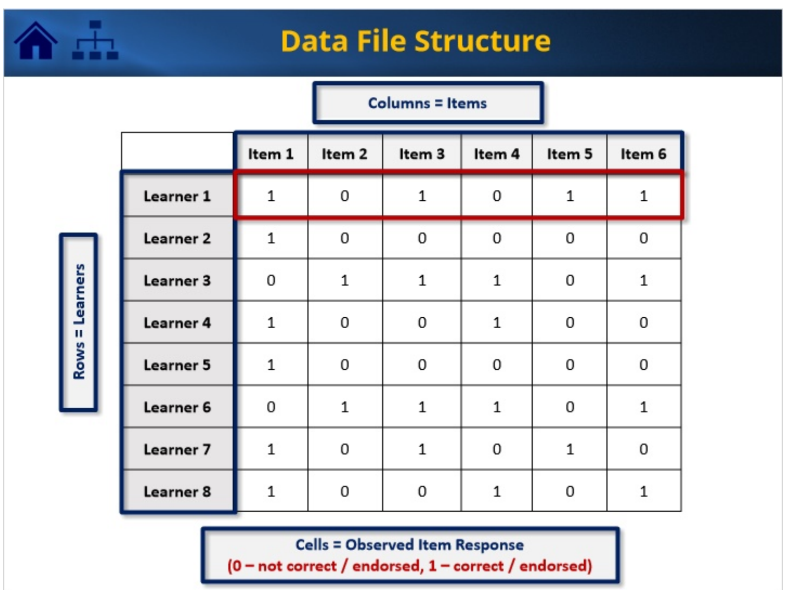
2.22 Bookend: Framework Comparison

2.23 Bookmark: CDM Outputs

2.24 Component Selection

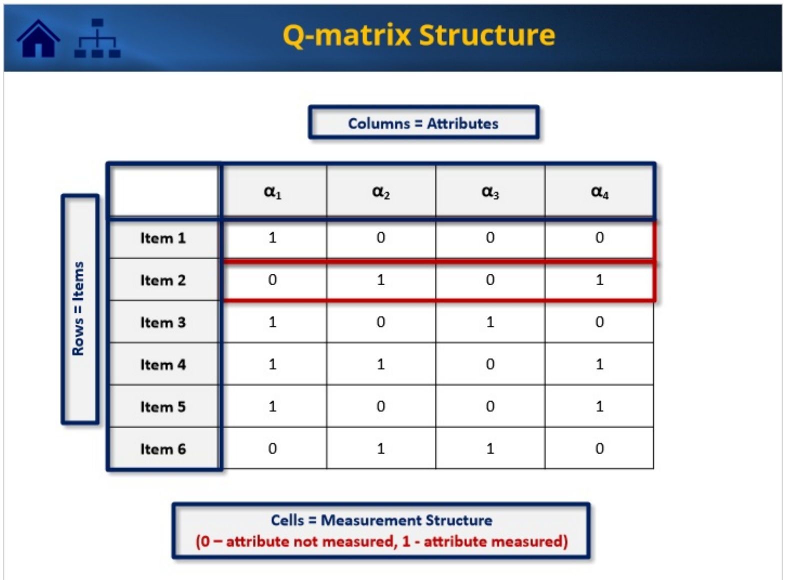


2.25 Data File Structure

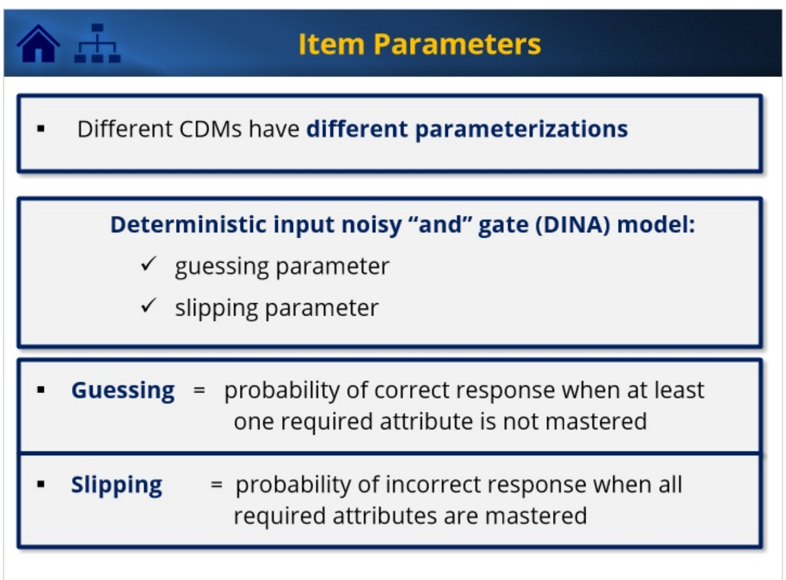


	Item 1	Item 2	Item 3	Item 4	Item 5	Item 6
Learner 1	1	0	1	0	1	1
Learner 2	1	0	0	0	0	0
Learner 3	0	1	1	1	0	1
Learner 4	1	0	0	1	0	0
Learner 5	1	0	0	0	0	0
Learner 6	0	1	1	1	0	1
Learner 7	1	0	1	0	1	0
Learner 8	1	0	0	1	0	1

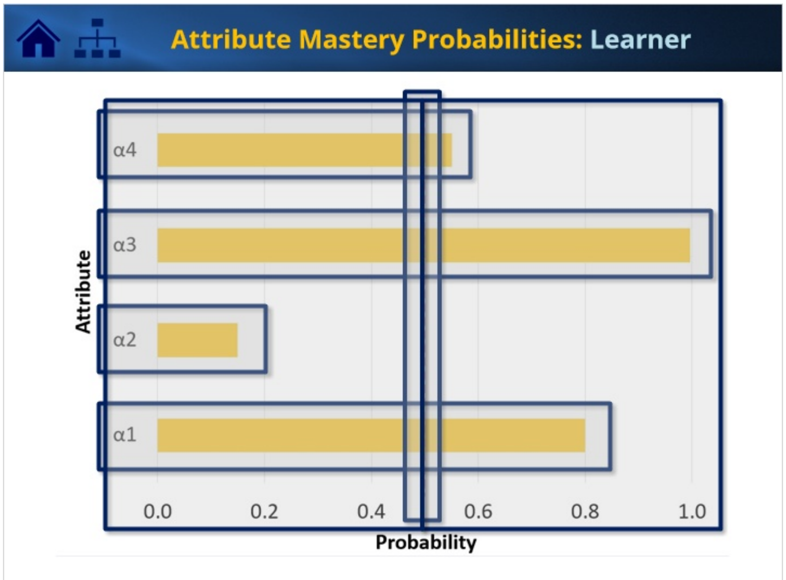
2.26 Q-matrix Structure



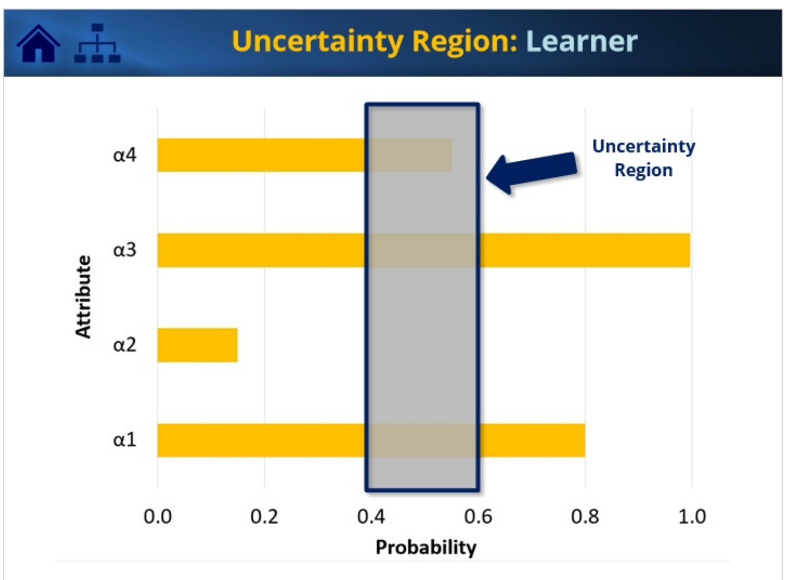
2.27 Item Parameters



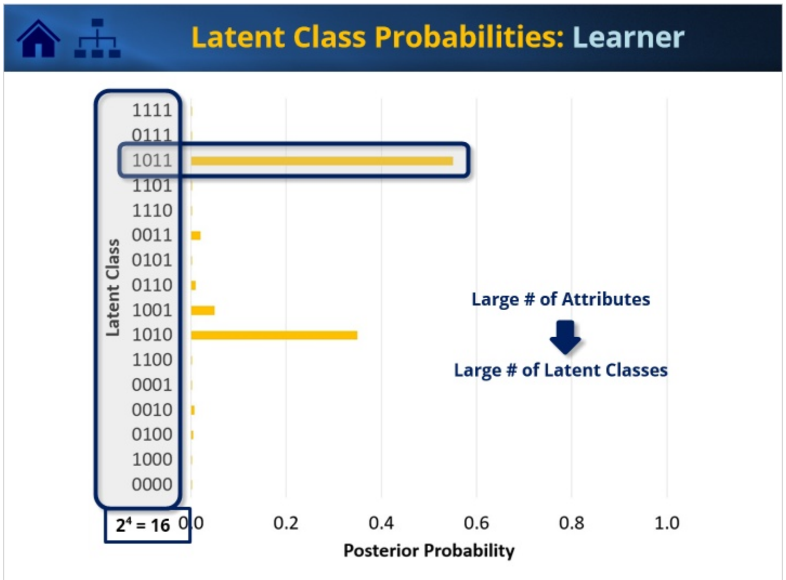
2.28 Attribute Probabilities (Person)



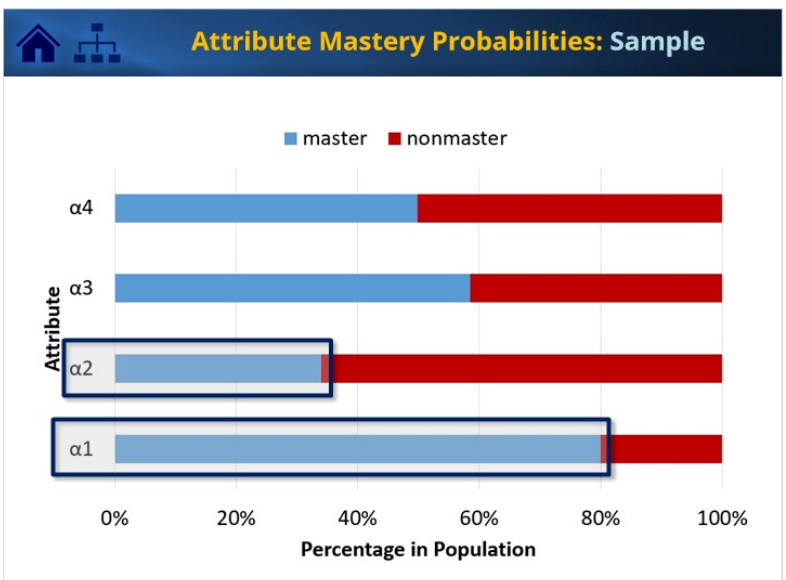
2.29 Attribute Uncertainties (Person)



