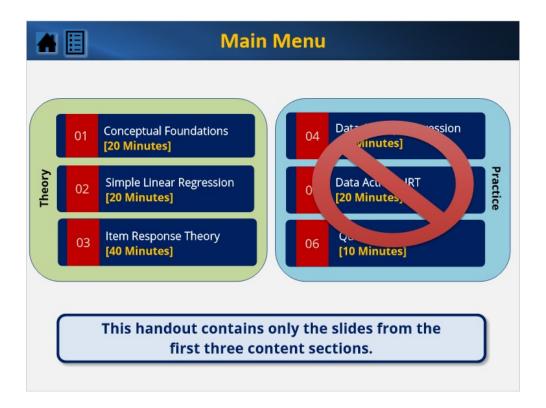
This document contains all core content slides from sections 1-3 with the exception of slides that show video screens. In the digital module all slides can be accessed individually.

Module Organization

The module starts with an introductory section that leads to the main menu from which learners can select individual content and activity sections:



DM06 HANDOUT Version

1. Module Overview

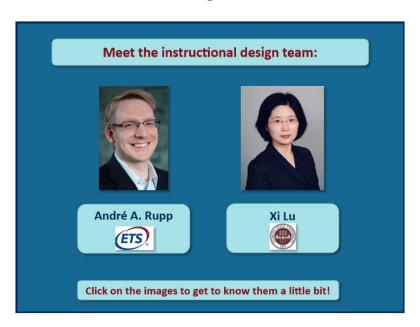
1.1 Module Cover



1.2 Content Team



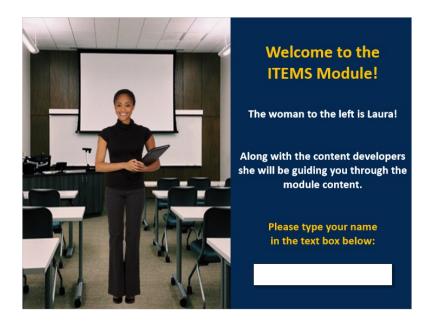
1.3 Design Team



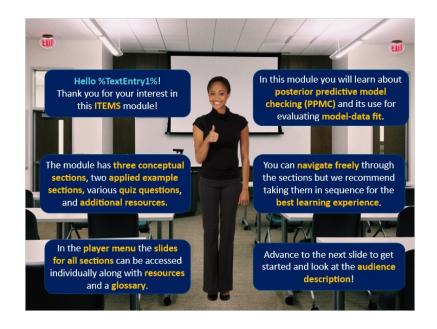
1.4 Support Team



1.5 Welcome



1.6 Overview



1.7 Target Audience

Target Audience

Anyone who would like a gentle statistical introduction to this topic:

- graduate students and faculty in Master's, Ph.D., or certificate programs
- psychometricians and other measurement professionals
- data scientists / analysts
- research assistants or research scientists
- · technical project directors
- assessment developers



However, we hope that you find the information in this module useful no matter what your official title or role in an organization is!

1.8 Expecations (I)







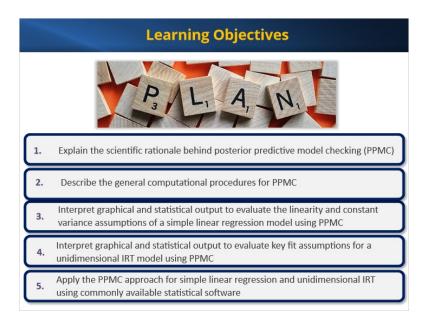


Let's discuss expectations....

1.9 Expectations (II)



1.10 Learning Objectives



1.11 Prerequisites

Prerequisites • Working knowledge of foundational measurement concepts: ✓ Construct definitions / latent variables ✓ Assessment formats ✓ Item / task types ✓ Scales and scale scores ✓ Basic aspects of assessment development • Working knowledge of foundational statistical concepts: ✓ Descriptive statistics for distributions ✓ Simple linear regression model ✓ Statistical inference with p-values

1.12 Resources



Resources 1 (Slide Layer)

Resources: Books

Bolstad, W. (2007). Introduction to Bayesian statistics.





McElreath, R. (2016). Statistical rethinking: A Bayesian Course with R and Stan.

Kaplan, D. (2014). Bayesian statistics for the social sciences.







Resources 2 (Slide Layer)

Resources: Books

Levy, R., & Mislevy, R. (2016). Bayesian psychometric modeling.



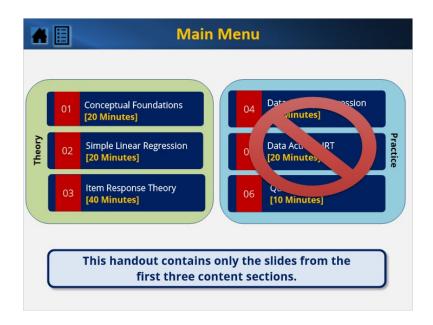


Jackman, S. (2009). Bayesian analysis for the social sciences.



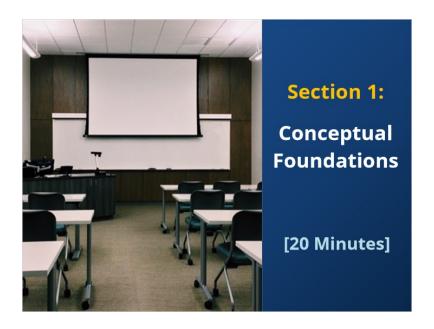


1.13 Main Menu



2. Conceptual Foundations

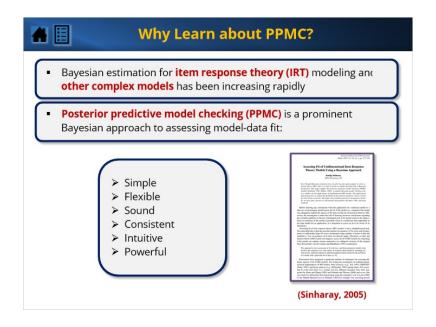
2.1 Cover: Foundations



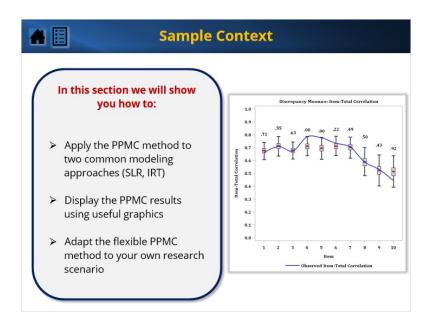
2.2 Objectives: Foundations



2.3 Motivation



2.4 Sample Context



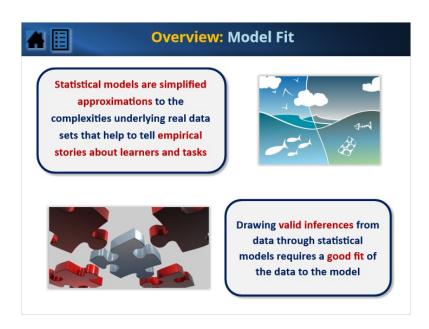
2.5 Topic Selection



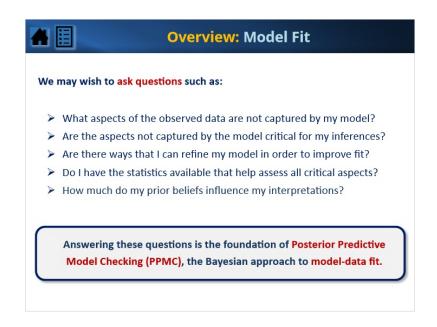
2.6 Bookmark: Theoretical Framework



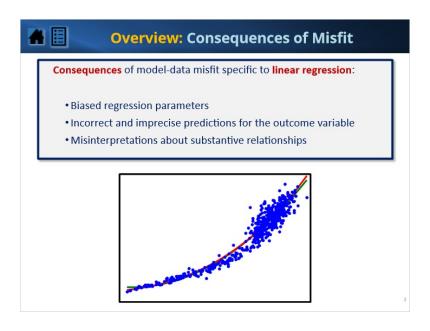
2.7 Overview: Model Fit



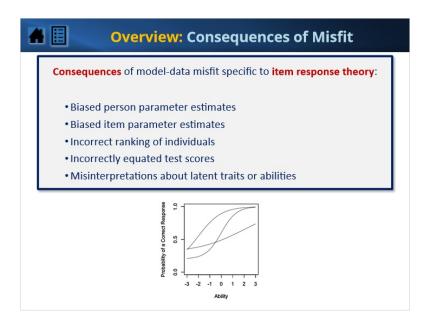
2.8 Overview: Model Fit



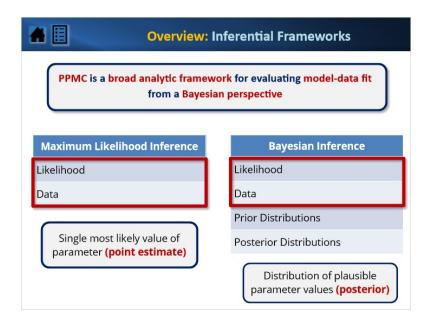
2.9 Consequences of Misfit (I)