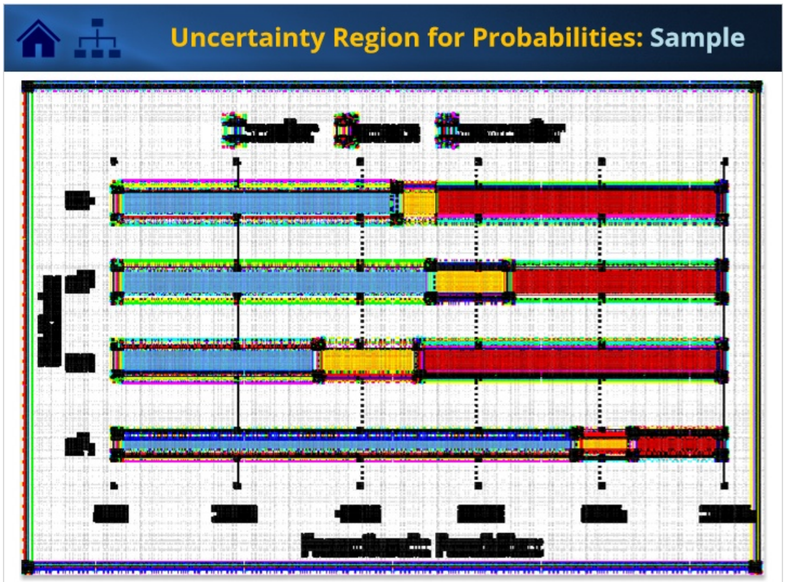
2.30 Latent Class Probabilities (Person)



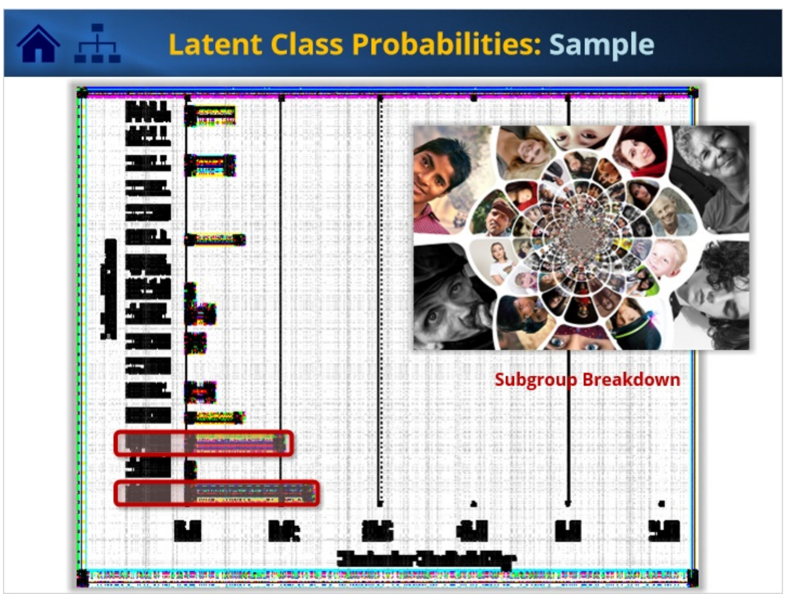
2.31 Attribute Probabilities (Sample)



2.32 Attribute Uncertainties (Sample)



2.33 Latent Class Probabilities (Sample)



2.34 Summary

Summary (I)

- CDMs aim to **classify learners** into different **latent classes** and provide **diagnostic feedback** to learners and teachers

- CDMs and IRT models are similar in some respects (**multidimensional structure**) but different in others (**discrete vs continuous latent variables**)

- CDMs are often used for **dichotomous responses and attributes** but can also handle **polytomous items and attributes**

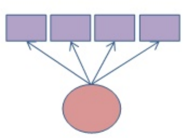
2.35 Summary

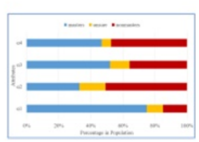
Summary (II)

- CDMs require that analysts provide **item responses** and **Q-matrix** and choose a **modeling framework** or **specific model**

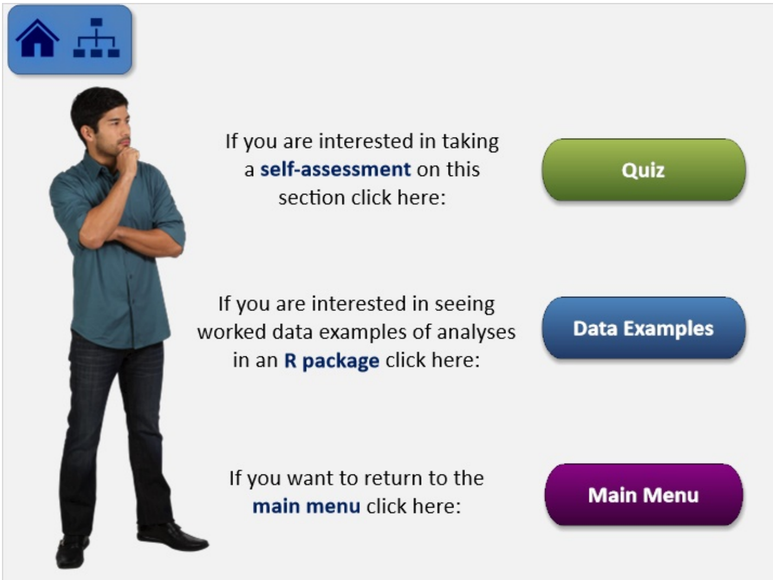
	+	-	x
5+3	✓		
6x2-5		✓	✓
9+4x2	✓		✓

- CDM analyses will produce **various outputs** including **item parameter estimates** to gauge item quality and **attribute profile estimates** to evaluate **learner characteristics**





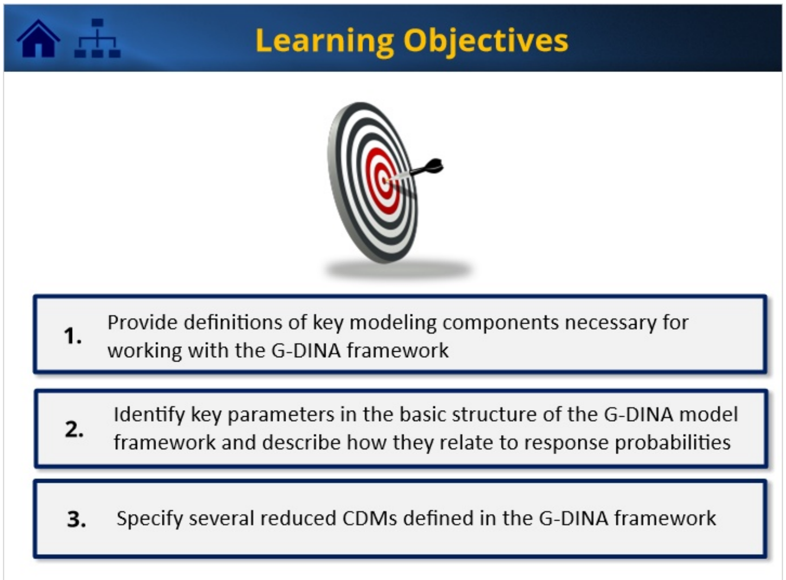
2.36 Bookend: Section 1



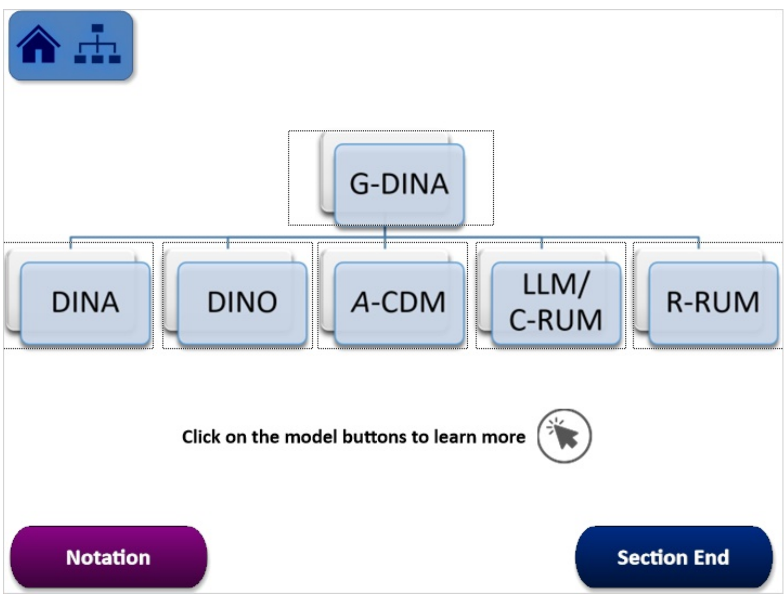
3. Section 2: GDINA Framework

3.1 Cover: Section 2

3.2 Objectives: Section 2

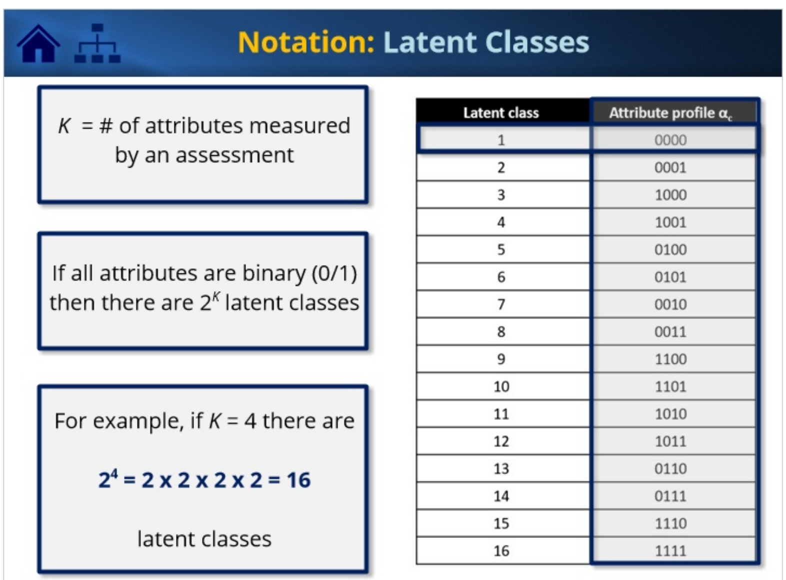


3.3 Model Selection



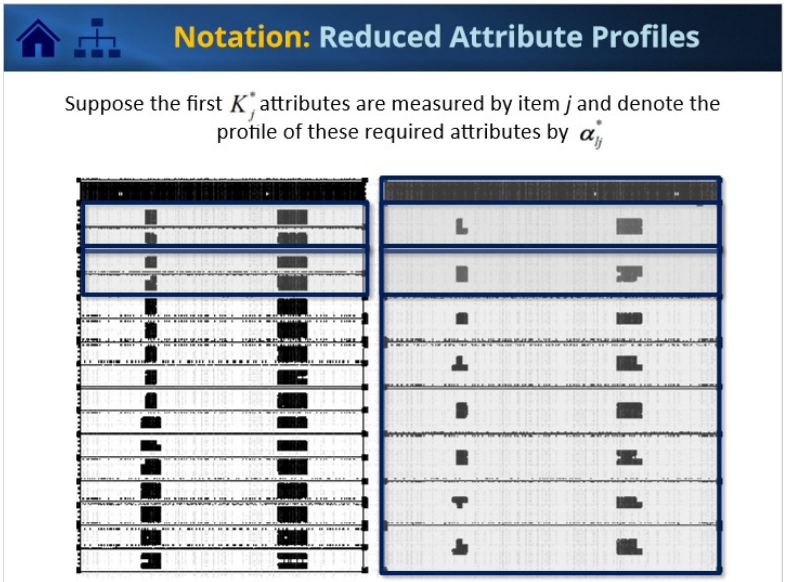
3.4 Bookmark: Notation

3.5 Latent Classes

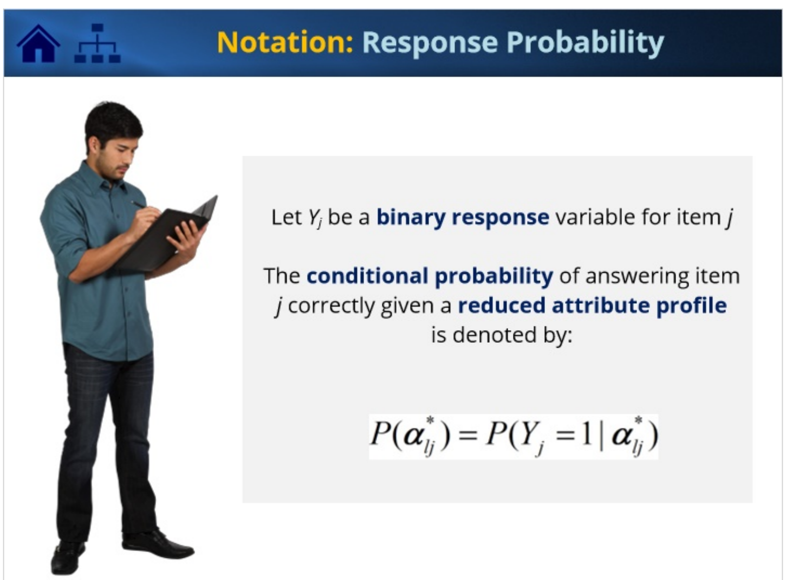


Latent class	Attribute profile α_c
1	0000
2	0001
3	1000
4	1001
5	0100
6	0101
7	0010
8	0011
9	1100
10	1101
11	1010
12	1011
13	0110
14	0111
15	1110
16	1111

3.6 Attribute Profiles



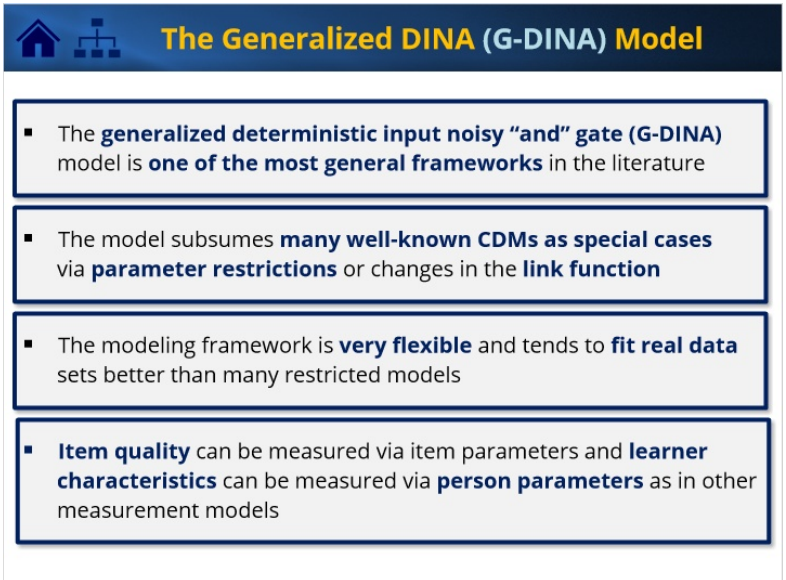
3.7 Response Probability



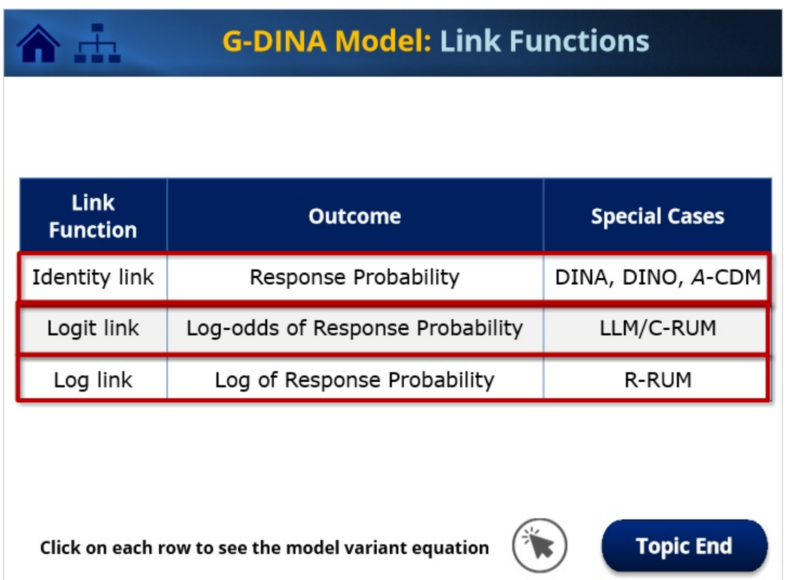
3.8 Bookend: Notation

3.9 Bookmark: G-DINA

3.10 Framework Properties

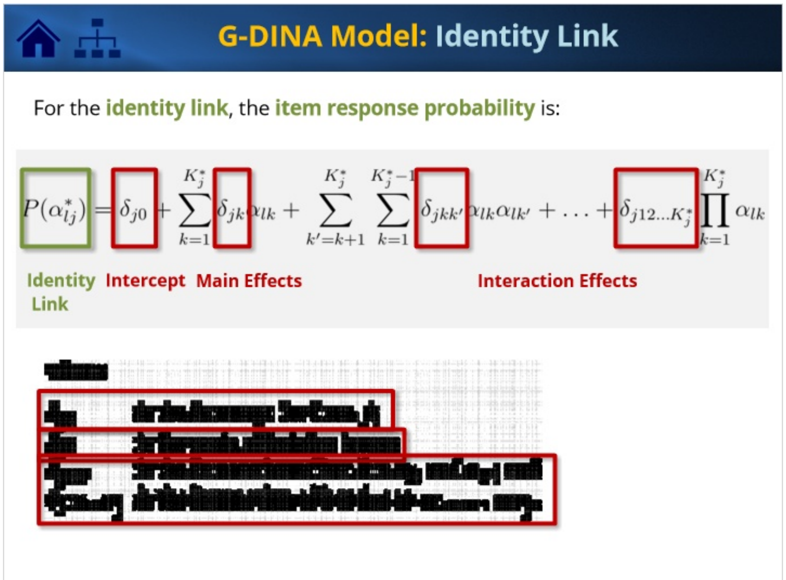


3.11 Link Function Selection



Link Function	Outcome	Special Cases
Identity link	Response Probability	DINA, DINO, *A*-CDM
Logit link	Log-odds of Response Probability	LLM/C-RUM
Log link	Log of Response Probability	R-RUM

3.12 Identity Link (I)



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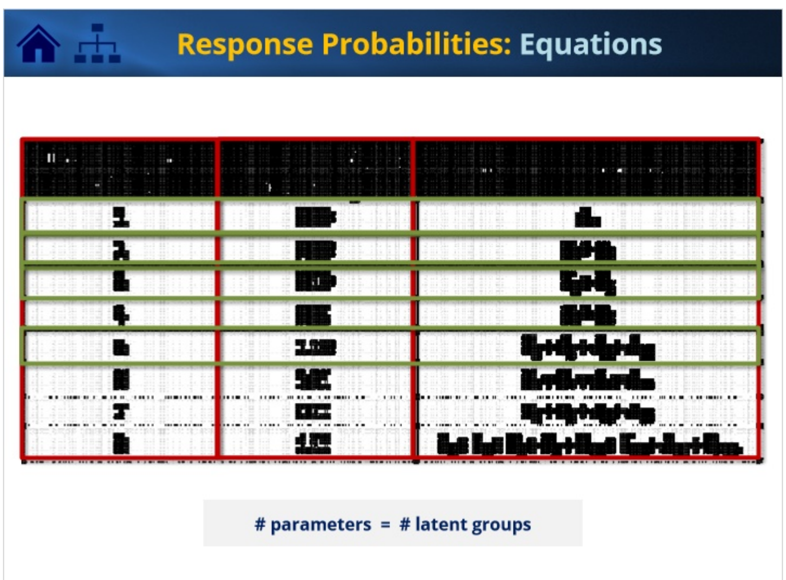
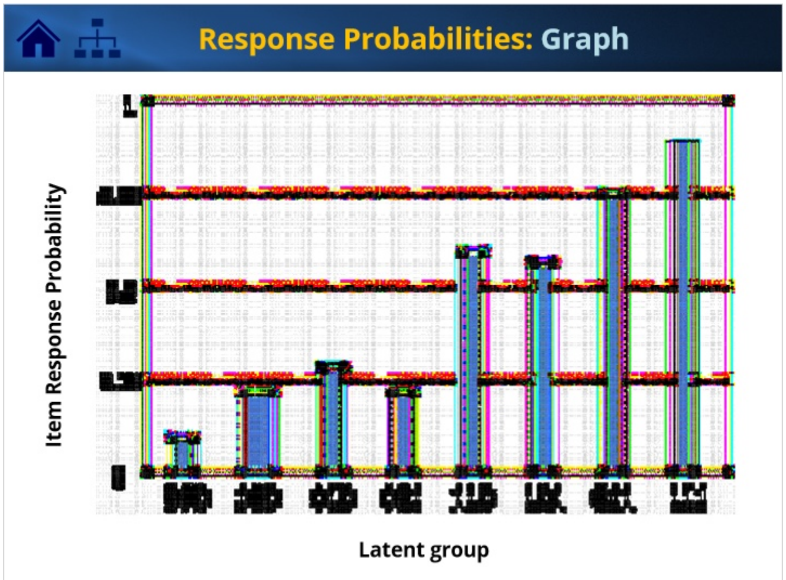
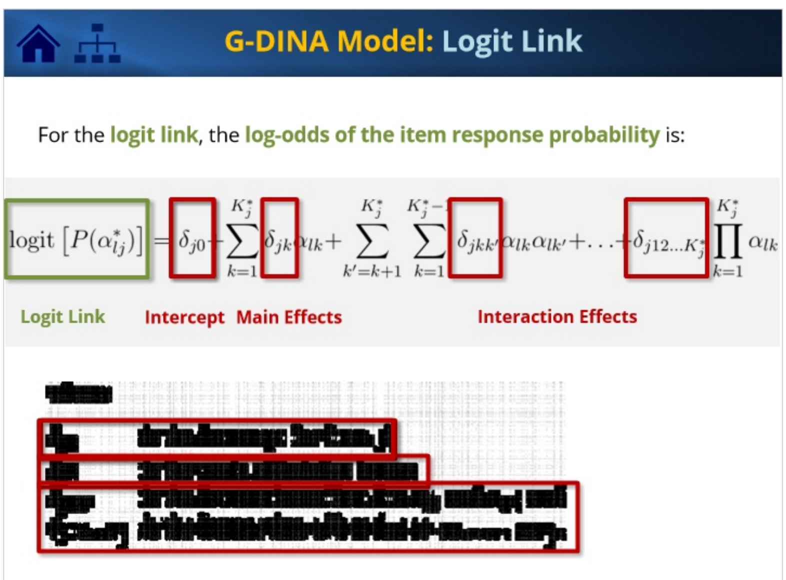


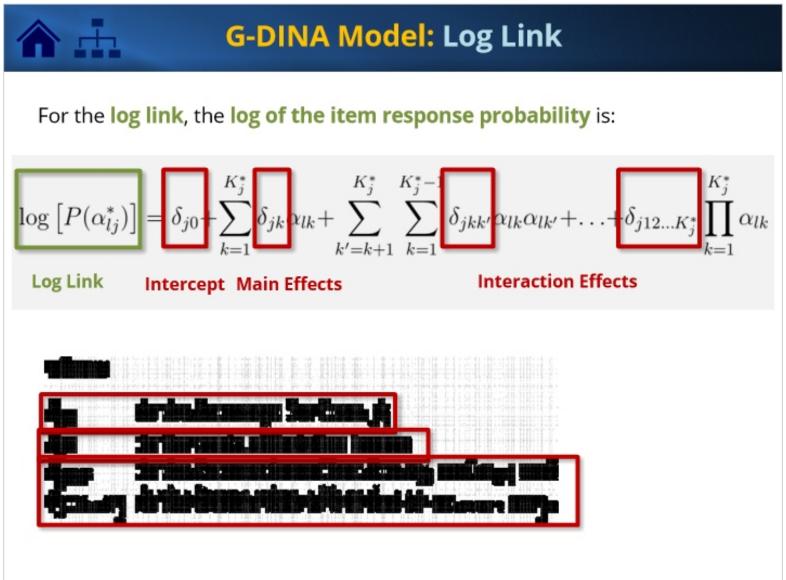
3.14 Identity Link (III)