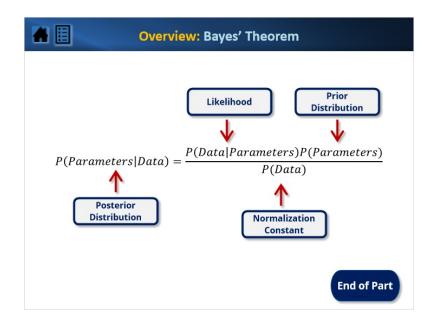
2.10 Consequences of Misfit (II)



2.11 Overview: Inferential Frameworks



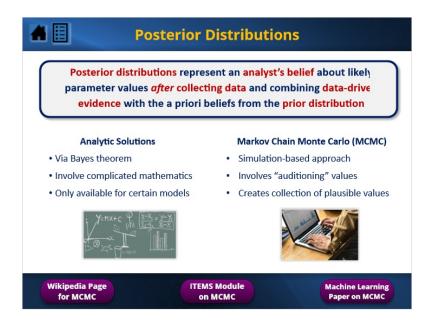
2.12 Overview: Bayes Theorem



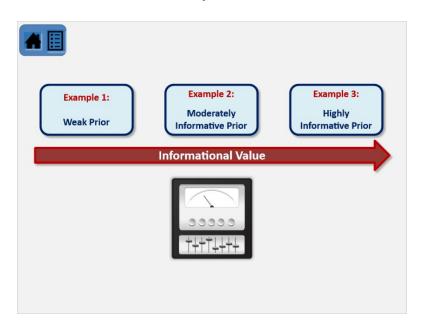
2.13 Bookmark: Posterior



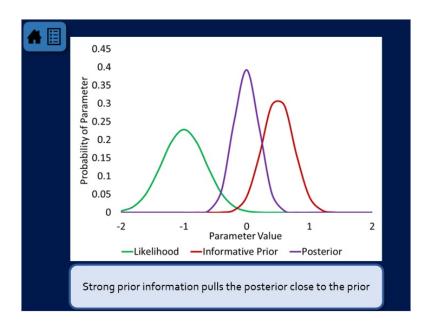
2.14 Posterior Distributions



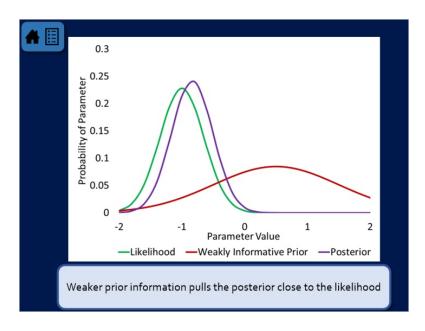
2.15 Example Selection



2.16 Posterior Example (I)



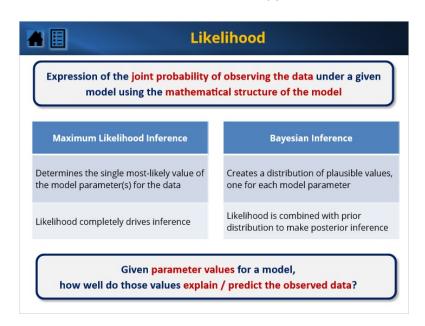
2.17 Posterior Example (II)



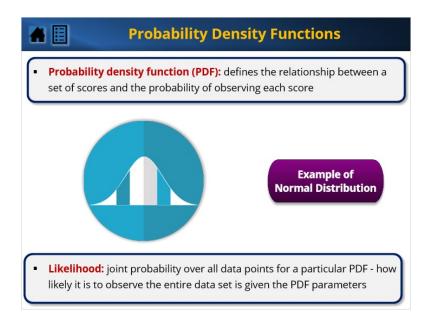
2.18 Bookmark: Likelihood



2.19 Likelihood (I)



2.20 Likelihood (II)



Example (Slide Layer)

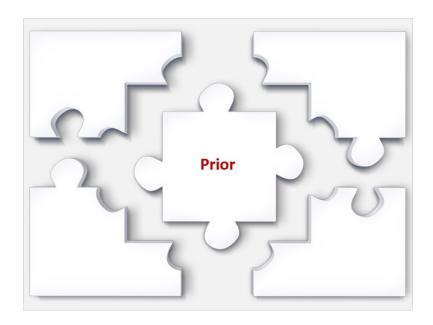


Substituting a value of x_i into the **normal distribution PDF** gives the **probability** (**likelihood**) **of observing this value** given a particular combination of **mean** (μ) and **standard deviation** (σ) parameters.

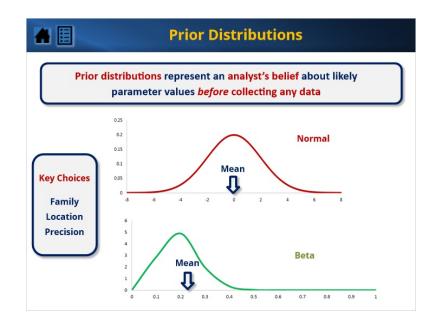
Different parameter values or different distributions would yield different probabilities for x,

Back to Main Slide

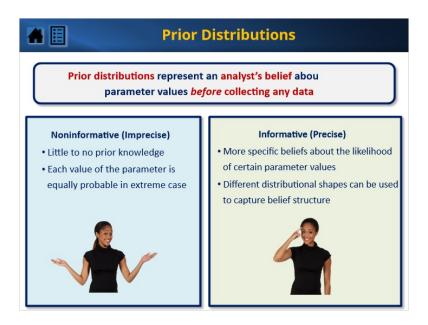
2.21 Bookmark: Prior



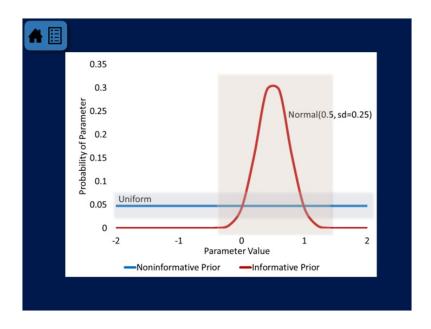
2.22 Prior Distributions (I)



2.23 Prior Distributions (II)



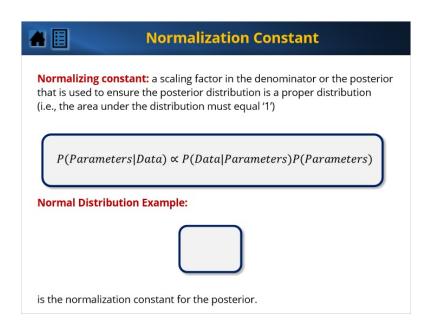
2.24 Prior Distributions (III)