3.15 Logit Link

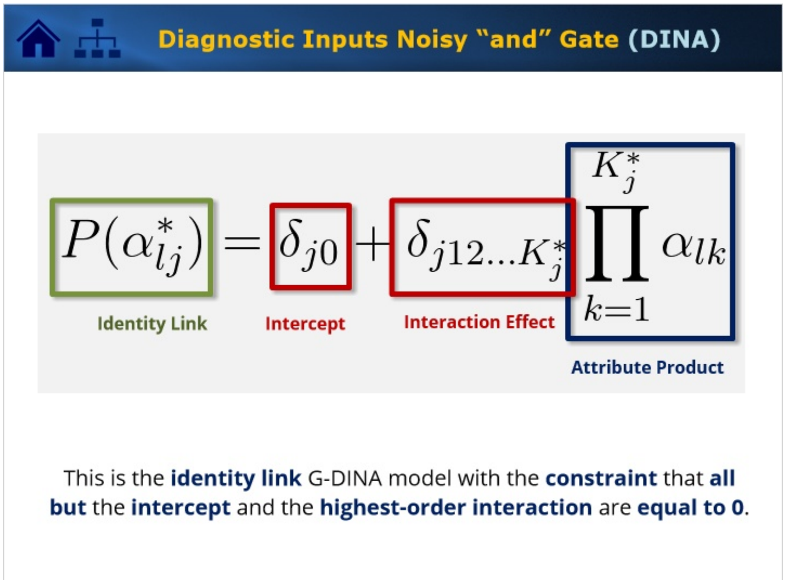


3.16 Log Link

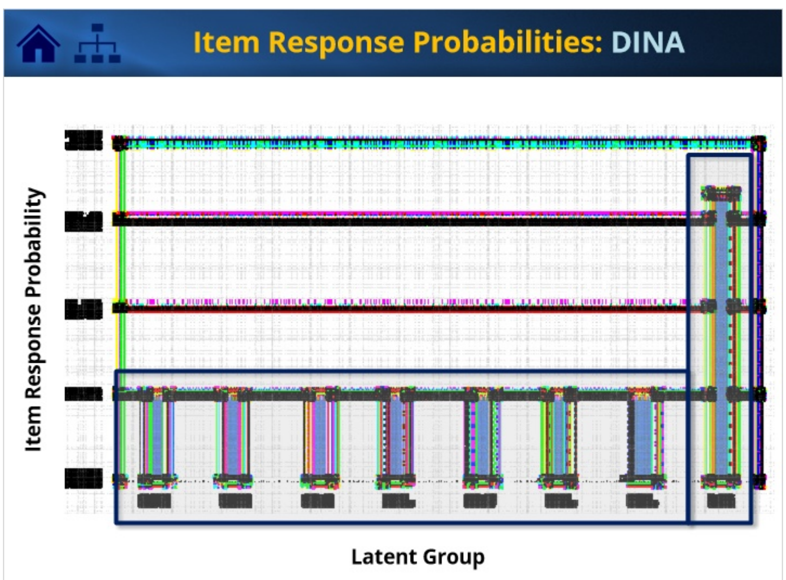


3.17 Bookmark: DINA

3.18 DINA Equation



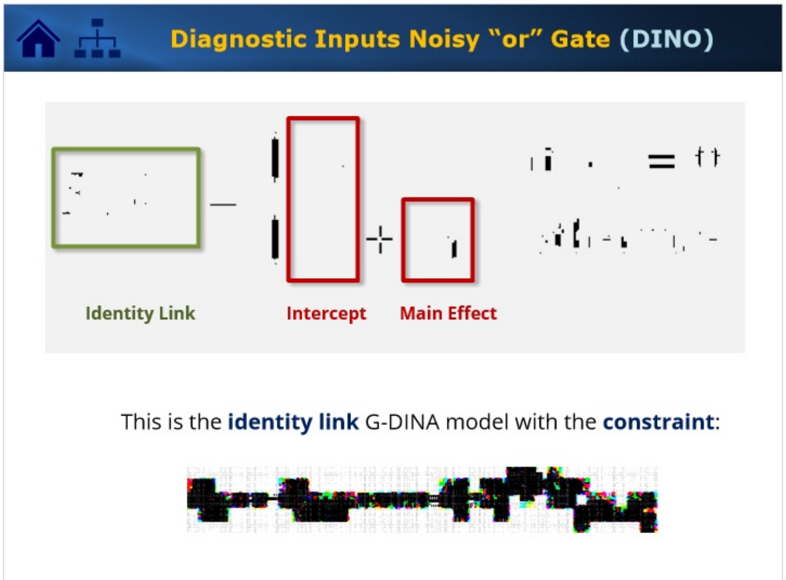
3.19 DINA Probabilities



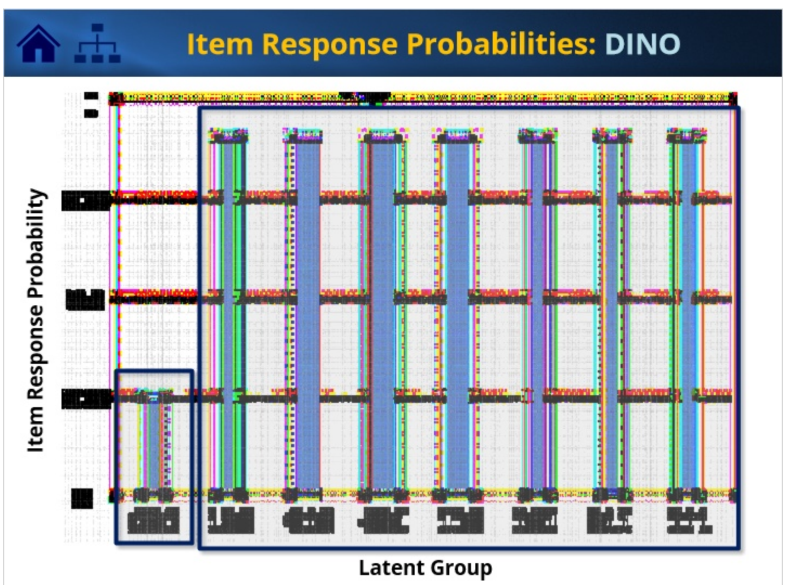
3.20 Bookend: DINA

3.21 Bookmark: DINO

3.22 DINO Equation



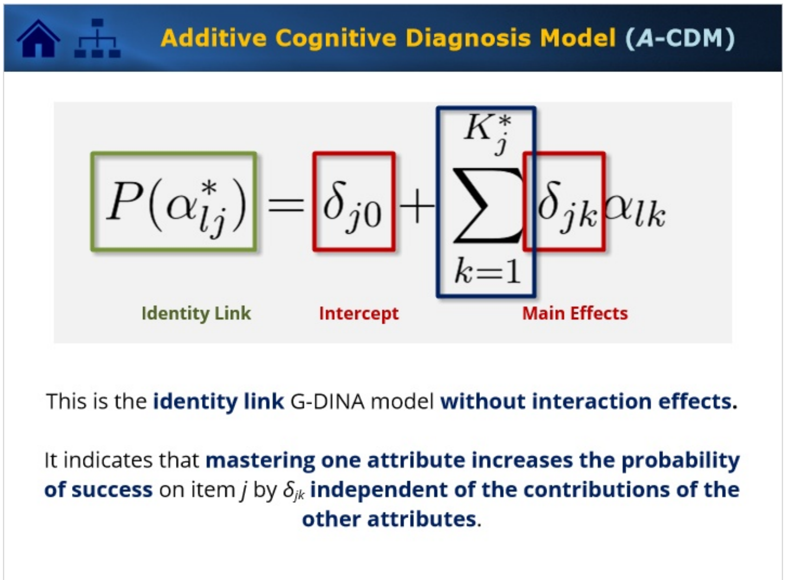
3.23 DINO Probabilities



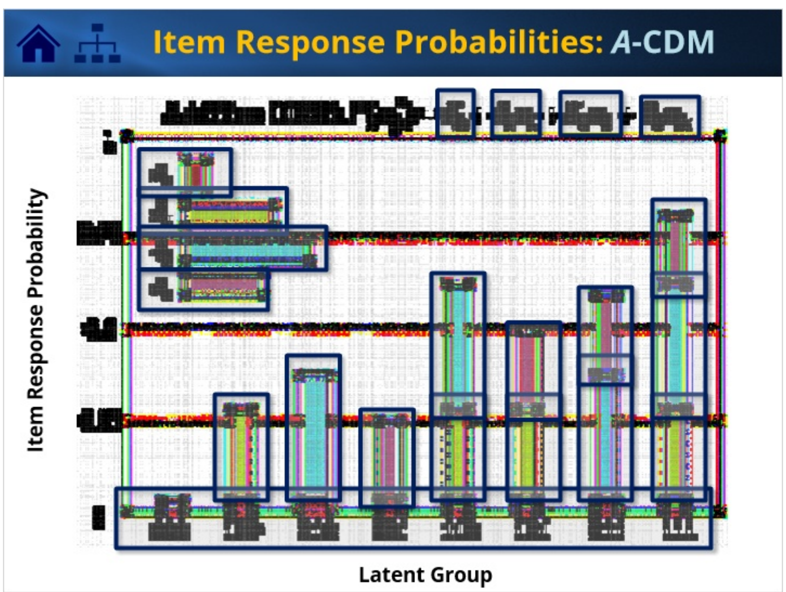
3.24 Bookend: DINO

3.25 Bookmark: A-CDM

3.26 A-CDM Equation

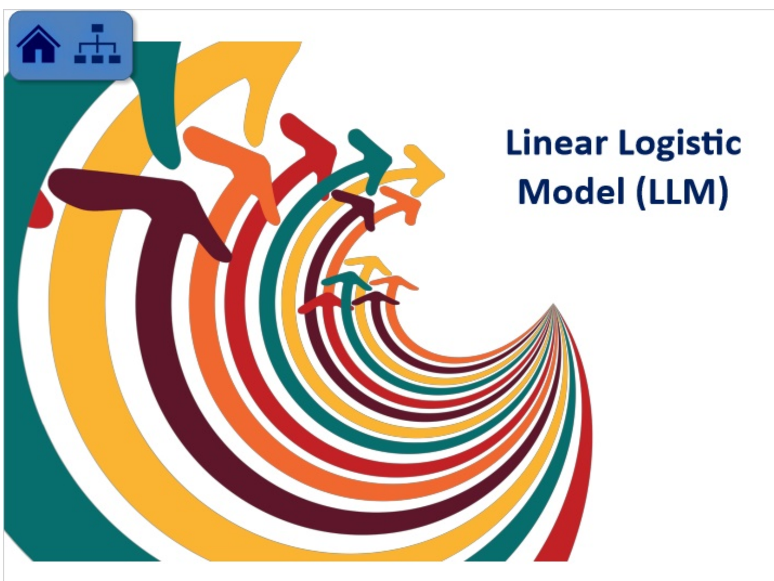


3.27 A-CDM Probabilities

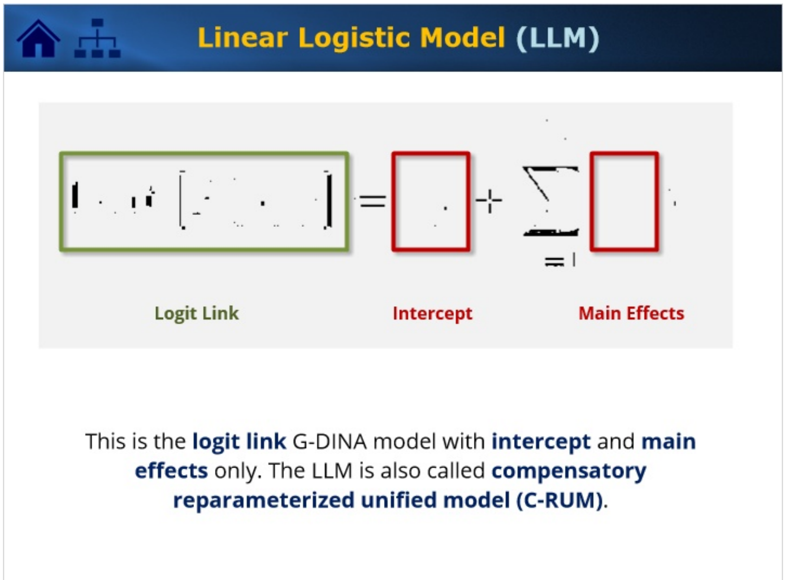


3.28 Bookend: ACDM

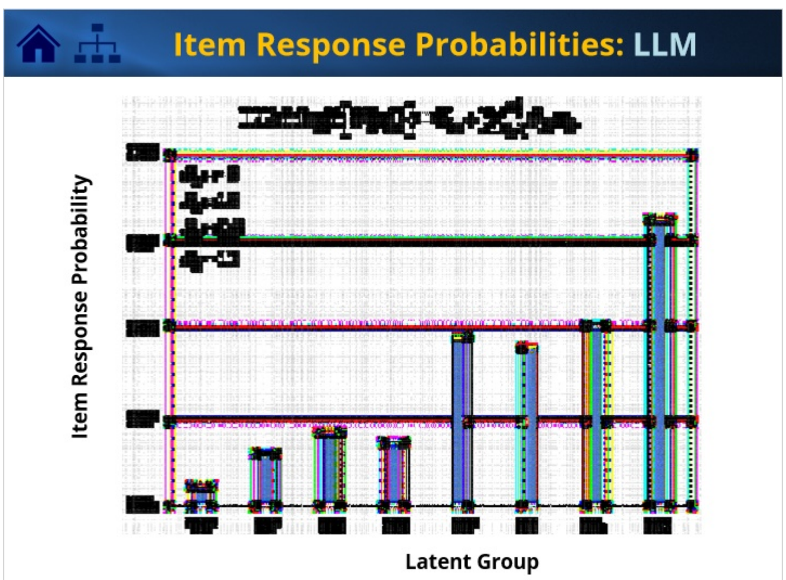
3.29 Bookmark: LLM



3.30 LLM Equation



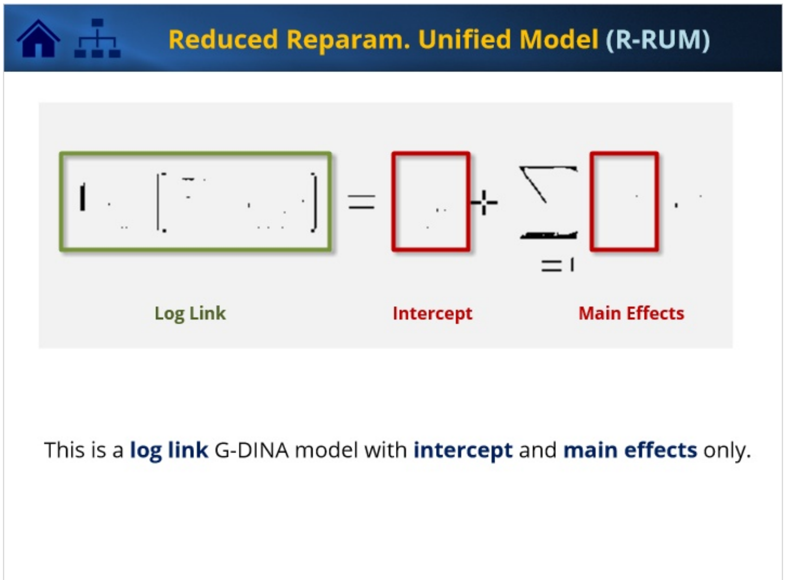
3.31 LLM Probabilities



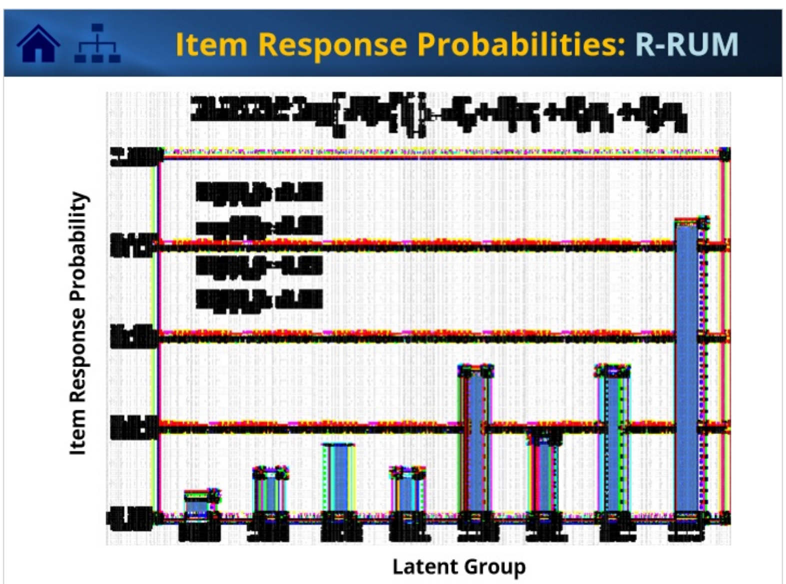
3.32 Bookend: LLM

3.33 Bookmark: R-RUM

3.34 R-RUM Equation

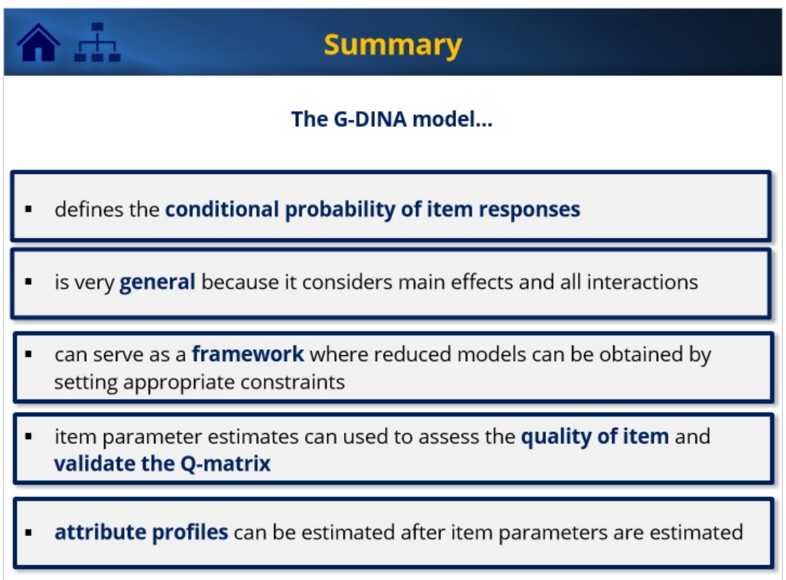


3.35 R-RUM Probabilities

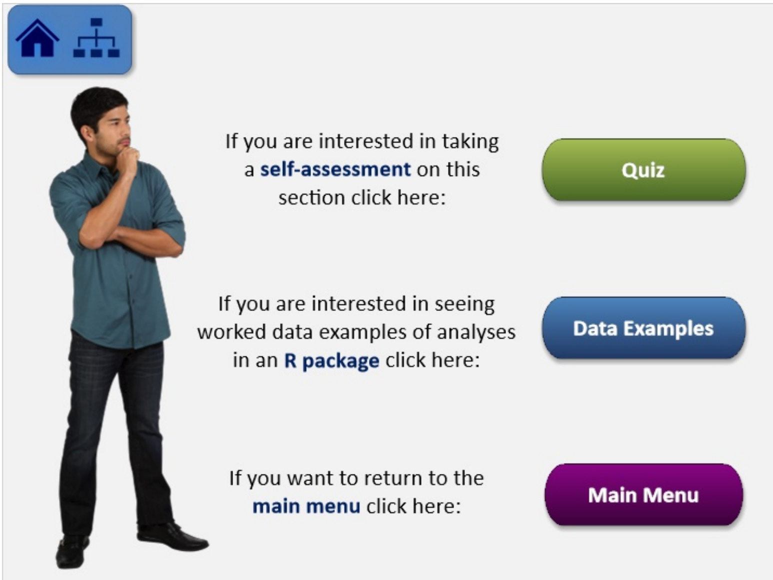


3.36 Bookend: R-RUM

3.37 Summary



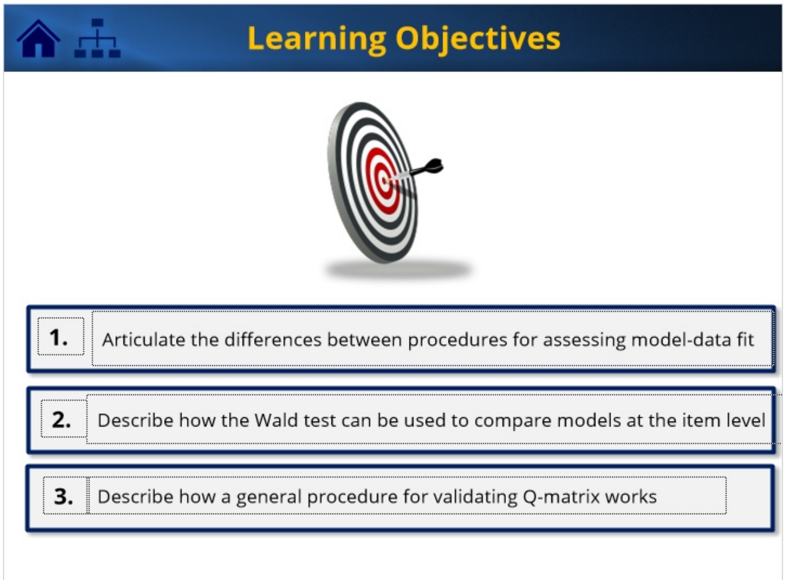
3.38 Bookend: Section 2



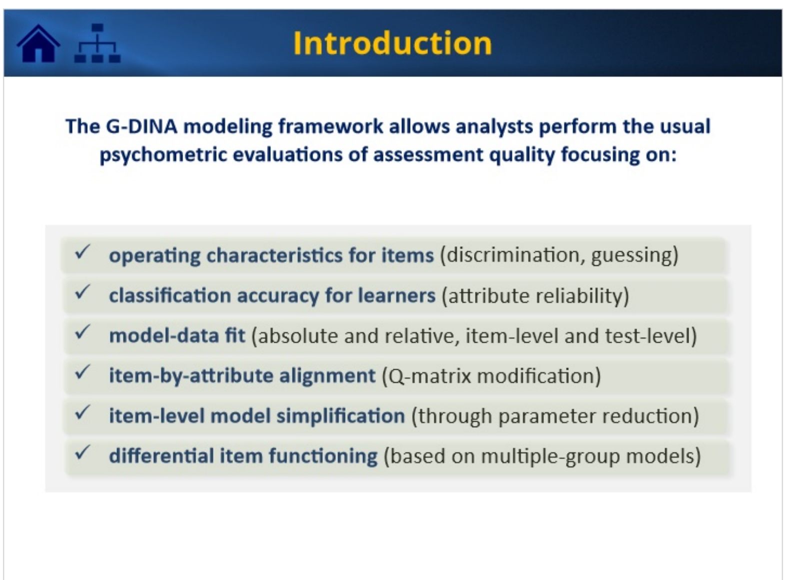
4. Section 3: Model Diagnostics

4.1 Cover: Section 3

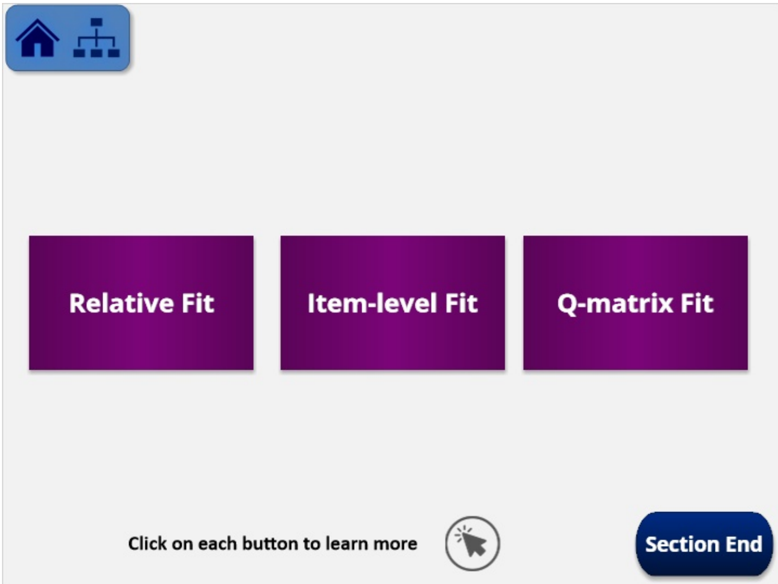
4.2 Objectives: Section 3



4.3 Introduction (I)

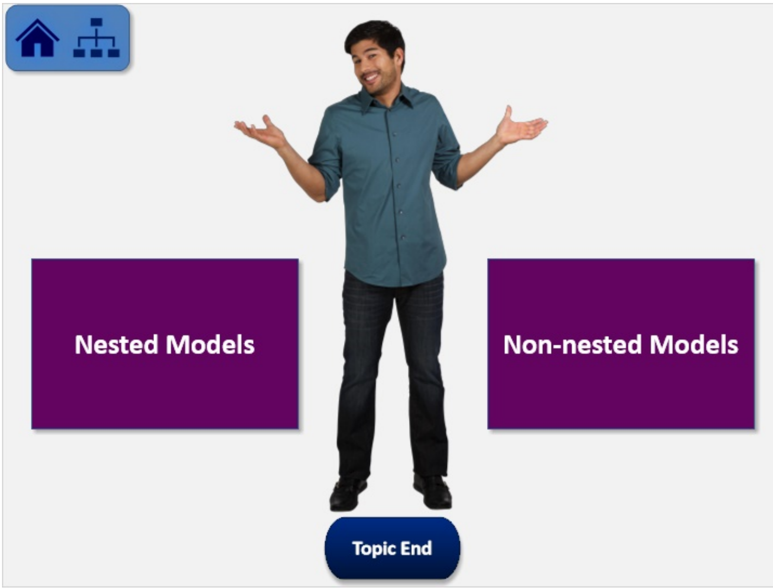


4.4 Model Fit Selection

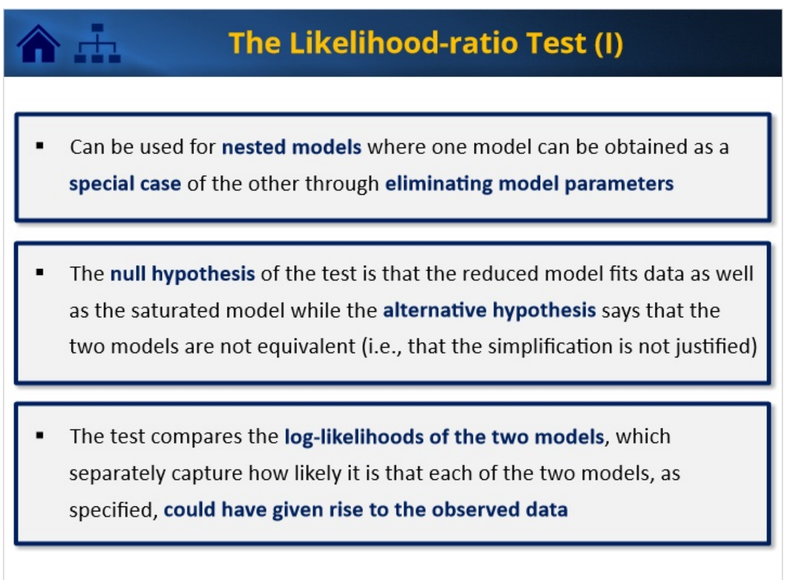


4.5 Bookmark: Relative Fit

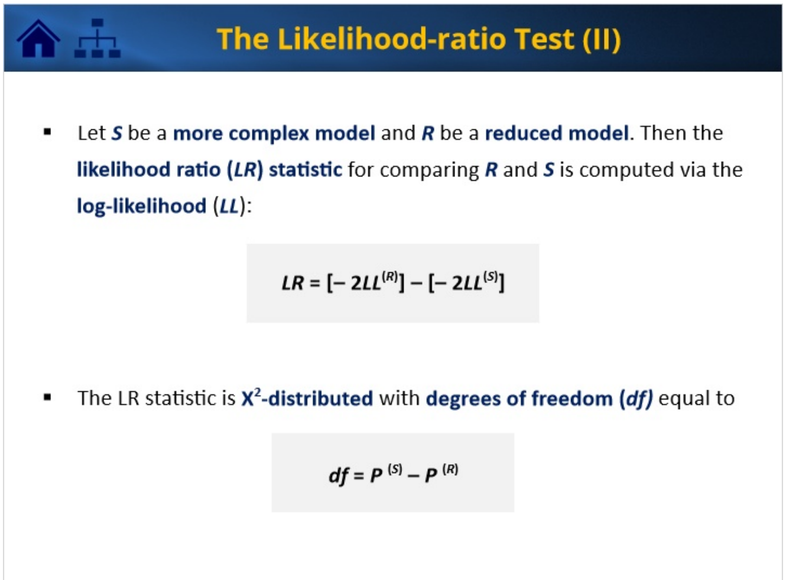
4.6 Model Selection



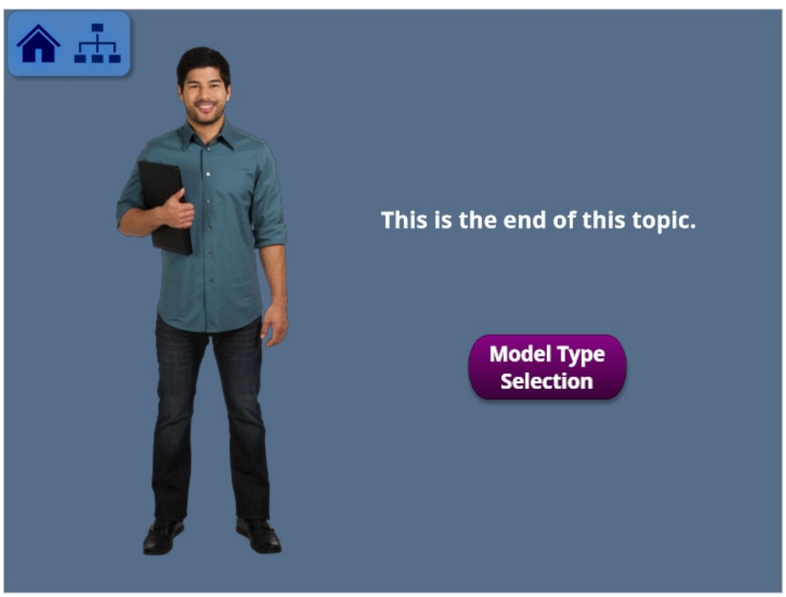
4.7 LR Test (I)



4.8 LR Test (II)



4.9 Bookend: Nested Models



4.10 Information Indices

Information Indices

Uses the observed marginalized **maximum likelihood (ML)** value and adjusts it via a **penalty term** that includes the **number of model parameters** and **sample size**:

Deviance (-2*LL*) = -2 x log-likelihood

Akaike information criterion (AIC) = $-2LL + 2P$

Bayesian information criterion (BIC) = $-2LL + P \times \ln(N)$

where ***P*** is the total number of **model parameters** and ***N*** is the **total sample size**

4.11 Information Indices Video

Number of Parameters

Q-matrix

	α_1	α_2	α_3