2.25 Bookmark: Normalization



2.26 Normalization Constant



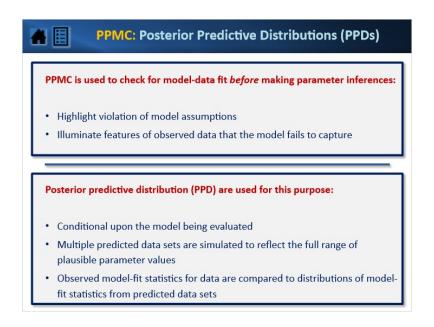
2.27 Bookend: Theoretical Framework



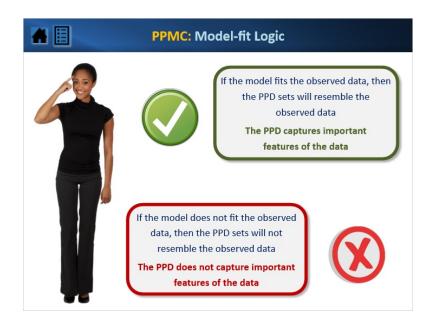
2.28 Bookmark: Computational Approach



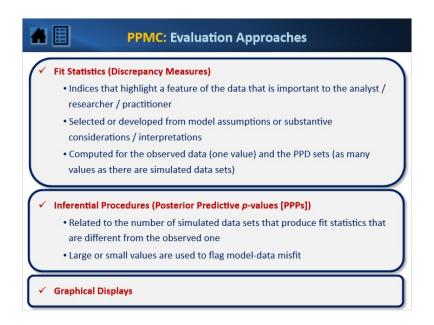
2.29 Posterior Predictive Distributions



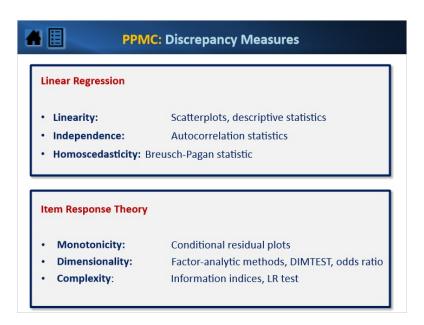
2.30 Model-fit Logic



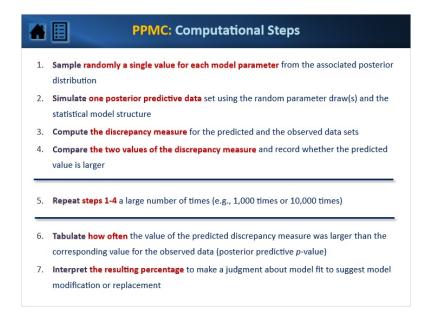
2.31 Evaluation Procedures



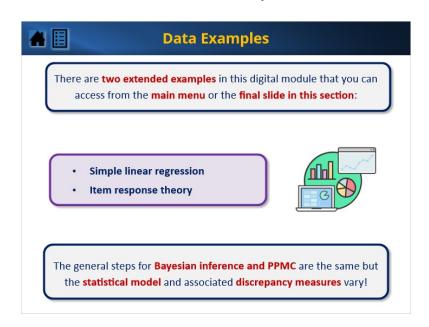
2.32 Discrepancy Measures



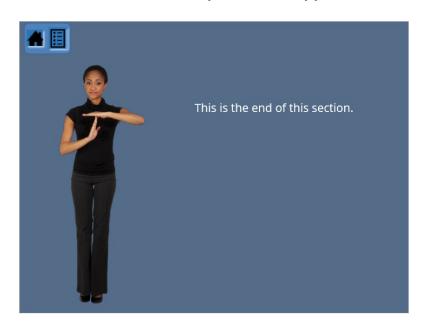
2.33 Computational Steps



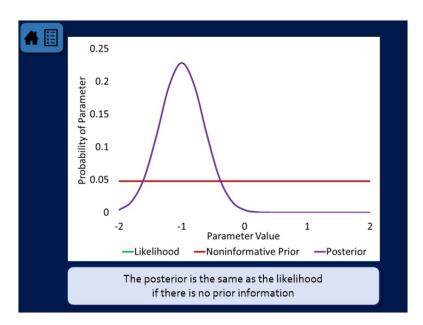
2.34 Data Examples



2.35 Bookend: Computational Approach

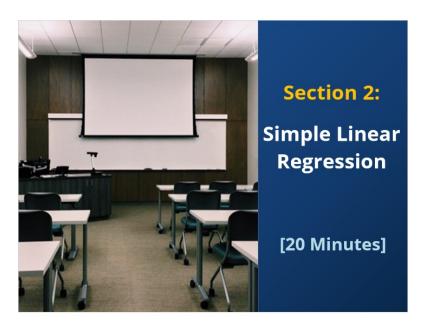


2.36 Posterior Example (III)

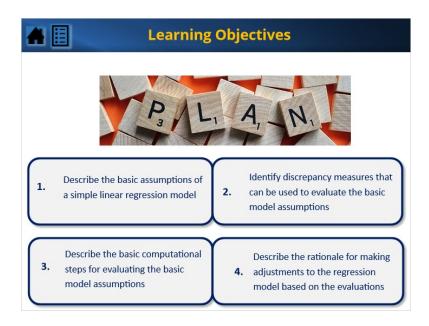


3. Simple Linear Regression

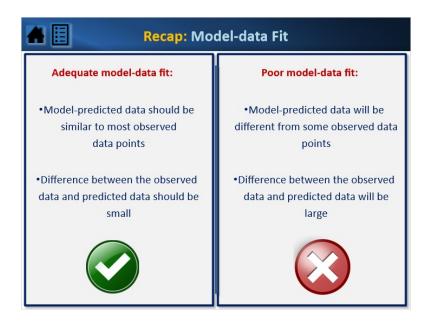
3.1 Cover: Regression



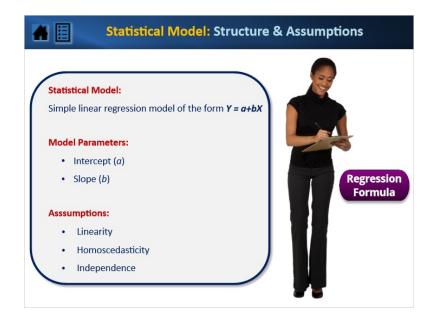
3.2 Learning Objectives: Regression



3.3 Model-data Fit (III)



3.4 Example: Overview



Regression Formula (Slide Layer)





Regression Formula

Structure

$$Y_i = a + bX_i + E_i$$

$$i = 1, \dots, n$$

n= number of observations

Assumptions

$$E_i \sim Normal(0, \sigma_E^2)$$

Error are mutually independent and identically normally distributed with a constant variance



3.5 Regression Example (II)





Statistical Model: Example Data

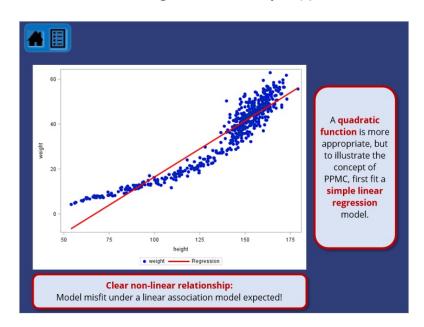
ullet Mean of the Normal distribution, μ_i , is specified as a linear function of the predictor variable and parameters of interest:

> $weight_i \sim Normal(\mu_i, \sigma^2)$ $\mu_i = a + b * height_i$

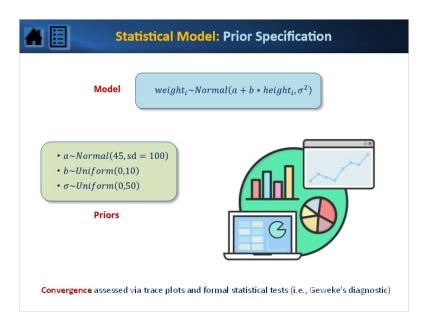
 $weight_i{\sim}Normal(a+b*height_i,\sigma^2)$

- $weight_i$ is the **dependent variable (outcome)** for the i^{th} individual $(i=1,2,\dots,n)$
- ullet height $_i$ is the independent variable (predictor) for the i^{th} individual
- a is the intercept
- b is the slope
- σ^2 is the variance (spread) of the data

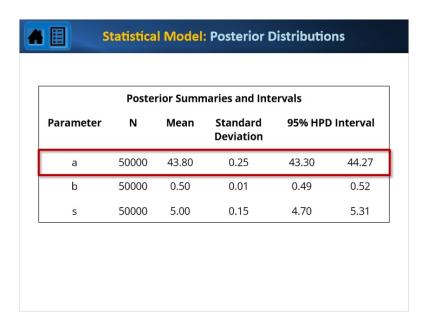
3.6 Regression Example (I)



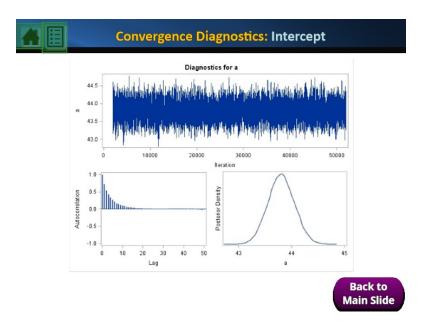
3.7 Regression Example (III)



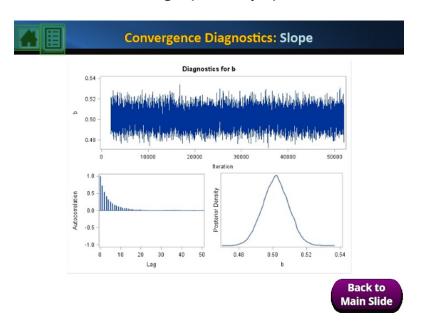
3.8 Regression Example (IV)



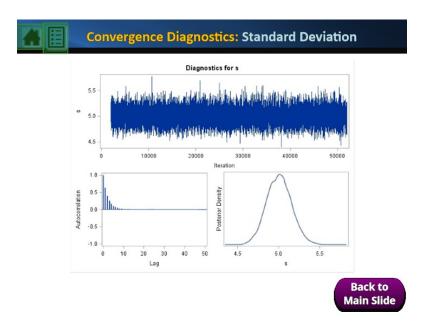
Diag a (Slide Layer)



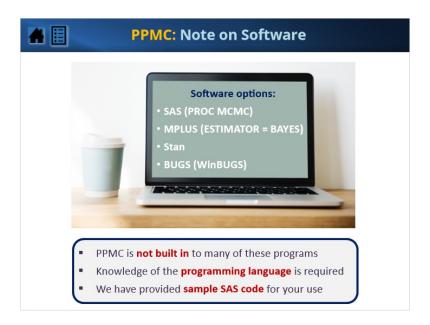
Diag b (Slide Layer)



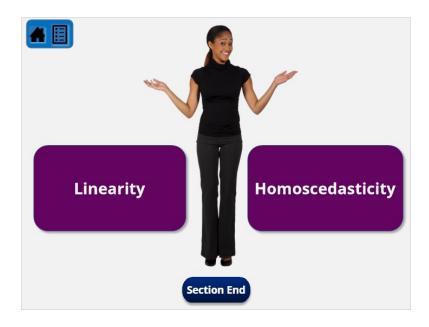
Diag s (Slide Layer)



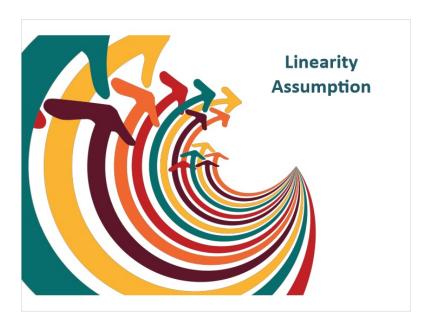
3.9 PPMC: Computational Steps (II)