Item 1	1	0	0
Item 2	0	1	0
Item 3	1	0	1
Item 4	1	1	1

Parameter Table

Model for all items	# of item parameters	# of joint attribute distribution parameters	Total # of parameters
DINA	2 + 2 + 2 + 2 = 8	$2^3 - 1 = 7$	15
ACDM	2 + 2 + 3 + 4 = 11	$2^3 - 1 = 7$	18
G-DINA	2 + 2 + 4 + 8 = 16	$2^3 - 1 = 7$	23

4.12 Bookend: Non-nested Models

4.13 Boomark: Item-level Fit

4.14 General Principles

General Principles

The **Wald test** can be used to compare the **saturated G-DINA model** and **reduced models** when items measure **more than one attribute**

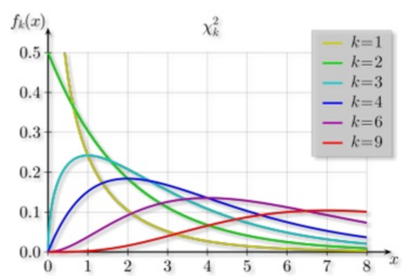
H_0: The reduced model fits data as well as the saturated G-DINA model
H_1: The reduced model fits data worse than the saturated G-DINA model

4.15 General Principles

General Principles

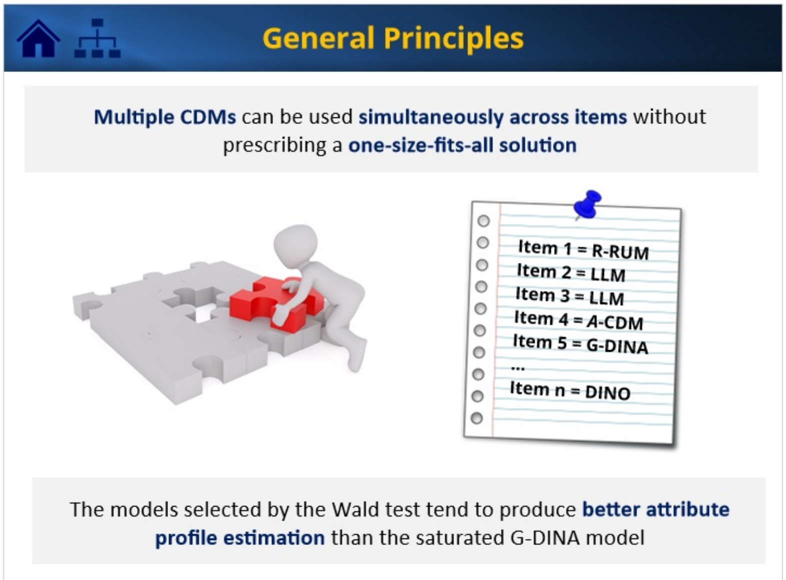
Wald test statistic is distributed as X^2 with degrees of freedom:

 # parameters of G-DINA model – # parameters of reduced model

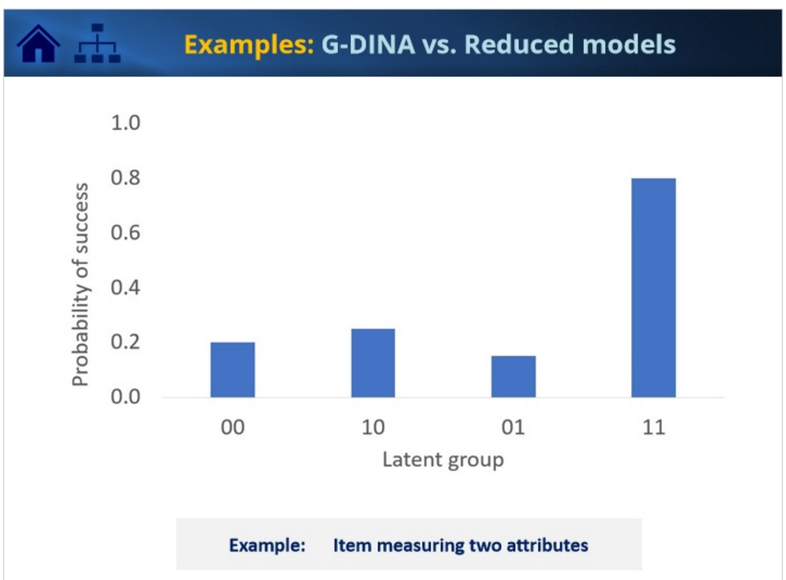


By Geek3 - Own work, CC BY 3.0, https://commons.wikimedia.org/w/index.php?curid=9884213