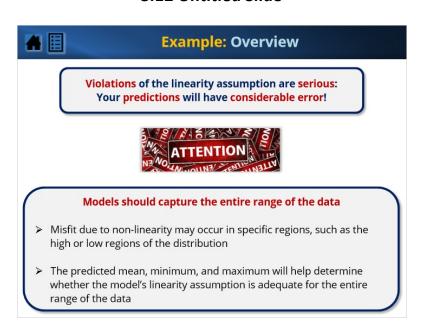
3.10 Topic Selection



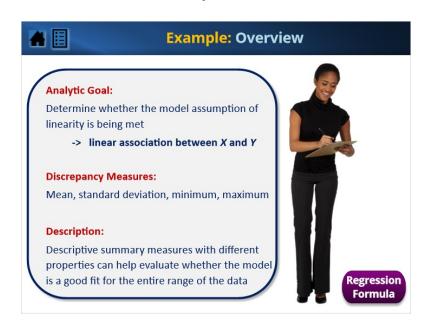
3.11 Bookmark: Linearity



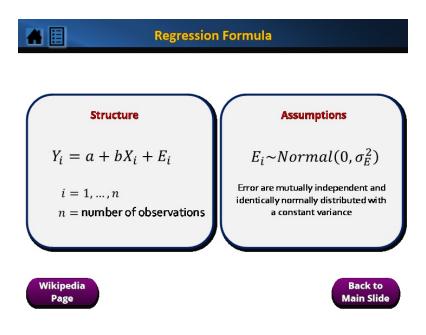
3.12 Untitled Slide



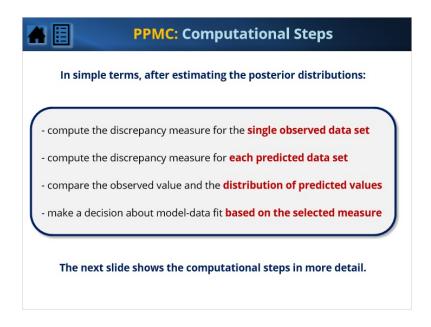
3.13 Example: Overview



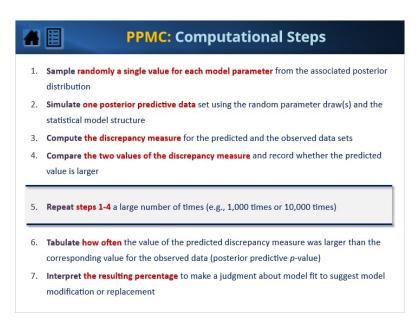
Regression Formula (Slide Layer)



3.14 PPMC: Computational Steps (I)



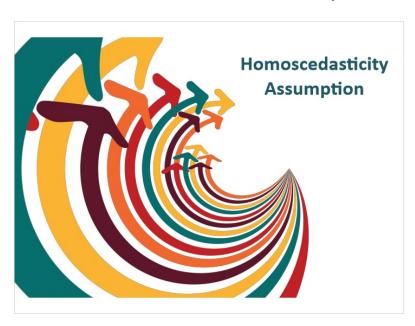
3.15 PPMC: Computational Steps (II)



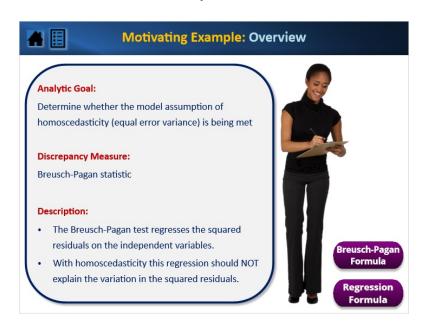
3.16 Bookend: Linearity



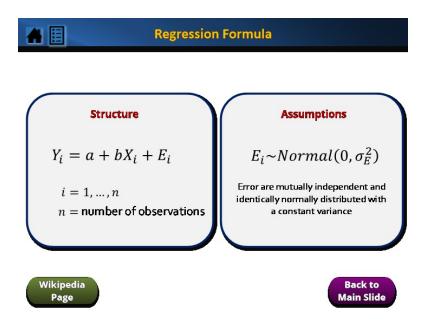
3.17 Bookmark: Homoscedasticity



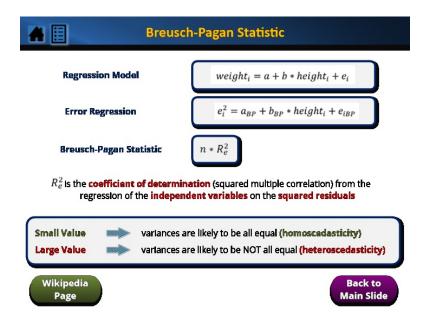
3.18 Example: Overview



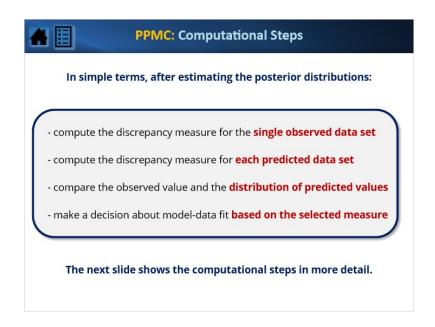
Regression Formula (Slide Layer)



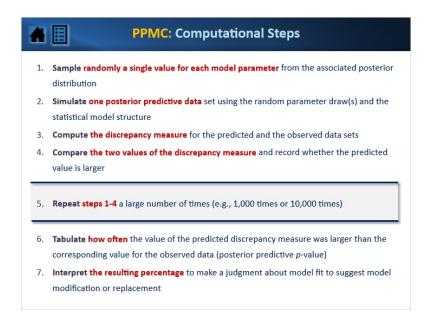
Breusch-Pagan Formula (Slide Layer)



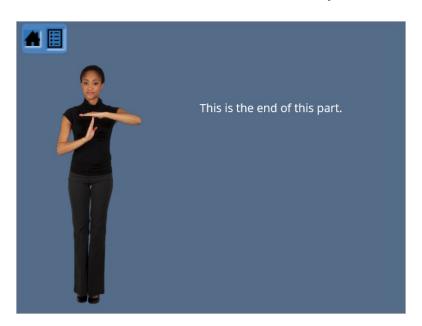
3.19 PPMC: Computational Steps (I)