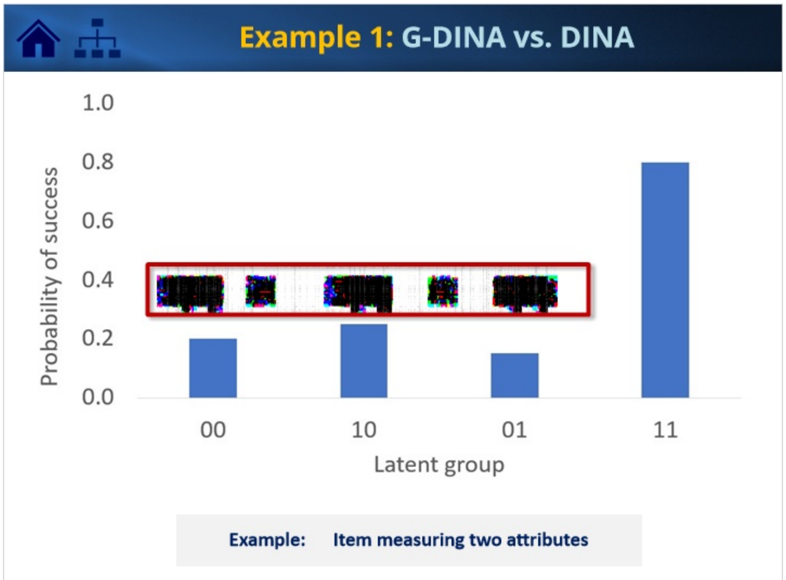
4.16 General Principles



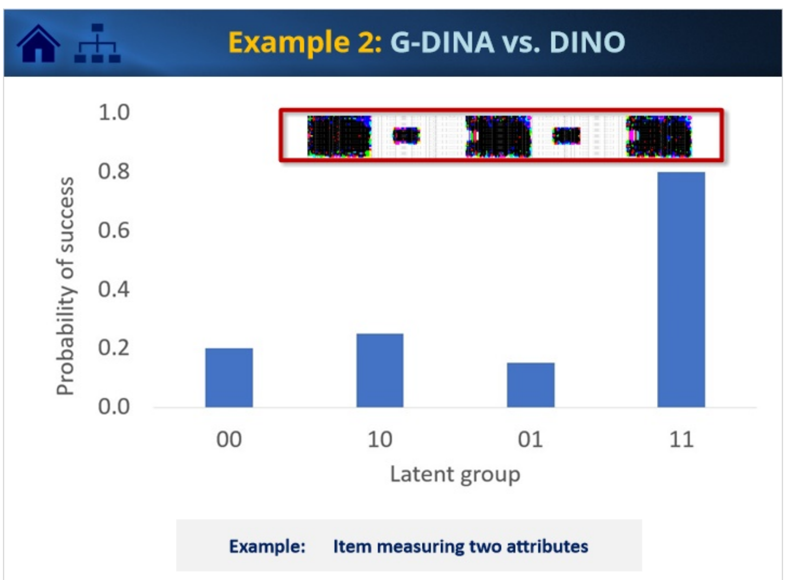
4.17 GDINA vs. DINA



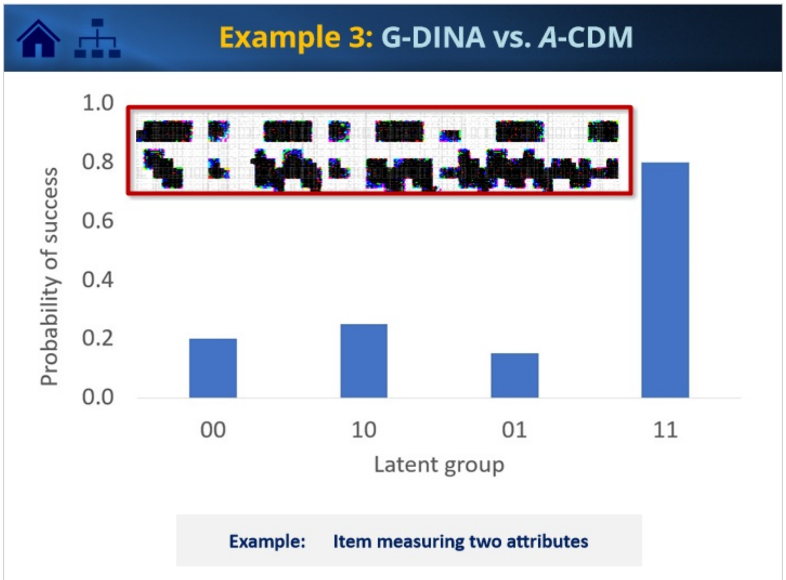
4.18 GDINA vs. DINA



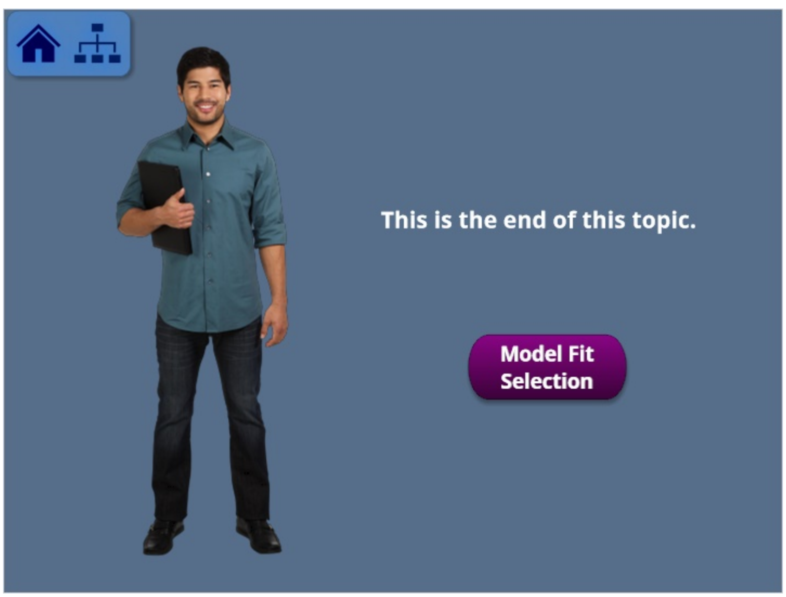
4.19 GDINA vs. DINO



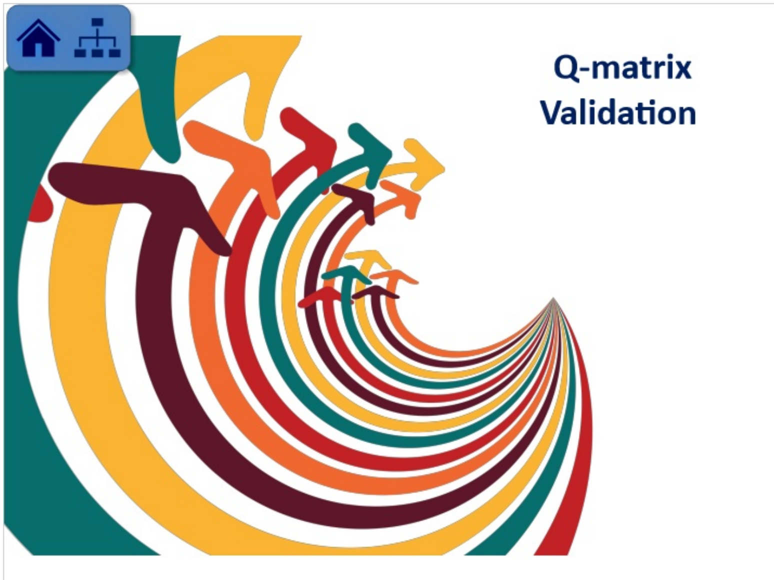
4.20 GDINA vs ACDM



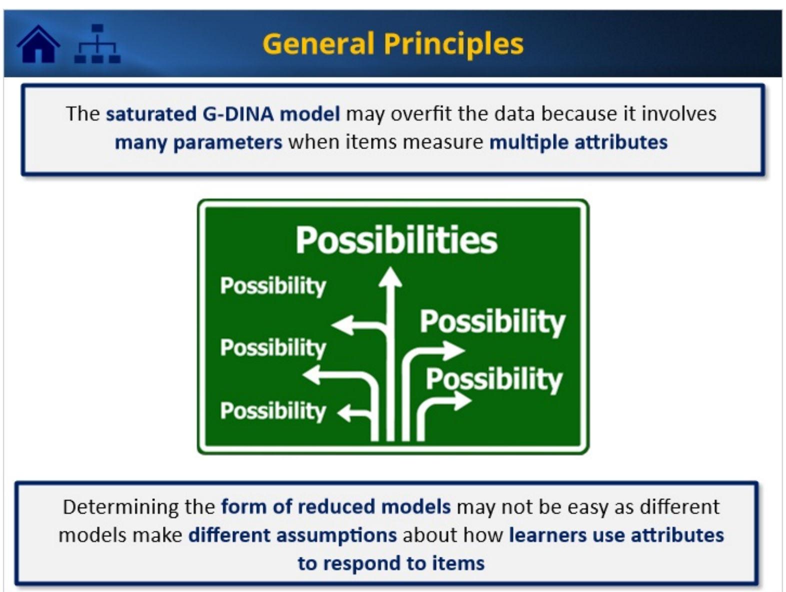
4.21 Bookend: Item-level Fit



4.22 Bookmark: Q-matrix Fit



4.23 General Principles



4.24 General Principles

General Principles

- A **Q-matrix specification** tends to be **partially subjective** in nature
- **Misspecifications** in the Q-matrix affect the **attribute classifications**
- The **following method** was designed specifically for the **G-DINA model**

Q-matrix

	α_1	α_2	α_3	α_4
Item 1	1	0	0	0
Item 2	0	1	0	1
Item 3	1	0	1	0
Item 4	1	1	0	1

4.25 Overview of Procedure (I)

Overview of Procedure (I)

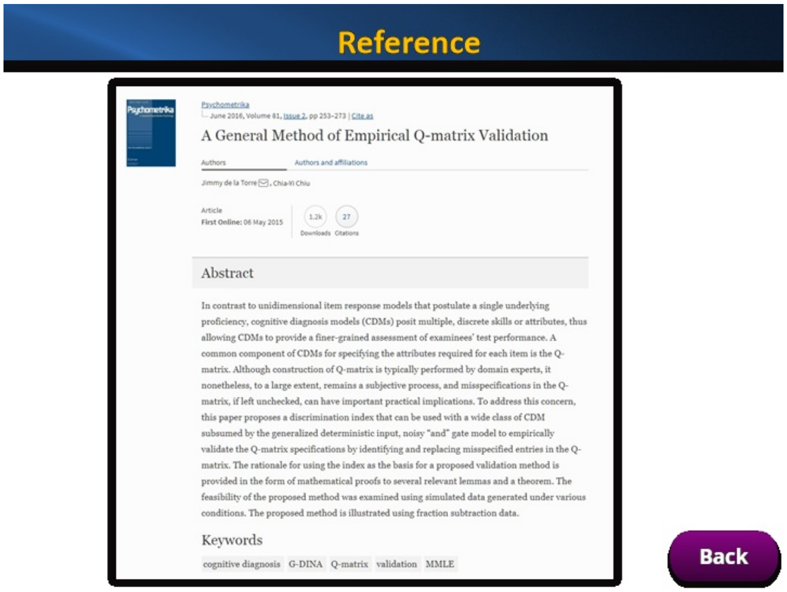
For each item, calculate the G-DINA discrimination index (GDI) for each possible *q*-vector (based on G-DINA estimates)

Properties of the GDI:

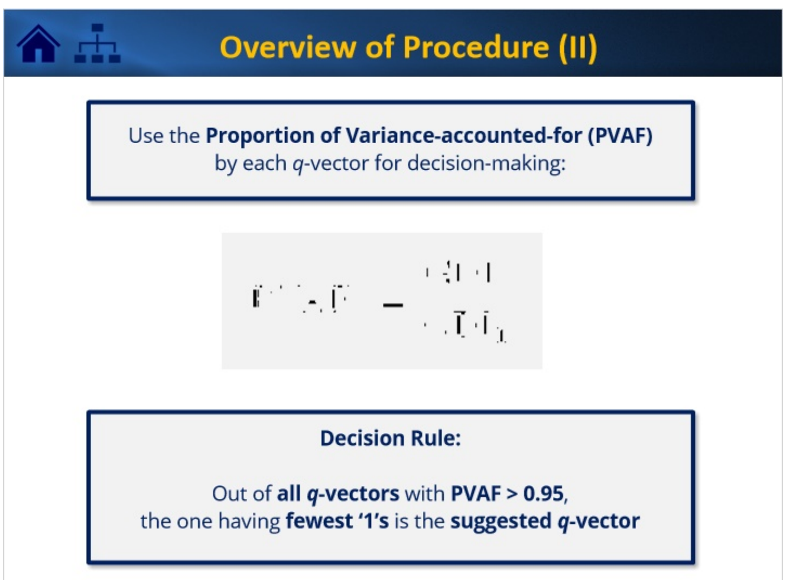
- the **true *q*-vector** and **over-specified *q*-vectors** have **identical and largest GDI** in principle
- **over-specified *q*-vectors** have **slightly larger GDI** than the **true *q*-vector**
- the **GDI increases** when there are **more '1's** in the *q*-vectors; the *q*-vector with **all '1's** has the **largest GDI**

Reference

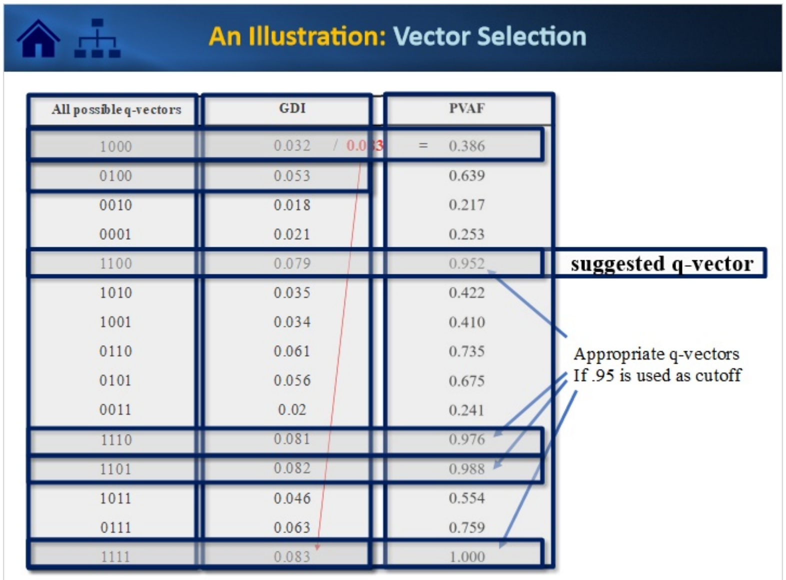
Reference (Slide Layer)



4.26 Overview of Procedure (II)

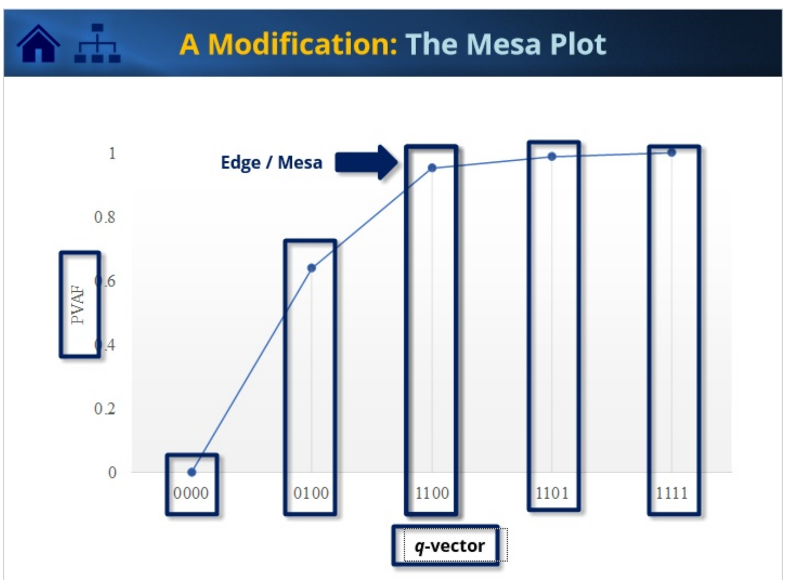


4.27 Illustration (I)

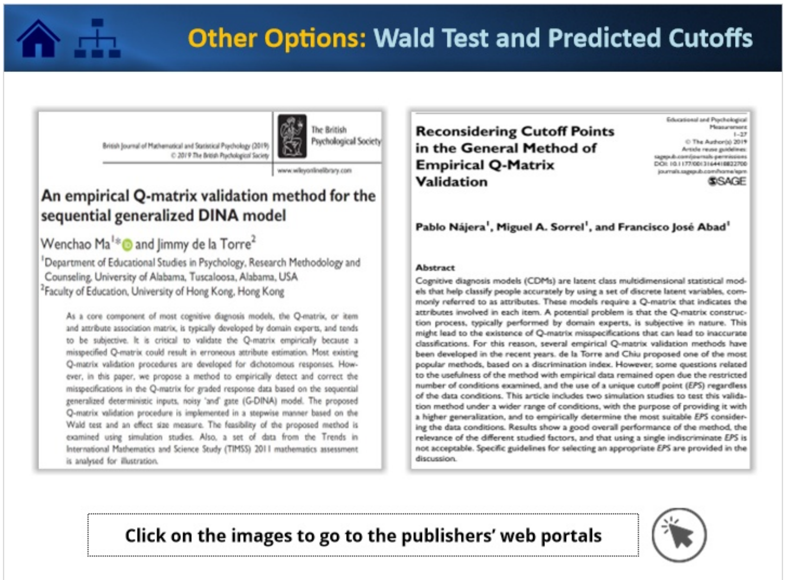


All possible q-vectors	GDI	PVAF
1000	0.032	0.386
0100	0.053	0.639
0010	0.018	0.217
0001	0.021	0.253
1100	0.079	0.952
1010	0.035	0.422
1001	0.034	0.410
0110	0.061	0.735
0101	0.056	0.675
0011	0.02	0.241
1110	0.081	0.976
1101	0.082	0.988
1011	0.046	0.554
0111	0.063	0.759
1111	0.083	1.000

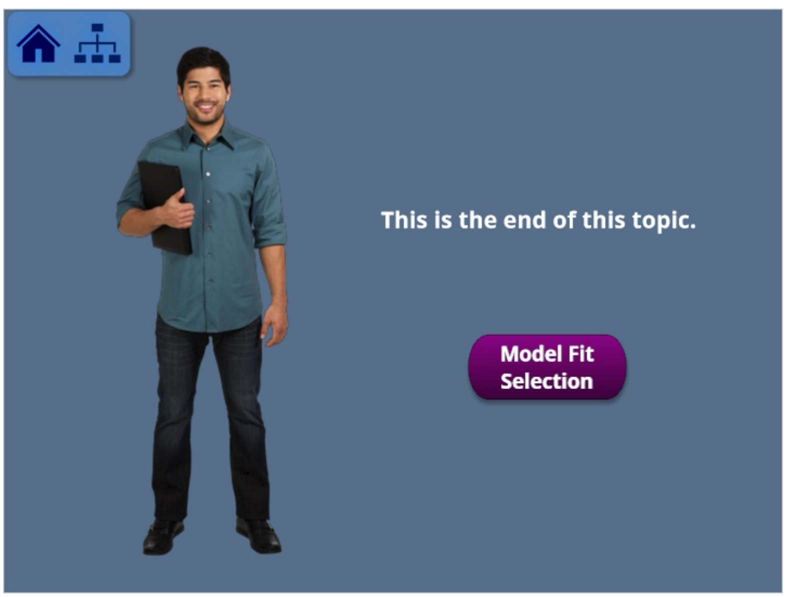
4.28 Illustration (II)



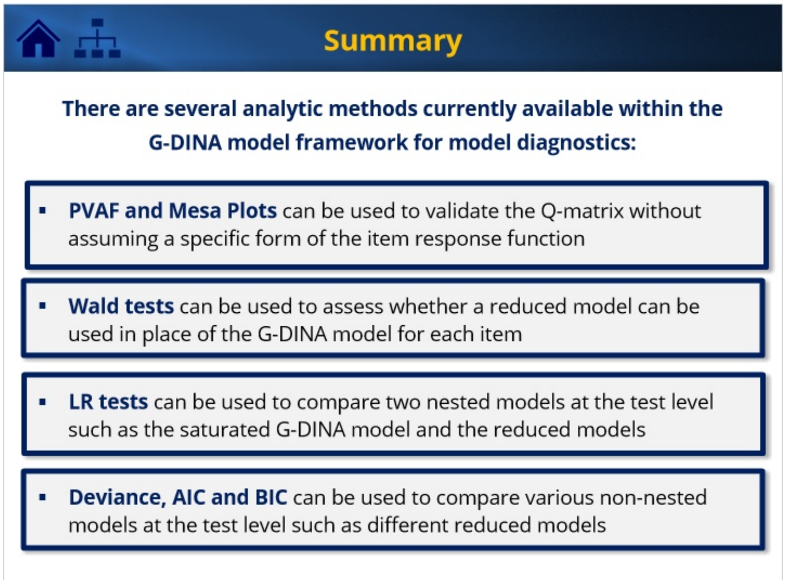
4.29 Other Options: Wald Test and Predicted Cutoffs



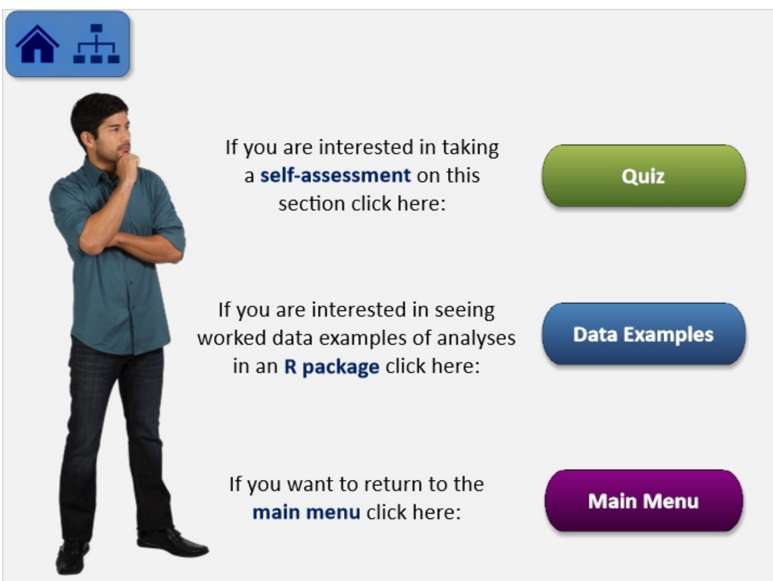
4.30 Bookend: Q-matrix Fit



4.31 Summary



4.32 Bookend: Section 3



4.33 Module Cover (END)