3.20 PPMC: Computational Steps (II)



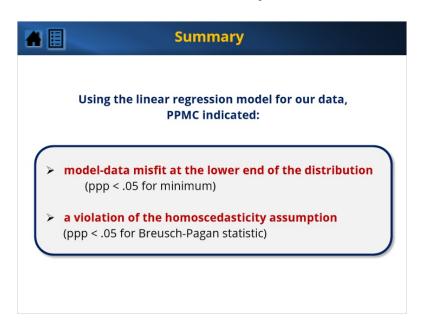
3.21 Bookend: Homoscedasticity



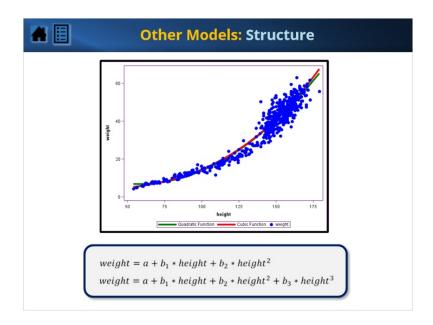
3.22 Bookmark: Summary



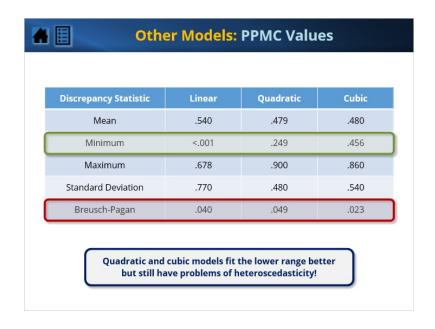
3.23 *Summary*



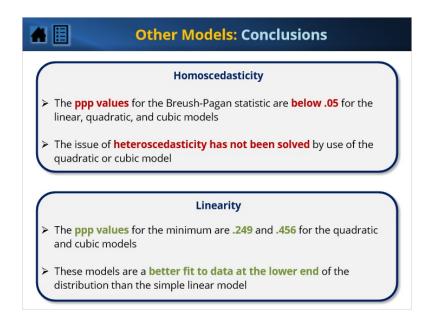
3.24 Other Models: Structure



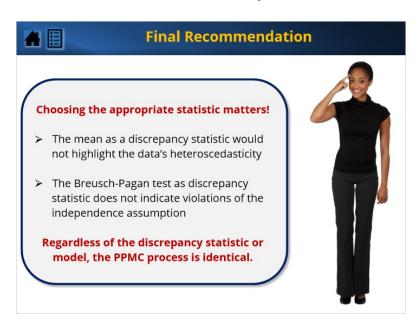
3.25 Other Models: PPMC Values



3.26 Other Models: Conclusions



3.27 Summary

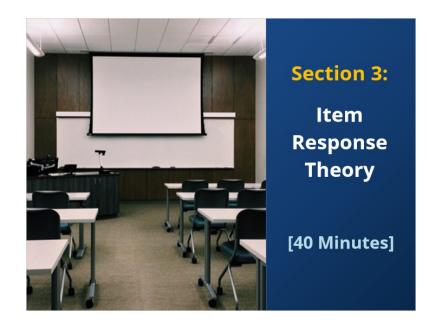


3.28 Bookend: Regression

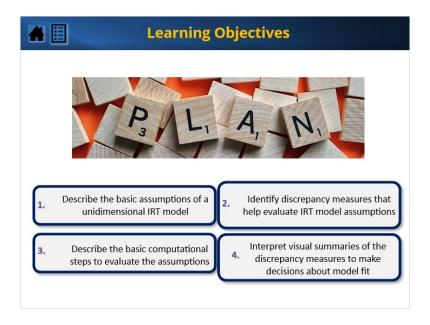


4. Item Response Theory

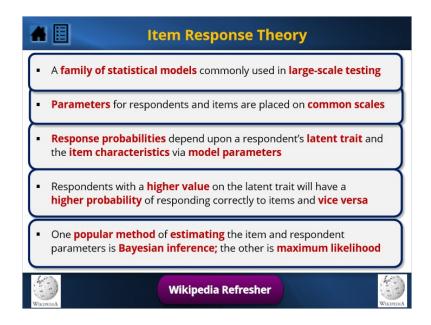
4.1 Cover: IRT



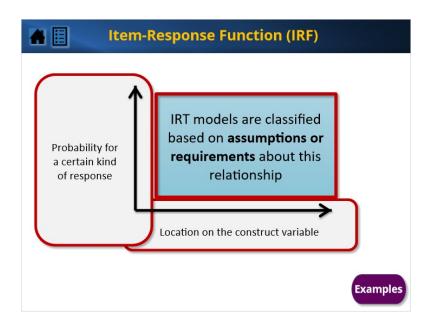
4.2 Objectives: IRT



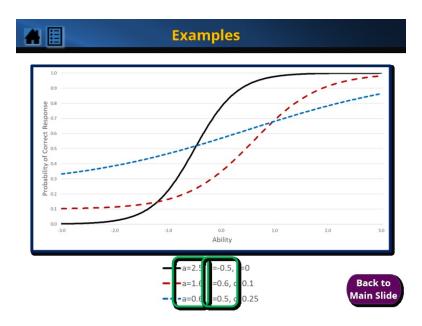
4.3 IRT Models (I)



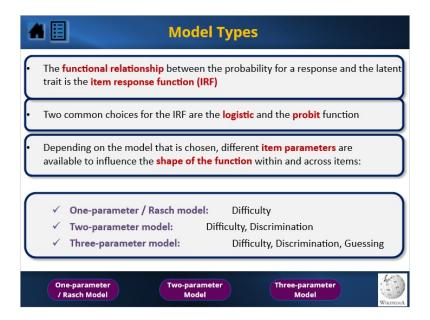
4.4 IRT Models (II)



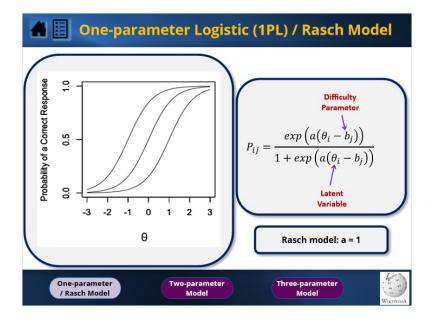
Examples (Slide Layer)



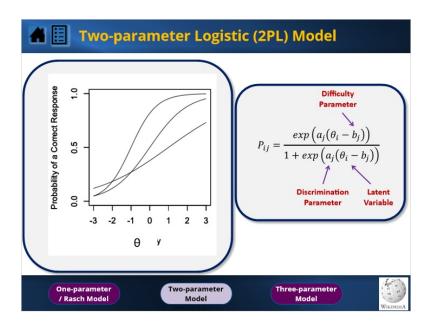
4.5 Parametric Item Response Theory



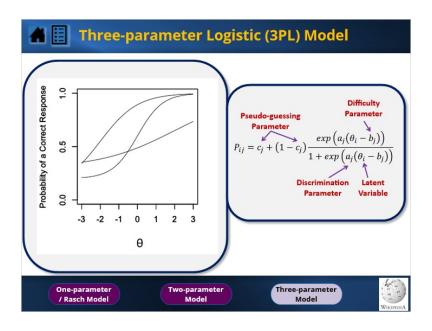
4.6 One-parameter Model



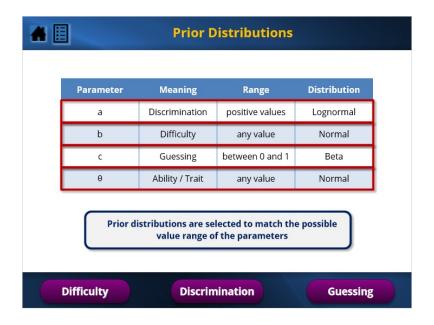
4.7 Two-parameter Model



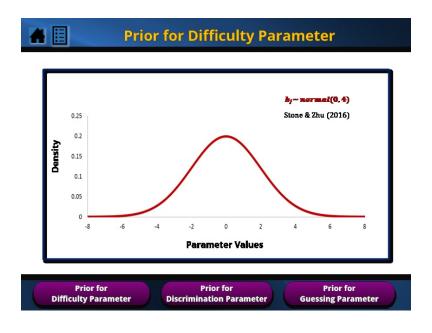
4.8 Three-parameter Model



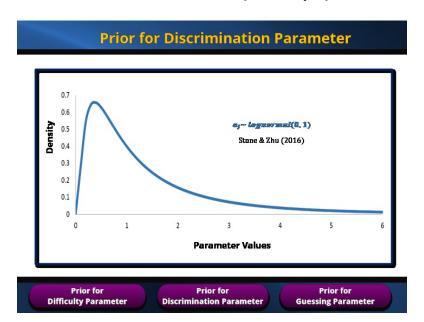
4.9 Prior Distributions (III)



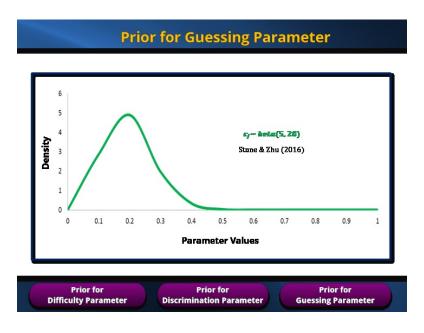
Prior Difficulty (Slide Layer)



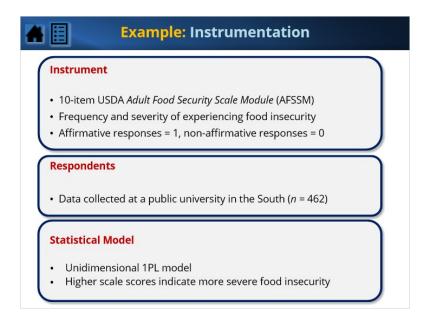
Prior Discrimination (Slide Layer)



Prior Guessing (Slide Layer)



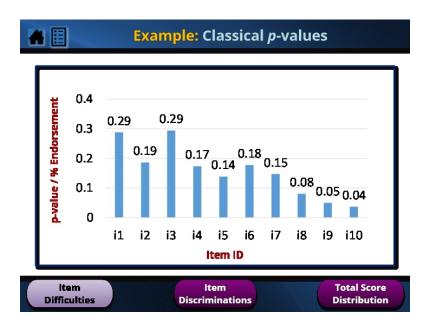
4.10 Example: Instrumentation



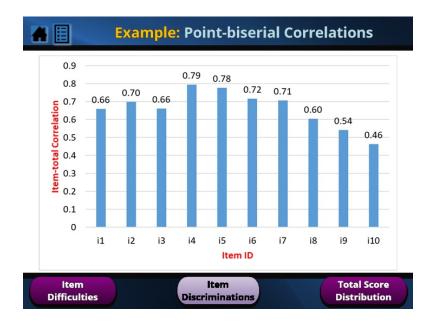
4.11 Example: Sample Items

♣ ■ •	cample: Item Types & Properties
Sample Item:	
In the last 12 months, did you ever eat less than you felt you should because there wasn't enough money for food?	
	[] Yes (1) [] No (0) [] Don't Know (0)
Full Survey:	https://www.ers.usda.gov/media/8282/short2012.pdf
Sample Statistics:	Access via buttons below
ltem Difficulties	Item Total Score Discriminations Distribution

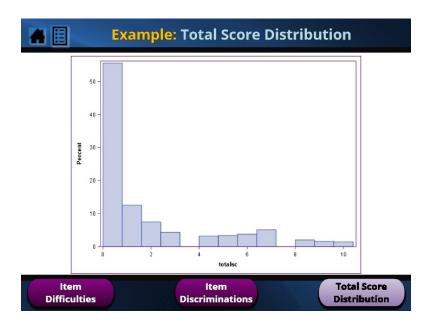
Difficulty (Slide Layer)



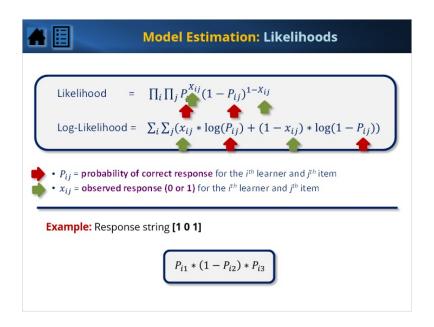
Discrimination (Slide Layer)



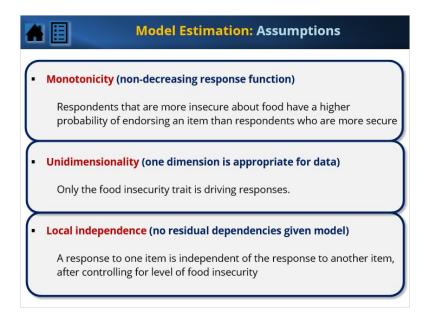
Total score distribution (Slide Layer)



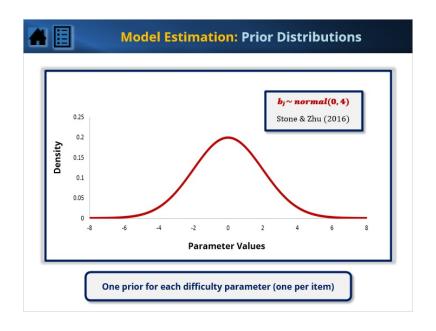
4.12 Model Estimation: Structure & Assumptions



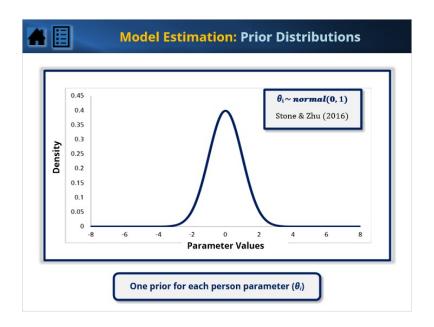
4.13 Model Estimation: Structure & Assumptions



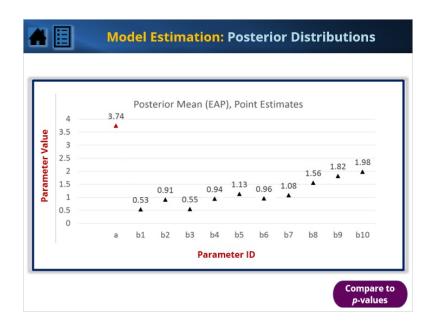
4.14 Model Estimation: Posterior Distributions



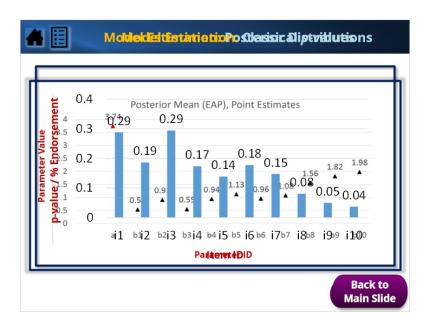
4.15 Model Estimation: Posterior Distributions



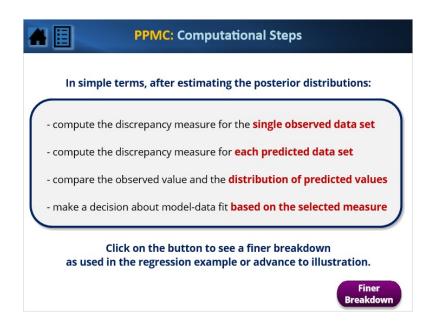
4.16 Model Estimation: Posterior Distributions



p-values (Slide Layer)



4.17 PPMC: Computational Steps (I)



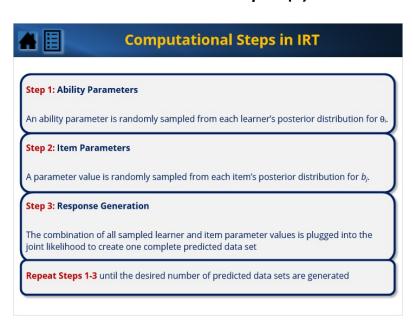
Finer Breakdown (Slide Layer)



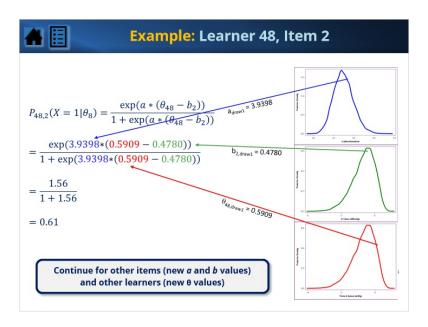
- Sample randomly a single value for each model parameter from the associated posterior distribution
- 2. **Simulate one posterior predictive data** set using the random parameter draw(s) and the statistical model structure
- 3. Compute the discrepancy measure for the predicted and the observed data sets
- Compare the two values of the discrepancy measure and record whether the predicted value is larger
- 5. Repeat steps 1-4 a large number of times (e.g., 1,000 times or 10,000 times)
- Tabulate how often the value of the predicted discrepancy measure was larger than the
 corresponding value for the observed data (posterior predictive p-value)
- 7. Interpret the resulting percentage to make a judgment about model fit to suggest model modification or replacement

 Return to Main Slide

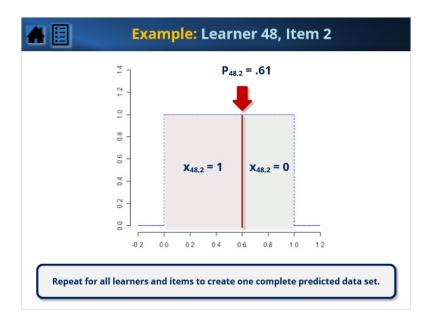
4.18 General Principles (II)



4.19 Computation (I)



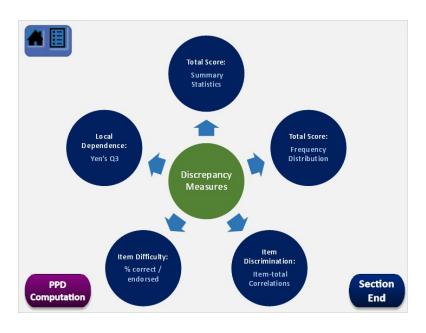
4.20 Computation (II)



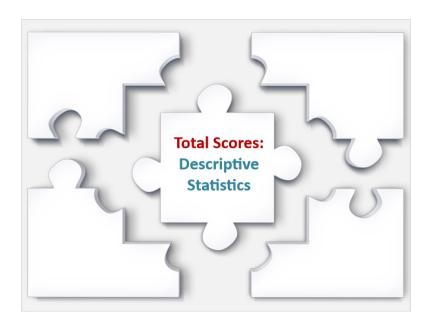
4.21 Untitled Slide



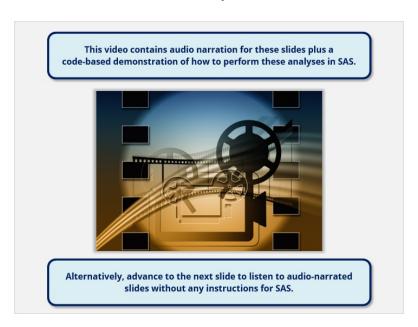
4.22 Discrepancy Measure Selection



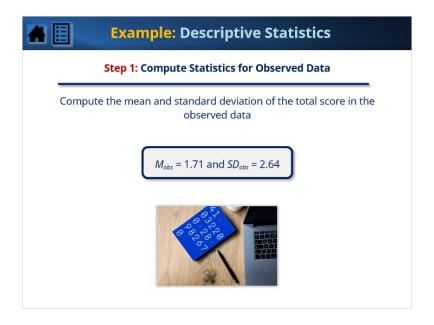
4.23 Bookmark: Descriptive Statistics



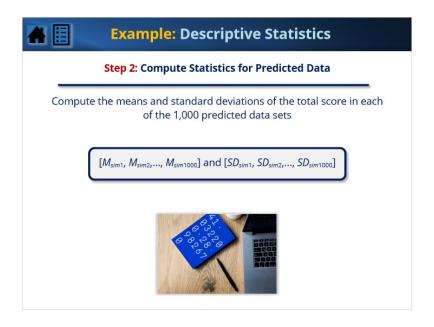
4.24 Video: Descriptive Statistics



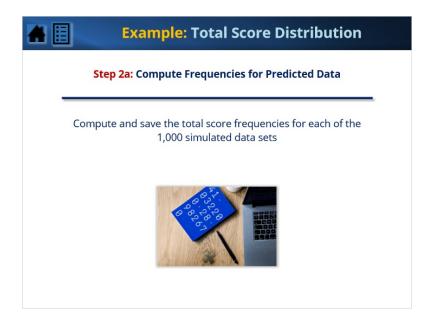
4.25 Descriptive Statistics (I)



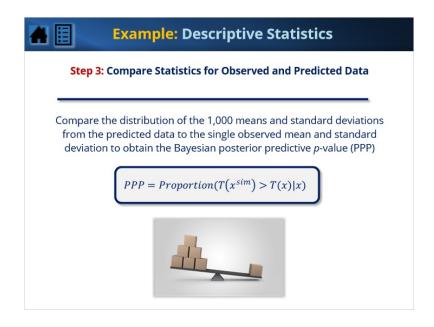
4.26 Descriptive Statistics (II)



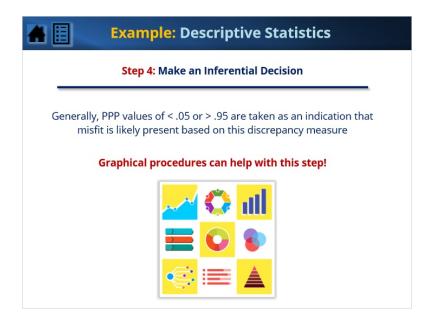
4.27 Total Score Distribution (II)



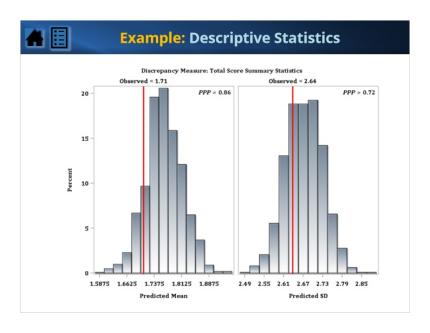
4.28 Descriptive Statistics (III)



4.29 Descriptive Statistics (IV)



4.30 Descriptive Statistics (V)



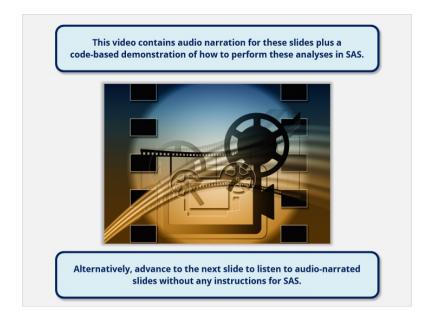
4.31 Bookend: Descriptive Statistics



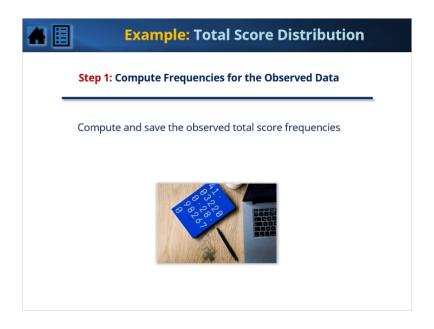
4.32 Bookmark: Total Score Distribution



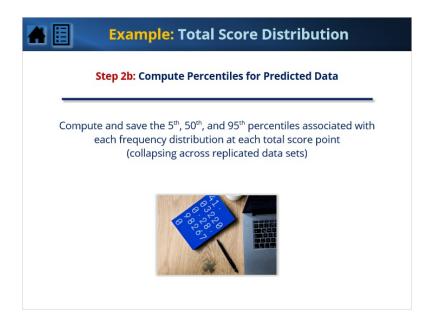
4.33 Video: Total Score Distribution



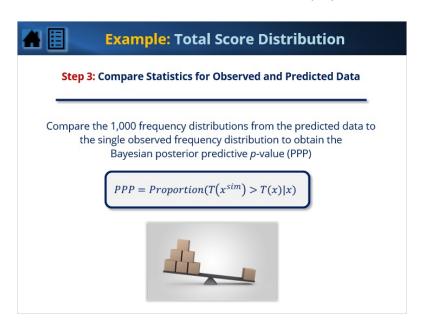
4.34 Total Score Distribution (I)



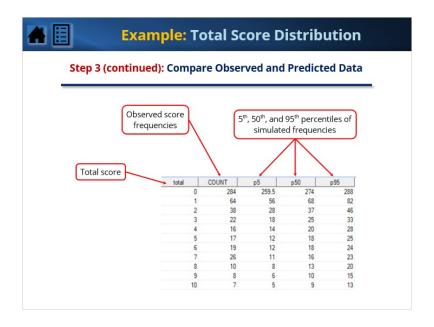
4.35 Total Score Distribution (III)



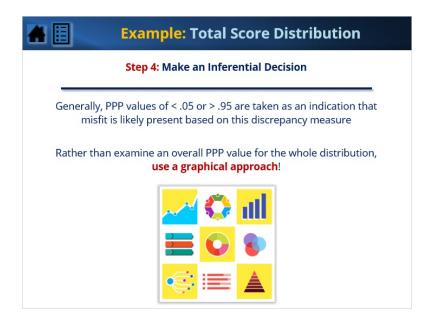
4.36 Total Score Distribution (IV)



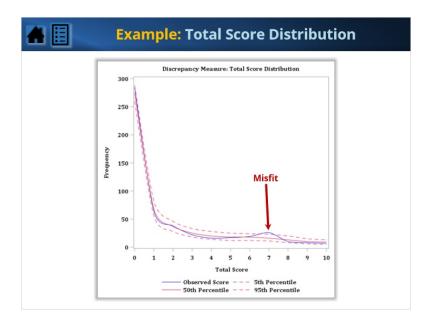
4.37 Total Score Distribution (V)



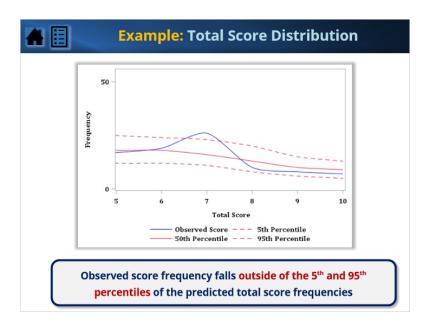
4.38 Total Score Distribution (VI)



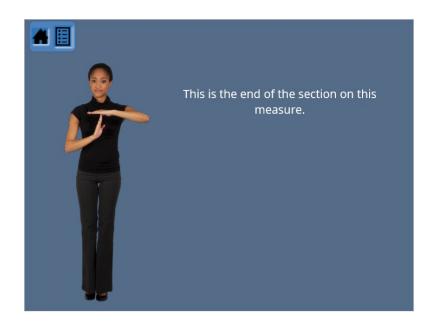
4.39 Total Score Distribution (VII)



4.40 Total Score Distribution (VIII)



4.41 Bookend: Total Score Distribution



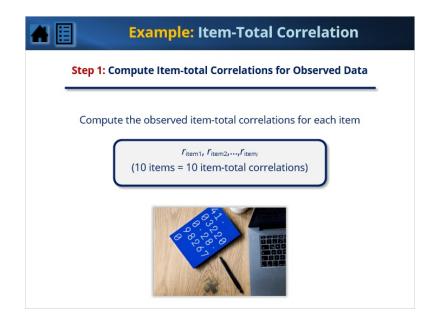
4.42 Bookmark: Item-total Correlations



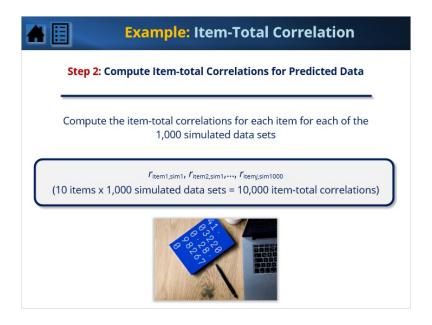
4.43 Video: Item-total Correlations



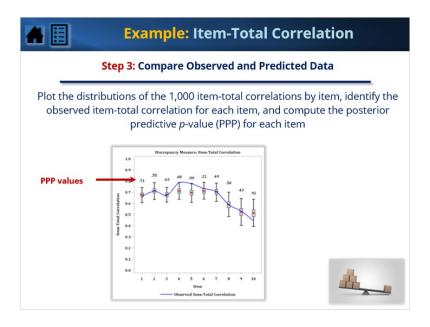
4.44 Item-total Correlations (I)



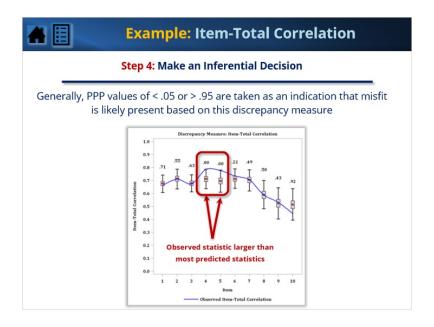
4.45 Item-total Correlations (II)



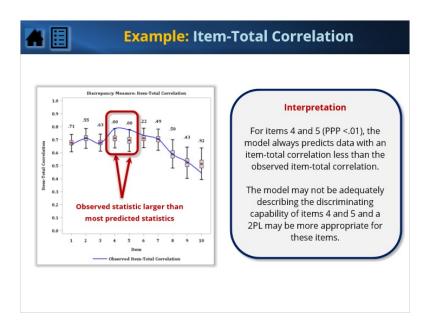
4.46 Item-total Correlations (III)



4.47 Item-total Correlations (IV)



4.48 Item-total Correlations (V)



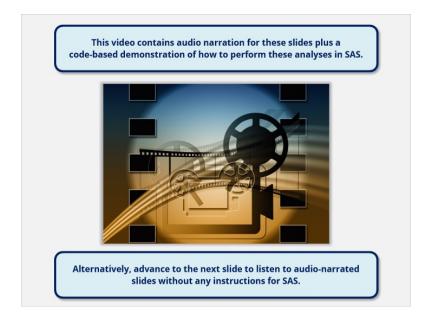
4.49 Bookend: Item-total Correlations



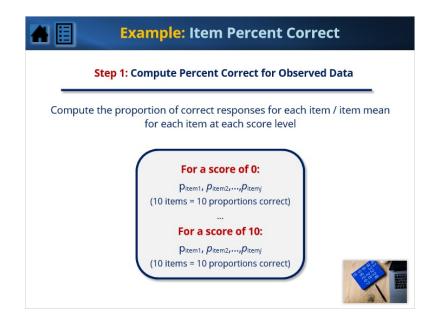
4.50 Bookmark: Percent Correct



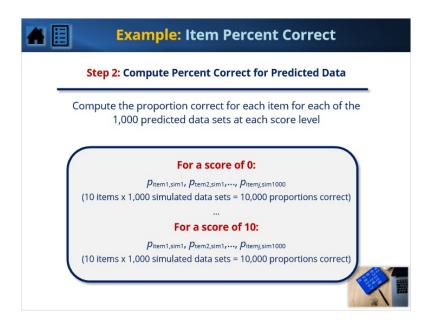
4.51 Video: Percent Correct



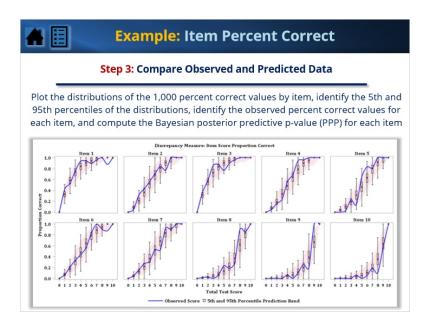
4.52 Percent Correct (I)



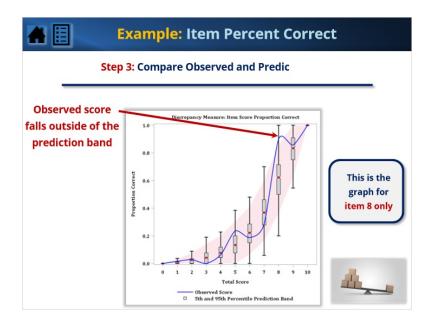
4.53 Percent Correct (II)



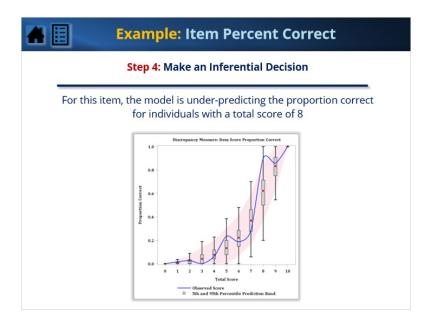
4.54 Percent Correct (III)



4.55 Percent Correct (IV)



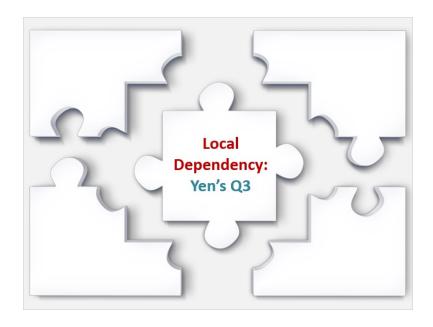
4.56 Percent Correct (V)



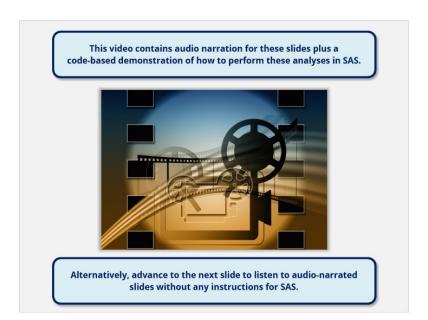
4.57 Bookend: Percent Correct



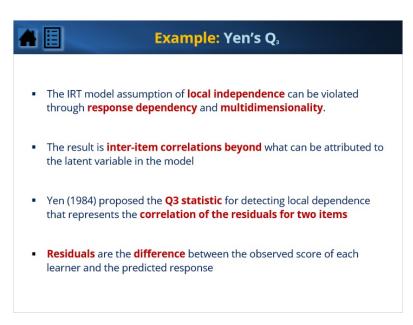
4.58 Bookmark: Yen's Q3



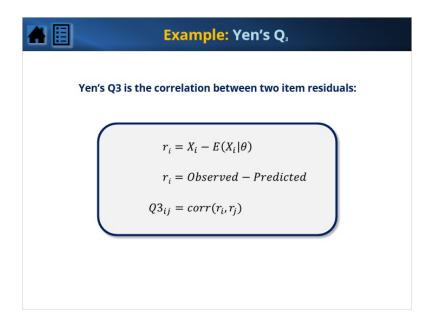
4.59 Video: Yen's Q3



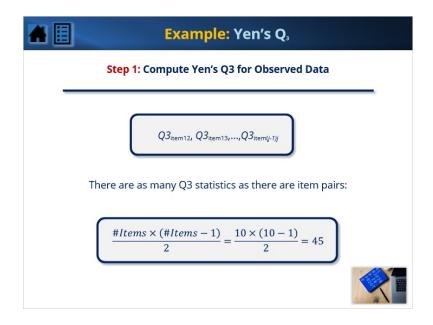
4.60 Yen's Q3 (I)



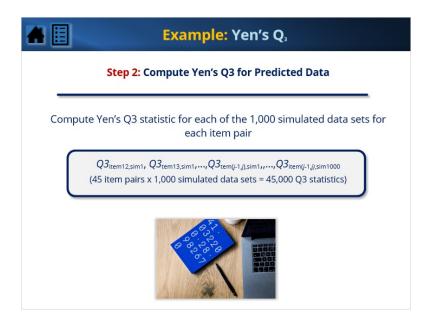
4.61 Yen's Q3 (II)



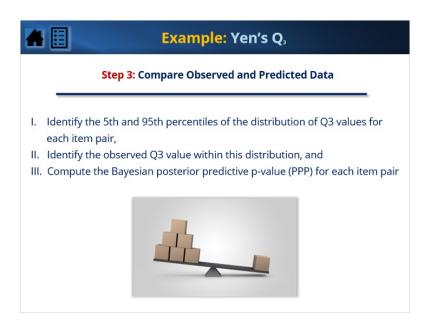
4.62 Yen's Q3 (III)



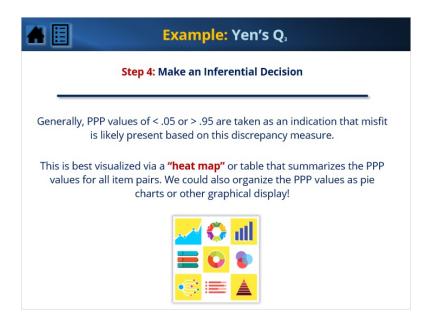
4.63 Yen's Q3 (IV)



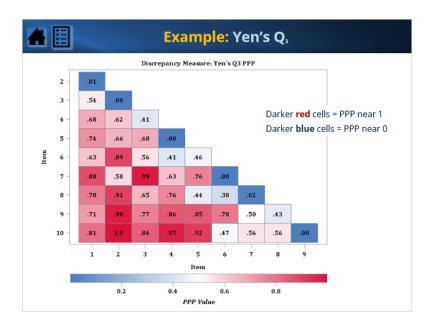
4.64 Yen's Q3 (V)



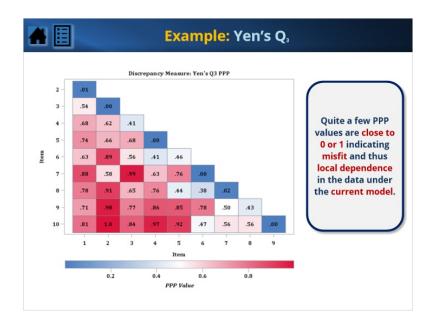
4.65 Yen's Q3 (VI)



4.66 Yen's Q3 (VII)



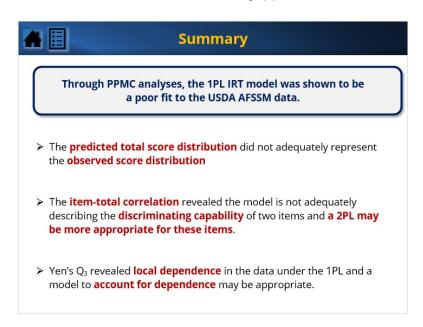
4.67 Yen's Q3 (VIII)



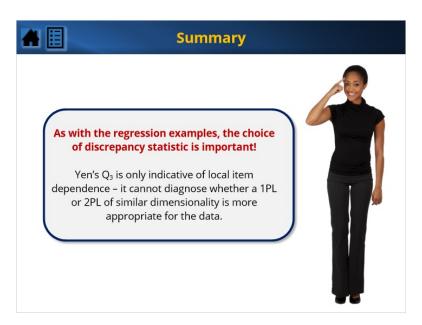
4.68 Bookend: Yen's Q3



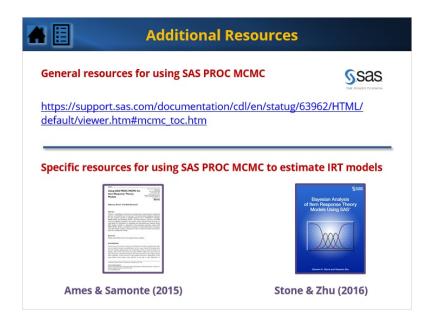
4.69 Summary (I)



4.70 Summary (II)



4.71 SAS Resources



4.72 Bookend: IRT

